Tilastollinen päättely I Harj. 3 vk 5 2012

3.1. $X_i \sim Exp(\theta) = Gamma(1, \theta)$ ks. Esimerkki 9.7

$M_{X_i}(t) = (1-\theta t)^{-1} = M(t)$

$M_{T_n}(t) = [M(t)]^n = [(1-\theta t)^{-1}]^n = (1-\theta t)^{-n}$ (seuraus 9.1)

$\therefore \quad T_n \sim Gamma(n, \theta)$

$M_{T_n/n}(t) = [M(\frac{t}{n})]^n = (1-\frac{\theta t}{n})^{-n}$ (seuraus 9.1)

$\therefore \quad T_n/n \sim Gamma(n, \frac{\theta}{n})$

3.2. a) $E(\bar{x}-\bar{y}) = E(\bar{x}) - E(\bar{y}) = \mu_1 - \mu_2$

$Var(\bar{x}-\bar{y}) = Var(\bar{x}) + (-1)^2 Var(\bar{y}) = \frac{\sigma^2}{n} + \frac{\sigma^2}{m}$

$\left[\begin{array}{l} E(\bar{x}) = E(\frac{\sum x_i}{n}) = \frac{1}{n} n E(x) = E(x) \\ Var(\bar{x}) = Var(\frac{\sum x_i}{n}) = \frac{1}{n^2} n Var(x) = \frac{Var(x)}{n} \end{array} \right]$

b) $P(|\bar{x}-\bar{y}| > \frac{\sigma}{4}) \geq 0.68$ $\quad \mu_1 = \mu_2 \quad n = m \quad E(\bar{x}-\bar{y}) = 0 \quad Var(\bar{x}-\bar{y}) = 2\frac{\sigma^2}{n}$

$P(|\bar{x}-\bar{y}| > \frac{\sigma}{4}) = P\left(\left|\frac{\bar{x}-\bar{y}}{\sqrt{2\sigma^2/n}}\right| > \frac{\sigma/4}{\sqrt{2\sigma^2/n}}\right) = P\left(|Z| > \frac{\sigma/4}{\sqrt{2}\sigma/\sqrt{n}}\right)$

$= P\left(|Z| > \frac{\sqrt{n}}{4\sqrt{2}}\right) = 2 \cdot \Phi\left(-\frac{\sqrt{n}}{4\sqrt{2}}\right) = 0.68$

$\therefore \quad \Phi\left(-\frac{\sqrt{n}}{4\sqrt{2}}\right) = 0.34$ $\quad$ qnorm (0.34) = −0.412

$-\frac{\sqrt{n}}{4\sqrt{2}} = -0.412 \iff \frac{\sqrt{n}}{4\sqrt{2}} = 0.412 \Rightarrow \underline{\underline{n = 6}}$

$(\frac{1}{3})^2 \cdot \frac{\sigma^2}{n_1} + (\frac{1}{3})^2 \frac{\sigma^2}{n_2} + (\frac{1}{3})^2 \frac{\sigma^2}{n_3} = (\frac{1}{3})^2 \sigma^2 (\frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3})$

3.3. $\bar{X}_i \sim N(\mu, \frac{\sigma^2}{n_i})$

$V_1 = \frac{1}{3}(\bar{X}_1 + \bar{X}_2 + \bar{X}_3)$ Lauseesta 9.10 $V_1 \sim N\left(\mu, \frac{\sigma^2}{9}(\frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3})\right)$

$V_2 = w_1 \bar{X}_1 + w_2 \bar{X}_2 + w_3 \bar{X}_3$, missä $w_i = \frac{n_i}{n_1+n_2+n_3}$ merk. $n = n_1+n_2+n_3$

$= \frac{n_1}{n}\bar{X}_1 + \frac{n_2}{n}\bar{X}_2 + \frac{n_3}{n}\bar{X}_3$

Lauseen 9.10 mukaan $V_2 \sim N\left(\sum_{i=1}^{3} \frac{n_i}{n}\mu_i, \sum_{i=1}^{n} (\frac{n_i}{n})^2 \frac{\sigma^2}{n_i}\right)$, missä

$\sum_{i=1}^{3} \frac{n_i}{n}\mu_i = \frac{n_1}{n}\mu + \frac{n_2}{n}\mu + \frac{n_3}{n}\mu = \frac{n_1+n_2+n_3}{n}\mu = \mu$ ja

$\sum_{i=1}^{n} (\frac{n_i}{n})^2 \frac{\sigma^2}{n_i} = \frac{n_1^2}{n^2}\frac{\sigma^2}{\cancel{n_1}} + \frac{n_2^2}{n^2}\frac{\sigma^2}{\cancel{n_2}} + \frac{n_3^2}{n^2}\frac{\sigma^2}{\cancel{n_3}} = \sigma^2\left(\frac{n_1+n_2+n_3}{n^2}\right) = \frac{\sigma^2}{n}$

