Tilastollinen päättely I   Harj. 9   vk 12   2012

(9.1.) Ks. 12.1.1 Napasuureet

Välin $[Y, aY]$ peitetodennäköisyys on

$$P(\theta \in [Y, aY]) = P(Y \le \theta \le aY) = P\left(\frac{1}{a} \le \frac{Y}{\theta} \le 1\right) = 1 - \left(\frac{1}{a}\right)^n$$

$n=5$ välin luottamustaso on siis

$$1 - \left(\frac{1}{a}\right)^5 = 0.9$$
$$\left(\frac{1}{a}\right)^5 = 0.1$$
$$\frac{1}{a} = \sqrt[5]{0.1} \Rightarrow a = \frac{1}{\sqrt[5]{0.1}} \approx 1.58$$

$Y = \max\{2.3, 1.9, 4.5, 3.8, 2.5\} = 4.5$

Joten 90%:n luottamusväli

$$[Y, aY] = [4.5, a \cdot 4.5] \approx [4.5, 7.13]$$

(9.2.) Ks. Esim. 12.5

$$P(a \le V \le b) = 1 - \alpha \qquad V = \frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$$

Nyt $1-\alpha = 0.8$ eli

$a = \chi^2_{0.1;\, n-1}$ ja $b = \chi^2_{0.9;\, n-1}$ $n=15$

Rillä $= qchisq(0.1, 14) = 7.79$ Rillä $= qchisq(0.9, 14) = 21.06$

$$S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 = \frac{1}{n-1}\left(\sum x_i^2 - \underbrace{n\bar{x}^2}_{n \cdot \frac{(\sum x_i)^2}{n^2}}\right) = \frac{1}{n-1}\left(\sum x_i^2 - \frac{(\sum x_i)^2}{n}\right)$$

$$= \frac{1}{15-1}\left(338 - \frac{63^2}{15}\right) \approx 5.24$$

Jos 80% lv. σ^2:lle on $\left[\frac{(n-1)S^2}{b}, \frac{(n-1)S^2}{a}\right]$ eli $\left[\frac{14 \cdot 5{,}24}{21{,}06}, \frac{14 \cdot 5{,}24}{7{,}79}\right]$

$$= [3.48, 9.42]$$

(9.3.) $\frac{2}{\theta}\sum x_i \sim \chi^2(2n)$

$$P\left(a \le \frac{2\sum x_i}{\theta} \le b\right) = P\left(\frac{1}{b} \le \frac{\theta}{2\sum x_i} \le \frac{1}{a}\right)$$

$$= P\left(\frac{2\sum x_i}{b} \le \theta \le \frac{2\sum x_i}{a}\right) = P\left(\frac{2n\bar{x}}{b} \le \theta \le \frac{2n\bar{x}}{a}\right)$$

$n = 20$, $\bar{x} = 3.2$ a ja b $\chi^2(2 \cdot 20)$ -jakaumasta

$a = \chi^2_{0.025;40} = qchisq(0.025, 40) = 24{,}433$

$b = \chi^2_{0.975;40} = qchisq(0.975, 40) = 59.342$

siis 95%:n lv θ:lle:

$$\left[\frac{2 \cdot 20 \cdot 3{,}2}{59{,}342}, \frac{2 \cdot 20 \cdot 3.2}{24{,}433}\right] = [2{,}16 \,;\, 5{,}24]$$

(9.4.) $100(1-\alpha)\%$:n lv. painon muutokselle (ks. alaluun 12.2 loppu)

$\bar{d} \pm t_{\alpha/2;n-1}\frac{S_d}{\sqrt{n}}$, missä $\bar{d}$ = havaintojen eli muutosten keskiarvo

S_d = muutosten hajonta

Laskettava painon muutos $n = 12$

Loppupaino	73.0	73.8	75.6	80.2	76.0	80.3	74.4	79.2	73.5	76.5	75.3	74.0
Alkupaino	71.0	74.3	74.2	82.4	75.7	81.1	70.7	79.3	72.9	76.3	74.4	74.1
Erotus (d)	2.0	-0.5	1.4	-2.2	0.3	-0.8	3.7	-0.1	0.6	0.2	0.9	-0.1

$$\bar{d} = \frac{2.0+(-0.5)+\ldots+(-0.1)}{12} = \frac{5.4}{12} = 0{,}45$$

$$S_d^2 = \frac{1}{12-1}\sum_{i=1}^{12}(d_i-\bar{d})^2 = \frac{1}{11}\left(\sum_{i=1}^{12}d_i^2 - 12\bar{d}^2\right)$$

$$= \frac{1}{11}\left(26.7 - 12\cdot 0.45^2\right) = \frac{1}{11}(26.7-2.43) = \frac{24.27}{11}$$

$$S_d = \sqrt{\frac{24.27}{11}} \approx 1.485$$

95% :n luottamusväli: $\alpha = 0.05$

$$0{,}45 \pm \underbrace{t_{0.025;11}}_{\text{t-jakaumantaulukostai tai qt(0.025,11) = 2,201}}\frac{1.485}{\sqrt{12}} = (-0.49,\ 1.39)$$

(9.5.) (ks. Luku 12.5)

havaintojen pienin arvo $X_{(1)} = 4.9$

havaintojen suurin arvo $X_{(12)} = 7.9$

Muotoa $(X_{(1)}, X_{(12)})$ oleva luottamusväli on siis $(4.9,\ 7.9)$

välin luottamustaso on

$$P(X_{(1)} < m < X_{(12)}) = 1-\left(\tfrac{1}{2}\right)^{12-1} = 1-\left(\tfrac{1}{2}\right)^{11} \approx \underline{\underline{0{,}9995}} \quad (s.332)$$

