Todennäköisyyslaskenta Harj. 4 vk 40 2011

4.1 X = miesten lkm otoksessa, Y = naisten lkm otoksessa

Valitaan 2 joukosta $\{N_1, N_2, N_3, M_1, M_2, M_3\}$

Osajoukkojen lkm $\binom{6}{2} = 15$

a) otosavaruus $\Omega = \{N_1N_2, N_1N_3, N_1M_1, N_1M_2, N_1M_3, N_2N_3, N_2M_1, N_2M_2, N_2M_3, N_3M_1, N_3M_2, N_3M_3, M_1M_2, M_1M_3, M_2M_3\}$

b) X:n todennäköisyysjakauma

$X: \Omega \to \{0,1,2\}$ ← alkeistapausten joukko, jossa $x=0$

$X^{-1}(0) = \{N_1N_2, N_1N_3, N_2N_3\}$

$X^{-1}(1) = \{N_1M_1, N_1M_2, N_1M_3, N_2M_1, N_2M_2, N_2M_3, N_3M_1, N_3M_2, N_3M_3\}$

$X^{-1}(2) = \{M_1M_2, M_1M_3, M_2M_3\}$

$P[X^{-1}(0)] = P(X=0) = \frac{3}{15} = \frac{1}{5}$

$P[X^{-1}(1)] = P(X=1) = \frac{9}{15} = \frac{3}{5}$

$P[X^{-1}(2)] = P(X=2) = \frac{3}{15} = \frac{1}{5}$

$F(x) = P(X \leq x)$

c) X:n todennäköisyysfunktio

$$f_X(x) = \begin{cases} \frac{1}{5}, & \text{kun } x=0 \text{ tai } x=2 \\ \frac{3}{5}, & \text{kun } x=1 \\ 0, & \text{muutoin} \end{cases}$$

s.46 (2.7.1) Valinta palauttamatta hypergeometrinen jakauma

4.2. $f_X(x) = \dfrac{\binom{3}{x}\binom{3}{2-x}}{15}$, $x = 0,1,2$ $\qquad f_Y(y) = \dfrac{\binom{3}{y}\binom{3}{2-y}}{15}$, $y = 0,1,2$

$$F_Y(y) = \begin{cases} 0, & y<0 \\ \frac{\binom{3}{2}}{15} = \frac{1}{5}, & 0 \leq y < 1 \\ \frac{\binom{3}{2}+\binom{3}{1}\binom{3}{1}}{15} = \frac{4}{5}, & 1 \leq y < 2 \\ 1, & y \geq 2 \end{cases}$$

siis

$$\boxed{F_Y(y) = \begin{cases} 0, & y<0 \\ \frac{1}{5}, & 0 \leq y < 1 \\ \frac{4}{5}, & 1 \leq y < 2 \\ 1, & y \geq 2 \end{cases}}$$

$F_X(x) = F_Y(y)$, kun y korvataan x:llä.

Y:n arvoalueen osajoukkojen todennäköisyydet

arvojoukko $S_Y = \{0, 1, 2\}$

$P(\{0\}) = F_Y(0)$, $\quad P(\{0,1\}) = F_Y(1)$, $\quad P(\{0,1,2\}) = F_Y(2)$

$P(\{1\}) = F_Y(1) - F_Y(0)$ $\quad P(\{2\}) = F_Y(2) - F_Y(1)$ $\quad P(\{1,2\}) = 1 - F_Y(0)$

jne..

(4.3.) 3 nimeä 3 kuraa

Y = oikeiden valintojen lkm

$S_Y = \{0, 1, 3\}$

$$f_Y(y) = \begin{cases} \frac{2}{6}, & \text{kun } y = 0 \\ \frac{3}{6}, & \text{kun } y = 1 \\ \frac{1}{6}, & \text{kun } y = 3 \\ 0, & \text{muutoin} \end{cases}$$

eli

$f_Y(0) = \frac{2}{6}$, $f_Y(1) = \frac{3}{6}$, $f_Y(3) = \frac{1}{6}$

Kura 1	2	3	Y
N_1	N_2	N_3	3
N_1	N_3	N_2	1
N_2	N_1	N_3	1
N_2	N_3	N_1	0
N_3	N_1	N_2	0
N_3	N_2	N_1	1

Valitaan 3 maljaa palauttaen, joihin helmet laitetaan. 3^3 mahdollista valintaa.

