

Subject Index

A

- added variable plots 164
- adding
 - variables in regression, BLUE 335
 - variables in regression, OLSE 161
- admissibility 259
- Aitken block-diagonalization formula 291
- Albert's theorem 305
- angle
 - between the regression line and the major axis 388
 - between two vectors 8
 - minimal 77, 134
 - principal 134
- ANOVA 31, 60, 186, 188
 - F -test 187
 - multivariate 234
- antieigenvalue 237, 419
 - the first antieigenvalue of $\Sigma_{2 \times 2}$ 387
- antieigenvector 237
- approximating
 - the centered data matrix 407
- approximation
 - best rank-1 406
 - best rank- k 401
 - by a matrix of lower rank 397
- AR(1)-structure *see* autocorrelation structure
- autocorrelation structure
 - determinant 209
 - efficiency of OLSE 241
 - inverse 209
 - prediction 208, 250
 - prediction error 209

B

- Banachiewicz–Schur form 295
- best linear predictor *see* BLP
- best linear unbiased estimator *see* BLUE
- best linear unbiased predictor *see* BLUP
- best predictor *see* BP
- best rank-1 approximation 406
- best rank- k approximation 401
- block design 31, 51
 - connected 348
- block-diagonalization 291
- Bloomfield–Watson–Knott inequality 238
- BLP 194–198
 - BLP($y; \mathbf{x}$) 196
 - BLP($\mathbf{z}; \mathbf{A}'\mathbf{z}$) 203
 - BLP($y; \mathbf{x}$) 195
 - $\mu_y + \sigma'_{xy} \Sigma_{xx}^{-1}(\mathbf{x} - \mu_x)$ 195
 - $\mu_y + \Sigma_{yx} \Sigma_{xx}^{-1}(\mathbf{x} - \mu_x)$ 196
- autocorrelation structure 250
- BLUE *see* covariance matrix, 215–222
 - definition 39
 - fundamental equations 41, 216
 - BLUE($\mathbf{M}_1 \mathbf{X}_2 \beta_2 \mid \mathcal{M}_{12.1}$) 328
 - BLUE($\mathbf{M}_1 \mathbf{X}_2 \beta_2 \mid \mathcal{M}_{12}$) 328
 - $\mathbf{A}(\mathbf{X} : \mathbf{V}\mathbf{X}^\perp) = (\mathbf{K}' : \mathbf{0})$,
 $\mathbf{A}\mathbf{y} = \text{BLUE}(\mathbf{K}'\beta)$ 216
 - $\mathbf{G}(\mathbf{X} : \mathbf{V}\mathbf{X}^\perp) = (\mathbf{X} : \mathbf{0})$,
 $\mathbf{G}\mathbf{y} = \text{BLUE}(\mathbf{X}\beta)$ 41, 216
 - BLUE's covariance matrix as a shorted matrix 312
 - equality of BLUE's subvectors under two models 333
 - equality of OLSE and BLUE, several conditions 218–222