(9.6.) $y_i \sim N(\beta x_i, 1)\quad i=1,\ldots,n$

$$l(\beta) = -\frac{n}{2}\log(2\pi) - \frac{1}{2}\sum_{i=1}^{\hat{n}}(y_i-\beta x_i)^2 \qquad (12.6.2)$$

ratkaistaan SUE

$$l'(\beta) = -\frac{1}{2}\cdot 2\cdot\sum(y_i-\beta x_i)^1\cdot(-x_i) = 0$$

$$\sum x_i y_i - \sum\beta x_i^2 = 0$$

$$\sum x_i y_i = \beta\sum x_i^2 \qquad \text{merk. } SS_x = \sum x_i^2$$

$$\beta = \frac{\sum x_i y_i}{\sum x_i^2} = \frac{\sum x_i}{\sum x_i^2}y_i \Rightarrow \hat{\beta} = \frac{\sum x_i}{SS_x}y_i$$

$$Var(\hat{\beta}) = \frac{1}{SS_x^2}\underbrace{\sum x_i^2}_{SS_x}\cdot\sum\underbrace{Var(y_i)}_{1} = \frac{1}{SS_x}$$

$$\hat{\beta} \sim N\left(\beta, \frac{1}{SS_x}\right) \qquad \frac{\hat{\beta}-\beta}{1/\sqrt{SS_x}} \sim N(0,1) \quad \Longrightarrow$$

$$P\left(-z_{\alpha/2} \le \frac{\hat{\beta}-\beta}{1/\sqrt{SS_x}} \le z_{\alpha/2}\right) = P\left(-z_{\alpha/2}\cdot\frac{1}{\sqrt{SS_x}} \le \hat{\beta}-\beta \le z_{\alpha/2}\frac{1}{\sqrt{SS_x}}\right)$$
$$= P\left(-\hat{\beta}-z_{\alpha/2}\frac{1}{\sqrt{SS_x}} \le -\beta \le -\hat{\beta}+z_{\alpha/2}\frac{1}{\sqrt{SS_x}}\right)$$
$$= P\left(\hat{\beta}+z_{\alpha/2}\frac{1}{\sqrt{SS_x}} \ge \beta \ge \hat{\beta}-z_{\alpha/2}\frac{1}{\sqrt{SS_x}}\right)$$
$$= P\left(\hat{\beta}-z_{\alpha/2}\frac{1}{\sqrt{SS_x}} \le \beta \le \hat{\beta}+z_{\alpha/2}\frac{1}{\sqrt{SS_x}}\right) = 1-\alpha$$

$100\cdot(1-\alpha)\%$:in lv. $\quad \alpha = 0.05 \quad z_{\alpha/2} = z_{0.025} = 1{,}96$

$$\hat{\beta} \pm 1{,}96\cdot\frac{1}{\sqrt{SS_x}}$$

9.7. a) $100(1-\alpha)\%$ luottamusväli π:lle

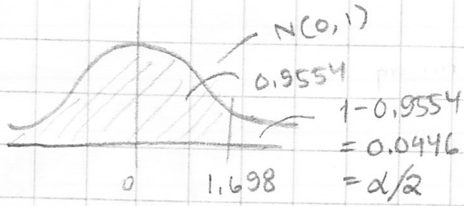
$$\hat{\pi} \pm z_{\alpha/2}\sqrt{\frac{\hat{\pi}(1-\hat{\pi})}{n}}$$ eli tässä 95% :n lv. $n=800$

$$0{,}48 \pm 1{,}96\sqrt{\frac{0{,}48\cdot 0{,}52}{800}} \qquad (0{,}445\,;\,0{,}515)$$ s.330

b) $$z_{\alpha/2}\cdot\sqrt{\frac{0{,}48\cdot 0{,}52}{800}} = 0{,}03$$ (Luku 12.4)

↖ maksimivirhe

$$\therefore\ z_{\alpha/2} = \frac{0{,}03}{\sqrt{\frac{0{,}48\cdot 0{,}52}{800}}} = 1{,}698$$



N(0,1) taulukosta $\Phi(1{,}698) \approx 0{,}9554$

$$\alpha/2 = 1-0{,}9554 \Rightarrow \alpha = 0{,}0892$$

Luottamustaso siis $1-\alpha = \underline{\underline{0{,}91}}$

9.8. 90% :n luottamusväli $\frac{\sigma_1}{\sigma_2}$:lle

$$\frac{S_2^2/\sigma_2^2}{S_1^2/\sigma_1^2} \sim F(12,15)$$, missä S_1^2 ja S_2^2 ovat otosvarianssejа

$$P\left(a \le \frac{S_2^2/\sigma_2^2}{S_1^2/\sigma_1^2} \le b\right) = 0{,}90$$

$$P\left(a\frac{S_1^2}{S_2^2} \le \frac{\sigma_1^2}{\sigma_2^2} \le b\frac{S_1^2}{S_2^2}\right) = 0{,}90$$

$$P\left(\sqrt{a}\frac{S_1}{S_2} \le \frac{\sigma_1}{\sigma_2} \le \sqrt{b}\frac{S_1}{S_2}\right)$$

$s_1 = 0{,}197$
$s_2 = 0{,}318$

$a = F_{0{,}05;12,15} = 0{,}382$
$b = F_{0{,}95;12,15} = 2{,}475$

$$\therefore\ \text{lv.}\ \left(\sqrt{0{,}382}\,\frac{0{,}197}{0{,}318},\ \sqrt{2{,}475}\,\frac{0{,}197}{0{,}318}\right)$$ eli $(0{,}383,\ 0{,}975)$