(4.4)

s. 48

Y = tyhjien maljojen lkm $S_Y = \{0, 1, 2\}$

$M_1 M_2 M_3$ voidaan valita 3! eri järjestyksessä

$$f_Y(y) = \begin{cases} \frac{3!}{27} = \frac{6}{27}, & \text{kun } y = 0 \\ \frac{18}{27}, & \text{kun } y = 1 \quad \left(1 - \frac{6}{27} - \frac{3}{27} = \frac{18}{27}\right) \\ \frac{3}{27}, & \text{kun } y = 2 \quad M_1M_1M_1,\ M_2M_2M_2,\ M_3M_3M_3 \\ 0, & \text{muulloin} \end{cases}$$

siis

$f_Y(0) = \frac{6}{27}$, $f_Y(1) = \frac{18}{27}$, $f_Y(2) = \frac{3}{27}$

$$F_Y(y) = \begin{cases} 0, & y < 0 \\ \frac{6}{27}, & 0 \le y < 1 \\ \frac{24}{27}, & 1 \le y < 2 \\ 1, & y \ge 2 \end{cases}$$

$P(\{0\}) = \frac{6}{27}$ $P(\{1\}) = \frac{18}{27}$, $P(\{2\}) = \frac{3}{27}$

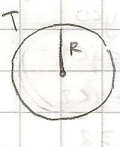
$P(\{0,1\}) = \frac{24}{27}$ $P(\{0,2\}) = \frac{9}{27}$

$P(\{1,2\}) = \frac{21}{27}$ $P(\{0,1,2\}) = 1$

Helmet 1	2	3	Y
M_1	M_1	M_1	2
M_1	M_1	M_2	1
M_1	M_1	M_3	1
M_1	M_2	M_1	1
M_1	M_3	M_1	1
M_2	M_1	M_1	1
M_3	M_1	M_1	1
M_2	M_2	M_2	2
M_2	M_2	M_1	1
M_2	M_2	M_3	1
M_2	M_1	M_2	1
M_2	M_3	M_2	1
M_1	M_2	M_2	1
M_3	M_2	M_2	1

Helmet 1	2	3	Y
M_3	M_3	M_3	2
M_3	M_3	M_1	1
M_3	M_3	M_2	1
M_3	M_1	M_3	1
M_3	M_2	M_3	1
M_1	M_3	M_3	1
M_2	M_3	M_3	1
M_1	M_2	M_3	0
M_1	M_3	M_2	0
M_2	M_1	M_3	0
M_2	M_3	M_1	0
M_3	M_1	M_2	0
M_3	M_2	M_1	0

(4.5.)



X = osumispisteen (x, y) etäisyys origosta

säde R

pisteen etäisyys origosta $= \sqrt{x^2 + y^2}$

$0 \le t \le R$

$x^2 + y^2 \le R^2$

ts. $X(s) = \sqrt{x^2 + y^2}$, siis

$0 \le X \le R$. Nyt siis

$$F_X(t) = P(X \le t) = \frac{\pi t^2}{\pi R^2} = \left(\frac{t}{R}\right)^2$$

siis

kertymäfunktio

$$F_X(t) = \begin{cases} 0, & t < 0 \\ \left(\frac{t}{R}\right)^2, & 0 \le t \le R \\ 1, & t > R \end{cases}$$

ympyrän pinta-ala πr^2

4.6. $\Omega = \{PP, PT, TP, TT\}$

$A \cap B = \{PP\}$

A = "Molemmat poikia" = $\{PP\}$

B = "Toinen lapsista poika" = $\{PP, TP, PT\}$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

tiedetään että ainakin toinen on poika

$f(x) = P(X=x)$

$F(x) = P(X \leq x)$

4.7. $f_X(0) = F(0) = 0$ $\quad P(X=0)$

$f_X(1) = F(1) - F(0) = \frac{1}{9}$

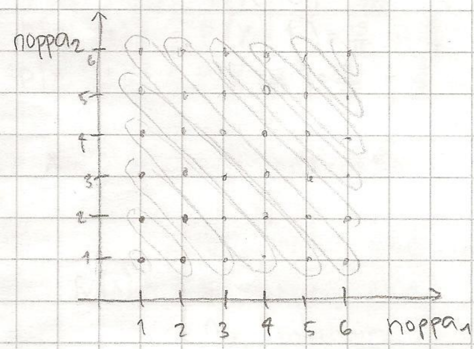
$f_X(2) = F(2) - F(1) = \frac{1}{6} - \frac{1}{9} = \frac{3}{18} - \frac{2}{18} = \frac{1}{18}$

$f_X(3) = F(3) - F(2) = \frac{1}{3} - \frac{1}{6} = \frac{2}{6} - \frac{1}{6} = \frac{1}{6}$

$f_X(4) = F(4) - F(3) = \frac{1}{2} - \frac{1}{3} = \frac{3}{6} - \frac{2}{6} = \frac{1}{6}$

$f_X(5) = F(5) - F(4) = 1 - \frac{1}{2} = \frac{1}{2}$

4.8.



X = nopan 1 ja nopan 2 silmälukujen summa

x	2	3	4	5	6	7	8	9	10	11	12
fr	1	2	3	4	5	6	5	4	3	2	1
$f_X(x)$	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

$$f_X(x) = \begin{cases} 1/36, & \text{kun } x = 2 \text{ tai } x = 12 \\ 2/36, & \text{kun } x = 3 \text{ tai } x = 11 \\ 3/36, & \text{kun } x = 4 \text{ tai } x = 10 \\ 4/36, & \text{kun } x = 5 \text{ tai } x = 9 \\ 5/36, & \text{kun } x = 6 \text{ tai } x = 8 \\ 6/36, & \text{kun } x = 7 \end{cases}$$