- of β 44
- of β_2 327
- of $\mathbf{K}'\beta$ 216
- of $\mathbf{X}\beta$ 39, 43
- Pandora's Box 223
- representations 43, 228, 324
- residual $\tilde{\varepsilon} = \mathbf{V}\mathbf{M}\mathbf{y}$ 44, 325, 339
- under intraclass structure 225
- when $\mathbf{X} = \mathbf{1}$ 224
- BLUP 245–249
 - fundamental equations 247
 - $\mathbf{A}(\mathbf{X} : \mathbf{V}\mathbf{X}^\perp) = (\mathbf{X}_f : \mathbf{V}_{21}\mathbf{X}^\perp)$,
 - $\mathbf{A}\mathbf{y} = \text{BLUP}(\mathbf{y}_f)$ 247
 - autocorrelation structure 250
 - examples 250
 - intraclass correlation 250
 - of \mathbf{y}_f 245
 - of γ 255
 - of y_* 167
 - Pandora's Box 248
 - representations 248
 - standardized random walk process
 - $y_t = \beta t + \varepsilon_t$ 251
 - unbiasedly predictable 246
- BP
 - $\text{BP}(y; \mathbf{x})$ 194, 198
 - $\text{E}(y | \mathbf{x})$ 194
- C
- C-matrix 348
- canonical correlations 241, 382
 - and efficiency 382
 - and proper eigenvalues 379
 - based on Σ^{-1} 387
 - between $\hat{\beta}$ and $\tilde{\beta}$ 241, 382
 - between $\mathbf{H}\mathbf{y}$ and $\mathbf{M}\mathbf{y}$ 241, 382
 - between $\mathbf{K}'\mathbf{Q}_L\mathbf{u}$ and $\mathbf{L}'\mathbf{Q}_K\mathbf{u}$ 387
 - geometrically 408
 - intraclass correlation 389
 - non-unit cc's between $\mathbf{K}'\mathbf{u}$ and $\mathbf{L}'\mathbf{u}$ 387
 - SVD 408
 - unit 136, 321, 326
- Cauchy–Schwarz inequality 77, 174, 224, 396, 415, 421
 - $(\mathbf{x}'\mathbf{y})^2 \leq \mathbf{x}'\mathbf{x} \cdot \mathbf{y}'\mathbf{y}$ 415
 - $|\mathbf{A}'\mathbf{B}|^2 \leq |\mathbf{A}'\mathbf{A}||\mathbf{B}'\mathbf{B}|$ 423
 - $\mathbf{A}'\mathbf{B}(\mathbf{B}'\mathbf{B})^{-1}\mathbf{B}'\mathbf{A} \leq_L \mathbf{A}'\mathbf{A}$ 423
 - determinantal version 423
 - matrix version 423
 - versions 416
- centering matrix, \mathbf{C} 36, 91, 172
 - basis for $\mathcal{C}(\mathbf{C})$ 104
- characteristic equation 14, 358
- characteristic polynomial 358
- chi-squared distribution 18, 354, 355, 364
- chi-squared from frequency table 49
- Cholesky decomposition 208
- Cochran's theorem 352, 354, 390
- coefficient of determination *see* multiple correlation
 - in the no-intercept model 97, 171
- cofactor 429
- coin tossing 261
- collinearity 162, 412
- column space 3
 - $\mathcal{C}(\mathbf{A})$, basic properties 57
 - $\mathcal{C}(\mathbf{A}) \cap \mathcal{C}(\mathbf{B})$, several properties 190
 - $\mathcal{C}(\mathbf{A}\mathbf{G}) = \mathcal{C}(\mathbf{A}\mathbf{F})$ 126
 - $\mathcal{C}(\mathbf{A}\mathbf{P}_B) = \mathcal{C}(\mathbf{A}\mathbf{B})$ 127
 - $\mathcal{C}(\mathbf{A}^+) = \mathcal{C}(\mathbf{A}')$ 110, 133
 - $\mathcal{C}(\mathbf{W}^{-1}\mathbf{X} : \mathbf{I} - \mathbf{W}^{-1}\mathbf{W})$ 140
 - $\mathcal{C}(\mathbf{X} : \mathbf{V}) = \mathcal{C}(\mathbf{X} : \mathbf{V}\mathbf{M})$ 123
 - $\mathcal{C}[\text{cov}(\tilde{\varepsilon})]$ 137
 - $\mathcal{C}[\text{cov}(\mathbf{X}\hat{\beta})]$ 137
 - $\mathcal{C}[\mathbf{X}'(\mathbf{V} + \mathbf{X}\mathbf{U}\mathbf{X}')^{-1}\mathbf{X}]$ 286
 - of covariance matrix 62
- completely symmetric matrix 227
- eigenvalues 366, 388
- completeness
 - linear 258
- condition number 412, 419
- conditional *see* multinormal distribution
 - $\text{var}(y) = \text{var}[\text{E}(y | x)] + \text{E}[\text{var}(y | x)]$ 188
 - expectation and the best predictor 194, 198
- confidence ellipse *see* contours of constant density
- confidence ellipsoid
 - for β_2 178
- confidence interval
 - for $\text{E}(y_*)$ 168
- connected block design 348
- consistency
 - of $\Sigma_{\mathbf{x}\mathbf{x}}\mathbf{B} = \Sigma_{\mathbf{x}\mathbf{y}}$ 64
 - of $\mathbf{A}\mathbf{Y}\mathbf{B} = \mathbf{C}$ 267
 - of $\mathbf{G}(\mathbf{X} : \mathbf{V}\mathbf{X}^\perp) = (\mathbf{X} : \mathbf{0})$ 217
 - of $\mathbf{X}'\mathbf{W}^{-1}\mathbf{X}\beta = \mathbf{X}'\mathbf{W}^{-1}\mathbf{y}$ 288
 - of $\mathbf{Y}(\mathbf{A} : \mathbf{B}) = (\mathbf{0} : \mathbf{B})$ 139
- consistent linear model 43, 125
- constant term
 - the role of the vector $\mathbf{1}$ 95–100
- constraints
 - to obtain unique $\hat{\beta}$ 347
- contingency table 46, 47, 413