$\therefore \quad V_2 \sim N(\mu, \frac{\sigma^2}{n})$, missä $n = n_1+n_2+n_3$

(3.4.) $X \sim N(0,1)$, $Y \sim N(1,1)$, $W \sim N(2,4)$ X, Y, W riippumattomia

Merk. $X^2 + (Y-1)^2 = U$ ja $(W-2)^2/4 = Z^2$

jolloin $U \sim \chi^2(2)$ seuraus 9.2

$Z^2 \sim \chi^2(1)$ seuraus 9.2

$$P\left(\frac{U}{U+Z^2} > a\right) = P\left(\frac{1}{1+\frac{Z^2}{U}} > a\right) = P\left(\frac{2}{2+\frac{Z^2}{U/2}} > a\right)$$

merk. $\frac{Z^2}{U/2} = F \sim F(1,2)$

$$P\left(\frac{2}{2+F} > a\right) = P\left(F < \frac{2}{a} - 2\right) = 0.05$$

R:llä
qf(0.05, 1,2)
= 0.005013

$\therefore \frac{2}{a} - 2 = 0.005013 \Rightarrow a = 0.9975$

(3.5.) $P\left(|\bar{Y} - \mu| \leq \frac{2S}{\sqrt{n}}\right) = P\left(\left|\frac{\bar{Y}-\mu}{S/\sqrt{n}}\right| \leq 2\right)$ merk. $\frac{\bar{Y}-\mu}{S/\sqrt{n}} = T \sim t(n-1)$

kun $n = 25$

$P(-2 \leq T \leq 2) = 2 \cdot pt(2, 24) - 1 = 0.943$ (R:llä)

$= pt(2,24) - pt(-2, 24)$

(3.6.) $P(a \leq S^2 \leq b) = P\left(\frac{(n-1)a}{\sigma^2} \leq \frac{(n-1)S^2}{\sigma^2} \leq \frac{(n-1)b}{\sigma^2}\right)$ $n = 10$, $\sigma^2 = 0.8$

$= P\left(\frac{9a}{0.8} \leq \frac{9S^2}{0.8} \leq \frac{9b}{0.8}\right)$ Merk. $\frac{9S^2}{0.8} = W \sim \chi^2(9)$ ks. s. 254 ja Lause 9.9

Nyt on määritettävä positiiviluvut a ja b, s.e.

$P\left(\frac{9a}{0.8} \leq W \leq \frac{9b}{0.8}\right) = 0.90$

Määritetään b s.e.

$P\left(W \leq \frac{9b}{0.8}\right) = 0.95$ ja $P\left(W \leq \frac{9a}{0.8}\right) = 0.05$

qchisq(0.95, 9) = 16.91898 qchisq(0.05, 9) = 3.325113

$\frac{9b}{0.8} = 16.919$ $\frac{9a}{0.8} = 3.325$

$\therefore b = 1.5$ $a = 0.3$

3.7 $T \sim t(10)$ $\quad Z \sim N(0,1)$

a) $Var(T) = \frac{10}{10-2} = \frac{10}{8}$ (ks. Luku 7 mtt-perusteet 2011)

b) $P(|T| \geq 2.228) = 1 - P(|T| \leq 2.228) = 1 - P(-2.228 \leq T \leq 2.228)$

R:llä $= 1 - (pt(2.228, 10) - pt(-2.228, 10)) = 0.05$

tai suoraan

$2*pt(-2,228, 10) = 0.05$

$P(|Z| \geq 2.28)$ vastaavasti

R:llä $\quad 2*pnorm(-2,228, 0, 1) = 0.0259$

c) $P(-0.260 < T < 2.764) = pt(2.764, 10) - pt(-0.260, 10) = 0.5899$

$P(-0.260 < Z < 2.764) = pnorm(2.764) - pnorm(-0.260) = 0.5997$

d)

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curve (dt (x, 10), -5,5)
curve (dnorm (x, 0, 1), -5, 5, col= "blue", add = TRUE)
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tiheysf. jakauma jakauman parametrit mille välille näytetään lisätään samaan kuvioon

3.8. $F \sim F(\overset{r}{3}, \overset{s}{10})$ kts. luku 7 mtt-perusteet 2011

a) $E(F) = \frac{s}{s-2} = \frac{10}{10-2} = \frac{10}{8} = 1.25$

$Var(F) = \frac{2 \cdot s^2 (r+s-2)}{r(s-2)^2 (s-4)} = \frac{2 \cdot 10^2 (3+10-2)}{3(10-2)^2 (10-4)} = \frac{2 \cdot 10^2 \cdot 11}{3 \cdot 8^2 \cdot 6} \approx 1,9$

b) mediaani $\quad qf(0.5, 3, 10) = 0.845$

25%:n piste $\quad qf(0.25, 3, 10) = 0.409$

75%:n piste $\quad qf(0.75, 3, 10) = 1,6028$

tai $\quad qf(c(0.25, 0.5, 0.75), 3, 10)$

c) $P(F \leq b) = 0.975 \quad qf(0.975, 3, 10) = 4.8256 = b$

$P(a < F \leq b) = 0.95$ eli $P(F \leq a) = 0.025$

$qf(0.025, 3, 10) = 0.06935 = a$

Tarkistus $\quad P(a < F \leq b) = pf(b, 3, 10) - pf(a, 3, 10) = 0.95$

d) $\quad curve(df(x, 3, 10), 0, 10)$