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Survey on some nonparametric and robust multivariate methods

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Rather than attempt an encyclopedic survey of nonparametric and robust multivariate methods, we instead focus on two leading and pervasive themes, *descriptive statistics* and *outlier identification*. These will be treated in Parts B and C, after beginning in Part A with *perspectives*. In Part D we conclude with discussion of *open issues and directions*.

A variety of questions will be raised. Is nonparametric inference the goal of nonparametric methods? Are nonparametric methods more important in the multivariate setting than in the univariate case? Should multivariate analysis be carried out componentwise, or with full dimensionality, or pairwise? Do multivariate depth, outlyingness, quantile, and rank functions represent different methodological approaches? Can we have a coherent series of nonparametric multivariate descriptive measures for location, spread, skewness, kurtosis, etc., that are robust and accommodate heavy tailed multivariate data? Can nonparametric outlier identifiers be defined that do not require ellipsoidal contours? What makes an outlier identifier itself robust against outliers? Does outlyingness of a data point with respect to location estimation differ from its outlyingness with respect to dispersion estimation? How do univariate L-functionals extend to the multivariate setting? Does the transformation-retransformation approach pose any issues? How might we conceptualize multivariate descriptive measures and outlier identification methods with respect to arbitrary data spaces, for applications such as functional data analysis, shape fitting, and text analysis? Do asymptotic and bootstrap methods present any special challenges?

Outline

A. Perspectives

1. The Goal of Nonparametric Methods is . . . ?
2. Multivariate Analysis Needs . . . ?

3. Multivariate Analysis is Best Done . . . ?
4. How Best to "Survey" Nonparametric and Robust Multivariate Methods?
5. How are Multivariate Depth, Outlyingness, Rank, and Quantile Functions Interrelated?

B. Descriptive Measures in $\mathbb{R}^d$

1. Nonparametric Descriptive Features
2. Moments (Classical Approach)
3. Expectations of Volumes (Oja, 1983)
4. Miscellaneous Unsystematic Efforts
5. Matrices of Univariate L-Functionals
6. Multivariate L-Functionals
7. Multiple Location and Scatter Measures (Oja, 2007)
8. The Ultimately Perfected Method (UPM) (Oja, 2031)

C. Outlier Identification

1. Outlyingness Functions in $\mathbb{R}^d$
2. Nonparametric Outlier Identification in $\mathbb{R}^d$
3. Depth and Outlyingness in Parameter Space

D. Nine Open Issues and Directions

1. The First
2. The Second

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9. The Ninth