- contours of constant density 56, 169,
202, 365, 388
the slope of the major axis 365, 387
- contrast 62
elementary 62, 348
- Cook Islands, Aitutaki 54
- Cook's distance 181, 341
and Mahalanobis distance 341
generalized 338
- Cook's trick 206
- coordinate-free considerations 221
- correlation *see* multiple correlation
basic properties of sample correlation
91–104
 $\text{cor}(x, y) = \text{cov}(x, y) / \sqrt{\text{var}(x) \text{var}(y)}$
17
 $\text{cor}_s(x, y) = \text{cor}_d(\mathbf{x}, \mathbf{y}) = \cos(\mathbf{C}\mathbf{x}, \mathbf{C}\mathbf{y})$
37, 91
 $r_{xy} = \frac{\mathbf{x}'\mathbf{C}\mathbf{y}}{\sqrt{\mathbf{x}'\mathbf{C}\mathbf{x} \cdot \mathbf{y}'\mathbf{C}\mathbf{y}}}$ 37, 91
between $\hat{\beta}_1$ and $\hat{\beta}_2$ 167
between $\hat{\beta}_i$ and $\hat{\beta}_j$ 178
between $\hat{\beta}_x$ and \bar{y} 167
between dichotomous variables 46
between residuals 101, 178, 179
intraclass 225, 250
partial 101, 178, 179
sample 22, 91
- correlation matrix
 $\text{cor}(\mathbf{x}) = \Sigma_\delta^{-1/2} \Sigma \Sigma_\delta^{-1/2}$ 17
 $\mathbf{R} = \mathbf{S}_\delta^{-1/2} \mathbf{S} \mathbf{S}_\delta^{-1/2}$ 23, 127
 $\mathbf{R}_{\mathbf{x}\mathbf{x}} = \text{cor}_d(\mathbf{X}_0)$ 127
“correlation matrix” 307
determinant 127, 308
rank 127
- correspondence analysis 413
- cosine 8
correlation as a cosine 37, 91
maximal 77, 134
- cosine of matrix, $\cos(\mathbf{V})$ 237
- Courant–Fischer minimax theorem 389
- covariance
 $\text{cov}(x, y) = \sigma_{xy} = E(x - \mu)(y - \nu)$ 17
sample 22
- covariance matrix
 $\text{cov}(\mathbf{x}) = E(\mathbf{x} - \boldsymbol{\mu})(\mathbf{x} - \boldsymbol{\mu})'$ 17
 $\text{cov}(\mathbf{x}, \mathbf{y}) = E(\mathbf{x} - \boldsymbol{\mu})(\mathbf{y} - \boldsymbol{\nu})'$ 17
 $\text{cov}(\mathbf{e}_{\mathbf{y} \cdot \mathbf{x}})$ 198
 $\text{cov}[\mathbf{z} - \text{BLP}(\mathbf{z}; \mathbf{A}'\mathbf{z})]$ 203
 $|\Sigma| = |\Sigma_{\mathbf{x}\mathbf{x}}|(\sigma_y^2 - \sigma_{\mathbf{x}\mathbf{y}} \Sigma_{\mathbf{x}\mathbf{x}}^{-1} \sigma_{\mathbf{y}\mathbf{x}})$ 297
 $\mathbf{S} = \text{cov}_d(\mathbf{U}) = \frac{1}{n-1} \mathbf{U}'\mathbf{C}\mathbf{U}$ 23
 $\mathbf{S}_{\mathbf{x}\mathbf{x}} = \text{cov}_d(\mathbf{X}_0) = \frac{1}{n-1} \mathbf{T}_{\mathbf{x}\mathbf{x}}$ 127
 $\mathbf{S}_{\mathbf{x}\mathbf{x}} = \frac{1}{n-1} \mathbf{X}_0'\mathbf{C}\mathbf{X}_0$ 127
- as a Schur complement 211
conditional: $\text{cov}(\mathbf{y} \mid \mathbf{x})$ under
multinormality 197
Mahalanobis distance 157
minimal 191
of $\hat{\beta}_2$ 161
of $\hat{\beta}$ 45
of $\mathbf{X}\hat{\beta}$ 45, 137, 261, 326
of BLUE's residual, $\text{cov}(\hat{\varepsilon})$ 45, 50,
137, 326
of the prediction error 198, 203, 205
- Craig–Sakamoto–Matusita theorem 19
- Cronbach's *alpha* 422
- ## D
- data matrix
preliminaries 19–24
as a random matrix, representing a
random sample 20
keeping observed data as a theoretical
distribution 27
statistical independence of the rows
21
- data sets
1000 observations 1, 56, 169
12 observations 1, 56
12 observations, orthogonal distances,
rotation 404
15 observations, best predictor 211
2 treatments 38
3 observations 1, 26, 56
3 treatments 30, 35, 188
3 treatments, estimability 60
 t treatments, b blocks 31
Canner's data 263
coin tossing 261
correlation 0, cosine ≈ 1 93
correlation 1, cosine 0 94
generating observations 301
heights of fathers and sons, Galton
201
linear model when $\mathbf{V} = \text{diag}(1, 2, 3)$
224
over-speeding 45
throwing two dice 201
- decomposition *see* eigenvalue decompo-
sition, *see* full rank decomposition,
see singular value decomposition
SST = SSR + SSE 168
Cholesky 208
Hartwig–Spindelböck 112
of characteristic polynomial 359
 $\hat{\beta}_1(\mathcal{M}_{12}) = \hat{\beta}_1(\mathcal{M}_1) - \mathbf{A}$ 161

