Matemaattisen tilastotieteen perusteet   Harj. 4   Vk 47   2011

4.1. Kts. Lause 5.10   s. 132

$X \sim Bin(60, 0.03)$

$X \xrightarrow[n\to\infty]{d} Poi(\lambda)$, missä $np = \lambda$ eli $\lambda = 60 \cdot 0.3 = 1{,}8$

$$P(X \le 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$$
$$= \frac{e^{-1,8}\, 1{,}8^0}{0!} + \frac{e^{-1,8}\, 1{,}8^1}{1!} + \frac{e^{-1,8} \cdot 1{,}8^2}{2!} + \frac{e^{-1,8} \cdot 1{,}8^3}{3!}$$
$$= e^{-1,8} + e^{-1,8} \cdot 1{,}8 + e^{-1,8} \cdot 1{,}62 + e^{-1,8} \cdot 0{,}972 = e^{-1,8} \cdot 5{,}392$$
$$\approx 0{,}8913.$$

tai R:llä

ppois(3, 1.8) ≈ 0,8913

Vrt. Teht. 3.5   pbinom(3, 60, 0.03) = 0.8943
phyper(3, 300, 9700, 60) = 0,8948

4.2. Lause 5.11 $X \mid X+Y = 10 \sim Bin\left(10, \frac{1}{1+3}\right)$

a) $E(X \mid X+Y=10) = 10 \cdot \frac{1}{4} = 2{,}5$

b) $Y \mid X+Y = 10 \sim Bin\left(10, \frac{3}{3+1}\right)$

$$P(Y > 5 \mid X+Y=10) = 1 - P(Y \le 5 \mid X+Y=10)$$
$$= 1 - \text{pbinom}\left(5, 10, \tfrac{3}{4}\right) = 0{,}92$$

4.3. $N(t)$ tulipalojen lkm

$\lambda = E[N(1)] = 5$, yksikkö kuukausi

$N(1) \sim Poi(5)$

$p = P(N(1)=0)$ tn, että kuukaudessa ei yhtään tulipaloa.

Olk.

$$X_i = \begin{cases} 1, & \text{kun ei tulipaloa (onnistuminen)} \\ 0, & \text{kun on tulipaloja} \end{cases}$$

$X = X_1 + X_2 + \ldots + X_{12}$ on niiden kuukausien lkm, jolloin ei tulipaloja

$X \sim Bin(12, p)$

$P(X=2) = \binom{12}{2} p^2 (1-p)^{10}$, tässä $p = \frac{e^{-5}\, 5^0}{0!} = 0{,}0067$

$= 0{,}0028$

R:llä

dbinom(2, 12, dpois(0, 5)) = 0.0028...

(4.4.) $X \sim Exp(\theta)$, $\quad F(x) = 1 - e^{-\frac{x}{\theta}}$, $\quad x \geq 0$ ja $F(x) = 0$, kun $x < 0$

$M(t) = \frac{1}{1-\theta t}$

a) $M'(t) = \frac{\theta}{(1-\theta t)^2}$

$M'(0) = \theta = E(X)$

b) s. 149 prosenttipiste $\pi_{0,50}$ on mediaani

$$F(3) = 1 - e^{-\frac{3}{\theta}} = 0.5 \Rightarrow e^{-\frac{3}{\theta}} = \frac{1}{2}$$

$$\therefore -\frac{3}{\theta} = \log \frac{1}{2}$$

$$-\frac{3}{\theta} = -\log 2 \Rightarrow \theta = \frac{3}{\log 2} = 4,328$$

(4.5.) a) 1°) $f(0) \geq 0$ ja $f(x)$ on kasvava välillä $(0,4)$,

siis $f(x) > 0$ välillä $(0,4)$

2°) $\int_{-\infty}^{\infty} f(x)dx = \int_0^4 f(x)\,dx = \frac{3}{88} \Big/_0^4 \left(\frac{x^3}{3} + \frac{x^2}{2}\right) = 1$

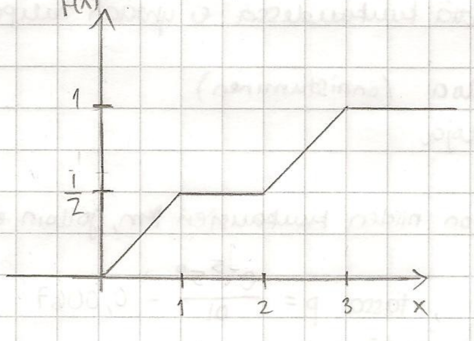
b)
$$F(x) = \begin{cases} 0, & x \leq 0 \\ \frac{3}{88}\left(\frac{x^3}{3} + \frac{x^2}{2}\right), & 0 < x < 4 \\ 1, & x \geq 4 \end{cases}$$

c) $P(2 \leq X \leq 3) = F(3) - F(2) = \frac{3}{88}\left[\frac{3^3}{3} + \frac{3^2}{2} - \frac{2^3}{3} - \frac{2^2}{2}\right] = \frac{3}{88} \cdot \frac{53}{6} =$

$= \frac{1}{88} \cdot \frac{53}{2} = \frac{53}{176} \approx 0,3011$

(4.6.)
$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{x}{2}, & 0 \leq x \leq 1 \\ \frac{1}{2}, & 1 < x < 2 \\ \frac{1}{2} + \frac{1}{2}(x-2), & 2 \leq x \leq 3 \\ 1, & x > 3 \end{cases}$$

$F(x) = P(X \leq x)$



vrt. Esimerkki 6.5

(4.7) $g(x) \geq 0$

$$c\int_0^2 g(x)\,dx = c\int_0^2 (x^2-x+1)\,dx = c\left[\Big/_0^2 \frac{x^3}{3}-\frac{x^2}{2}+x\right] = c\left(\frac{2^3}{3}-\frac{2^2}{2}+2\right)$$

$$= c\,\frac{2^3}{3} = 1 \Rightarrow c = \frac{3}{8}$$

$$P(0{,}5 \leq x \leq 1) = \frac{3}{8}\int_{0.5}^{1}(x^2-x+1)\,dx = \frac{3}{8}\Big/_{0.5}^{1}\left(\frac{x^3}{3}-\frac{x^2}{2}+x\right) =$$

$$= \frac{3}{8}\left(\frac{1}{3}-\frac{1}{2}+1\right) - \frac{3}{8}\left(\frac{1}{3}\cdot\left(\frac{1}{2}\right)^3-\frac{1}{2}\left(\frac{1}{2}\right)^2+\frac{1}{2}\right)$$

$$= \frac{3}{8}\cdot\frac{5}{6} - \frac{3}{8}\cdot\frac{10}{24} = \frac{3}{8}\left(\frac{10}{12}-\frac{5}{12}\right) = \frac{3}{8}\cdot\frac{5}{12} = \frac{1}{8}\cdot\frac{5}{4} = \frac{5}{32}$$

$$\approx 0{,}15625$$

(4.8) $X \sim Tas(0,1)$ Ks. Alaluku 6.2.1

a) $E(e^x) = \int_0^1 e^x\,dx = \Big/_0^1 e^x = e^1 - e^0 = e-1$ $\approx 1{,}718$

b) $P(e^X \leq \frac{4}{3}) \Leftrightarrow P(X \leq \log(\frac{4}{3}))$

$= F_X(\log(\frac{4}{3})) = \log(\frac{4}{3})$ $\approx 0{,}28$