- $\tilde{\beta}_1(\mathcal{M}_{12}) = \tilde{\beta}_1(\mathcal{M}_1) - \mathbf{B}$ 336
 defenceman 29, 252
 deleting
 q observations in regression 188
 an observation in regression 180, 338, 341
 variables in regression, BLUE 335
 variables in regression, OLSE 161
 derivative vi, 366
 determinant
 $|\Sigma| = |\Sigma_{xx}|(\sigma_y^2 - \sigma'_{xy}\Sigma_{xx}^{-1}\sigma_{xy})$ 297
 cofactor 429
 generalized variance, $|\mathbf{S}|$ 131, 132
 historical development 300
 Laplace expansion 429
 multiplicative on the Schur compl. 182, 294
 of autocorrelation matrix 209
 of correlation matrix 127, 308
 of partitioned matrix 297
 product of eigenvalues 358
 deviant
 how? 420
 DFBETA $_i(\mathbf{V}) = \tilde{\beta} - \tilde{\beta}_{(i)}$ 338, 341
 DFBETA $_i = \hat{\beta} - \hat{\beta}_{(i)}$ 181
 diagonal increments 301
 diagonalizability 359
 diagonalization
 block 291
 simultaneous 370, 387
 dice
 throwing two dice 201
 direct sum decomposition 5
 discriminant function 374
 disjointness
 several equivalent conditions for
 $\mathcal{C}(\mathbf{A}) \cap \mathcal{C}(\mathbf{B}) = \{\mathbf{0}\}$ 343
 $\mathcal{C}(\mathbf{A}') \cap \mathcal{C}(\mathbf{B}')$ 351
 $\mathcal{C}(\mathbf{A}) \cap \mathcal{C}(\mathbf{B})$ 123, 139
 $\mathcal{C}(\mathbf{VH}) \cap \mathcal{C}(\mathbf{VM})$ 135, 321
 $\mathcal{C}(\mathbf{X}'_{(1)}) \cap \mathcal{C}(\mathbf{X}'_{(2)})$ 189
 $\mathcal{C}(\mathbf{X}) \cap \mathcal{C}(\mathbf{VM})$ 123
 $\mathcal{C}(\mathbf{X}) \cap \mathcal{C}(\mathbf{V}^\perp)$ 132
 $\mathcal{C}(\mathbf{X}_1) \cap \mathcal{C}(\mathbf{X}_2)$ 160
 $\mathcal{C}[(\mathbf{V} : \mathbf{X})'] \cap \mathcal{C}[(\mathbf{X}' : \mathbf{0})']$ 125
 $\mathbf{i}_i \in \mathcal{C}(\mathbf{X})$ 130, 144
 $\mathbf{x}_{(i)} \notin \mathcal{C}(\mathbf{X}'_{(i)})$ 130, 144
 distance *see* Cook's distance, *see* Mahalanobis distance
 statistical 372
 distribution *see* Hotelling's T^2 , *see* multinomial
 F -distribution 18, 176, 187, 341
 t -distribution 18, 341
 chi-squared 18, 354
 discrete uniform 53
 invariance of the F -test 342
 multinomial 49, 50, 120
 of $\mathbf{y}'(\mathbf{H} - \mathbf{J})\mathbf{y}/\sigma^2$ 185
 of $\mathbf{y}'(\mathbf{I} - \mathbf{H})\mathbf{y}/\sigma^2$ 185
 Wishart-distribution 26, 231
 double-centered matrix 48–50, 120, 413
 doubly-stochastic matrix 355
 Duncan formula 301
 Dürer's *Melencolia I* 54
- E**
- Eckart–Young theorem 204, 400
 symmetric \mathbf{A} 401
 efficiency
 of OLSE 234–241
 the Watson efficiency,
 $\text{eff}(\hat{\beta}) = |\text{cov}(\hat{\beta})|/|\text{cov}(\hat{\beta})|$
 238
 $\eta = \text{tr}[\text{cov}(\mathbf{X}\hat{\beta}) - \text{cov}(\mathbf{X}\tilde{\beta})]$ 240
 $\psi = \frac{1}{2}\|\mathbf{HV} - \mathbf{VH}\|^2$ 240
 and canonical correlations 241, 382
 of $\hat{\beta}_2$ 332
 of OLSE under autocorrelation structure 241
 eigenspace 359
 eigenvalue decomposition 14, 357, 393
 eigenvalues 357–390
 $(\lambda, \mathbf{P}_B \mathbf{w})$ as an eigenpair for $\mathbf{B}^+ \mathbf{A}$ 378
 (λ, \mathbf{t}) as an eigenpair for (\mathbf{A}, \mathbf{I}) 358
 (λ, \mathbf{t}) as an eigenpair for \mathbf{A} 358
 (λ, \mathbf{w}) as an eigenpair for $\mathbf{B}^{-1} \mathbf{A}$ 375
 (λ, \mathbf{w}) as an eigenpair for (\mathbf{A}, \mathbf{B}) 368
 $\text{ch}(\mathbf{A})$ 359
 $\text{ch}(\mathbf{A}, \mathbf{B}) = \text{ch}(\mathbf{B}^{-1} \mathbf{A})$ 368, 375
 $\text{ch}(\Sigma_{2 \times 2})$ 364
 $\text{ch}_1(\mathbf{A}, \mathbf{B}) = \text{ch}_1(\mathbf{B}^+ \mathbf{A})$ 377
 $|\mathbf{A} - \lambda \mathbf{B}| = 0$ 368
 $|\mathbf{A} - \lambda \mathbf{I}| = 0$ 358
 $\text{nzch}(\mathbf{A}, \mathbf{B})$, \mathbf{B} singular 376
 $\text{nzch}(\mathbf{A}, \mathbf{B}) = \text{nzch}(\mathbf{B}^+ \mathbf{A})$,
 $\mathcal{C}(\mathbf{A}) \subset \mathcal{C}(\mathbf{B})$ 376
 $\text{nzch}(\mathbf{AB}) = \text{nzch}(\mathbf{BA})$ 299, 360
 algebraic multiplicity 359
 and determinant 358
 and trace 358
 geometric multiplicity 359
 intraclass correlation 226, 310, 366, 388

- of $(\mathbf{H}\mathbf{V}\mathbf{H})^{-1}\mathbf{H}\mathbf{V}\mathbf{M}(\mathbf{M}\mathbf{V}\mathbf{M})^{-1}\mathbf{M}\mathbf{V}\mathbf{H}$ 383
 - of $\alpha^2\mathbf{I} + \mathbf{a}\mathbf{1}' + \mathbf{1}\mathbf{a}'$ 342
 - of $\mathbf{C}\mathbf{P}_X\mathbf{C}\mathbf{P}_Y$ 413
 - of $\mathbf{P}_{C_X}\mathbf{P}_{C_Y}$ 413
 - of $\mathbf{P}_{Q_Y}\mathbf{X}\mathbf{P}_{Q_X}\mathbf{Y}$ 413
 - of $\mathbf{P}_K\mathbf{P}_L - \mathbf{P}_F$ 387
 - of $\mathbf{P}_X\mathbf{P}_Y$ 413
 - of $\mathbf{P}_A - \mathbf{P}_B$ 386
 - of $\mathbf{P}_A\mathbf{P}_B$ 78, 133
 - of the BLUE's covariance matrix 378
 - proper eigenvalues 78, 368
 - proper eigenvalues and canonical correlations 383
 - proper eigenvalues, $\mathbf{B} \geq_L \mathbf{0}$ 374
 - the slope of the major axis 365, 387
 - unit eigenvalues of $\mathbf{P}_{V^{1/2}\mathbf{H}}\mathbf{P}_{V^{1/2}\mathbf{M}}$ 135, 244, 383
 - unit eigenvalues of $\mathbf{P}_A\mathbf{P}_B$ 133, 134
 - elementary column operation 389
 - elementary contrast 348
 - elephants in rows and columns 200
 - equality
 - of OLSE and BLUE 215
 - of OLSE and BLUE, several conditions 218–222
 - $\mathbf{X}\hat{\beta} = \mathbf{X}\beta$ 218–222
 - $\text{BLUP}(\mathbf{y}_f) = \text{BLUE}(\mathbf{X}_f\beta)$ 249
 - $\text{BLUP}(\mathbf{y}_f) = \text{OLSE}(\mathbf{X}_f\beta)$ 249
 - $\text{COOK}_i(\mathbf{I}) = \text{COOK}_i(\mathbf{V})$ 341
 - $\text{SSE}(\mathbf{I}) = \text{SSE}(\mathbf{V})$ 340
 - $\mathbf{A}'\mathbf{A} = \mathbf{B}'\mathbf{B}$ 396
 - $\mathbf{P}_X\mathbf{v}_1 = \mathbf{P}_X\mathbf{v}_2$ 88
 - $\hat{\beta}_1(\mathcal{M}_{12}) = \tilde{\beta}_1(\mathcal{M}_{12})$ 333
 - $\hat{\beta}(\mathcal{M}_{(i)}) = \tilde{\beta}(\mathcal{M}_{(i)})$ subject to $\hat{\beta}(\mathcal{M}) = \tilde{\beta}(\mathcal{M})$ 341
 - $\hat{\beta}_1(\mathcal{M}_{12}) = \tilde{\beta}_1(\mathcal{M}_{12})$ subject to $\hat{\beta}_1(\mathcal{M}_1) = \tilde{\beta}_1(\mathcal{M}_1)$ 341
 - $\hat{\beta}_2(\mathcal{M}_{12}) = \tilde{\beta}_2(\mathcal{M}_{12})$ 332
 - $\hat{\beta}_1(\mathcal{M}_{12}) = \tilde{\beta}_1(\mathcal{M}_{12}^*)$ subject to $\hat{\beta}_1(\mathcal{M}_1) = \tilde{\beta}_1(\mathcal{M}_1^*)$ 337
 - $\tilde{\beta}_1(\mathcal{M}_{12}) = \tilde{\beta}_1(\mathcal{M}_1)$ 336
 - of BLUPs of γ under two mixed models 278
 - of BLUPs of \mathbf{y}_f under two models 279
 - of OLSE and BLUE under multivariate linear model 231
 - of the BLUEs of $\mathbf{X}_1\beta_1$ under two models 333
 - of the BLUEs under two models 269
 - of the OLSE and BLUE of the subvectors 332
 - estimability
 - definition 37
 - and invariance 284
 - in a simple ANOVA 60
 - of β_k 162
 - of δ in the extended model 130
 - of $\mathbf{K}_2\beta_2$ 159
 - of β_2 160
 - of $\mathbf{K}'\beta$ 37, 284
 - of $\mathbf{X}_2\beta_2$ 160, 345
 - expected frequency 48
 - expression
 - wonderful 191
 - externally Studentized residual 182, 186
 - generalized 341
- F**
- factor analysis viii, 213, 401
 - factor scores 213
 - factorization
 - triangular 207
 - finiteness matters 53
 - frequency table 46, 47, 413
 - Frisch–Waugh–Lovell theorem 99, 163, 180, 328
 - generalized 328
 - Frucht–Kantorovich inequality 418
 - full rank decomposition 349
 - proof of RCR 354
- G**
- Galton's discovery of regression 199
 - Gauss–Markov model *see* linear model
 - Gauss–Markov theorem
 - generalized version 216
 - simple version 41
 - generalized inverse 4
 - general representation 111
 - least squares \mathbf{A}_ℓ^- 117
 - minimum norm $(\mathbf{X}')_{m(\mathbf{V})}^-$ 42
 - minimum norm \mathbf{A}_m^- 115
 - minimum seminorm 116, 315
 - nonnegative definite 316
 - reflexive 5, 119, 316
 - through SVD 109, 408
 - generalized normal equation 44, 82, 288
 - generalized variance, $|\mathbf{S}|$ 131, 132
 - generating observations 301
 - geometric illustration
 - SST = SSR + SSE 183
 - correlation 93
 - high cosine, no correlation 96

large R^2 but small r_{1y} and r_{2y} 184
 minimizing orthogonal distances 403
 partial correlation 95
 partial correlation zero 96
 projecting \mathbf{y} onto $\mathcal{C}(\mathbf{X})$ 183
 reduced model 184
 group inverse 355
 Grubbs's test 342

H

Hadamard product vi
 Hadamard's determinantal inequality 298
 Hartwig–Spindelböck decomposition 112
 hat matrix, \mathbf{H} 33
 remember \mathbf{H} and \mathbf{M} 34
 h_{ii} as a determinant 182
 and Mahalanobis distance 157, 421
 heights of fathers and sons 199
 Henderson's mixed model equation 255
 Hogg–Craig theorem 355
 hopelessness 121
 Hotelling's T^2 26, 53, 187, 234
 Householder transformation 355, 390
 hypothesis
 $\beta_k = 0$ under the intraclass correlation 342
 $\delta = 0$, outlier testing 182, 341
 $\mu_1 = \dots = \mu_g$ 187
 $\mu_1 = \mu_2$ 187
 $\mathbf{K}'\mathbb{B} = \mathbf{D}$ 231
 $\mathbf{K}'\beta = \mathbf{0}$ 176
 $\beta_2 = \mathbf{0}$ 176
 $\beta_{\mathbf{x}} = \mathbf{0}$ 176
 $\delta = \mathbf{0}$, outlier testing 189
 $\mu_1 = \mu_2$ 233
 $\varrho_{xy} = 0$ 187
 linear 175
 outlier testing when $\mathbf{X} = (\mathbf{1} : \mathbf{i}_n)$ 186

I

idempotent matrix
 $\mathcal{C}(\mathbf{A}) \cap \mathcal{C}(\mathbf{I} - \mathbf{A}) = \{\mathbf{0}\}$ 10
 $\text{rk}(\mathbf{A}) = \text{tr}(\mathbf{A})$ 350
 full rank decomposition 350
 incidence matrix 52
 independence
 linear 3
 statistical 19
 $\text{cov}(\hat{\beta}_{\mathbf{x}}, \bar{y}) = \mathbf{0}$ 167
 $\text{cov}(\mathbf{e}_{\mathbf{y}-\mathbf{x}}, \mathbf{x}) = \mathbf{0}$ 198

$\text{cov}(\mathbf{x}, \mathbf{y} - \mathbf{A}\mathbf{x}) = \mathbf{0}$ 64
 $\text{cov}(\mathbf{x}, \mathbf{y} - \Sigma_{\mathbf{y}\mathbf{x}}\Sigma_{\mathbf{x}\mathbf{x}}^{-1}\mathbf{x}) = \mathbf{0}$ 192
 of \bar{y} and s_y^2 184
 of $\bar{\mathbf{u}}$ and $\mathbf{U}'\mathbf{C}\mathbf{U}$ 26
 of SSR and SSE 185
 of dichotomous variables 47
 of quadratic forms 19
 Indian Statistical Institute 90
 inequality
 $\text{ch}_i[(\mathbf{X}'\mathbf{B}\mathbf{X})^{-1}\mathbf{X}'\mathbf{A}\mathbf{X}] \leq \text{ch}_i(\mathbf{B}^{-1}\mathbf{A})$ 411
 $\text{ch}_{i+k}(\mathbf{A}) \leq \text{ch}_i(\mathbf{A} - \mathbf{B})$ 411
 $\text{ch}_i(\mathbf{A}\mathbf{A}') \leq \text{ch}_i(\mathbf{A}\mathbf{A}' + \mathbf{B}\mathbf{B}')$ 386
 $\text{ch}_i(\mathbf{G}'\mathbf{A}\mathbf{G}) \leq \text{ch}_i(\mathbf{A})$ 398
 $|\text{tr}(\mathbf{A}\mathbf{B})| \leq \sum_{i=1}^k \text{sg}_i(\mathbf{A})\text{sg}_i(\mathbf{B})$ 410
 $\|\mathbf{a}\|_{\mathbf{V}} \leq \|\mathbf{a} + \mathbf{b}\|_{\mathbf{V}}$ 76
 $\kappa(\mathbf{X}_1 : \mathbf{M}_1\mathbf{X}_2) \leq \kappa(\mathbf{X}_1 : \mathbf{X}_2)$ 412
 $\lambda_n \leq \mathbf{x}'\mathbf{A}\mathbf{x}/\mathbf{x}'\mathbf{x} \leq \lambda_1$ 362
 $\text{sg}_{i+k}(\mathbf{A}) \leq \text{sg}_i(\mathbf{A} - \mathbf{B})$ 411
 $\text{tr}(\mathbf{A}\mathbf{B}) \leq \text{ch}_1(\mathbf{A}) \cdot \text{tr}(\mathbf{B})$ 386
 $\text{tr}(\mathbf{P}_\mathbf{A}\mathbf{P}_\mathbf{B}) \leq \text{rk}(\mathbf{P}_\mathbf{A}\mathbf{P}_\mathbf{B})$ 386
 Bloomfield–Watson–Knott 238
 Cauchy–Schwarz 77, 174, 396, 415, 421
 Cauchy–Schwarz, matrix version 423
 Cronbach's *alpha* 422
 Frucht–Kantorovich 418
 Hadamard 298
 Kantorovich 235, 418
 Kantorovich, matrix version 237, 424, 426
 Poincaré separation theorem 398
 Samuelson 420
 Sylvester 143
 triangle 8
 von Neumann 410
 Wielandt 419
 Wielandt, matrix version 425, 426
 inertia 299
 inner product 6, 11
 semi 8
 standard 6, 11
 intercept
 the role of the vector $\mathbf{1}$ 95–100
 intercept $\hat{\beta}_0$ 166
 intersection
 $\mathcal{C}(\mathbf{A}) \cap \mathcal{C}(\mathbf{B})$, several properties 190
 $\mathcal{C}(\mathbf{A}) \cap \mathcal{C}(\mathbf{B}) = \mathcal{C}[\mathbf{A}(\mathbf{A}'\mathbf{Q}_\mathbf{B})^\perp]$ 137
 intraclass correlation 225, 234
 F -test 342
 BLUP 250
 canonical correlations 389
 eigenvalues 226, 310, 366, 388
 multiple correlation 310

invariance
 and estimability 284
 of $MHLN^2(\mathbf{x}, \boldsymbol{\mu}_x, \boldsymbol{\Sigma}_{xx})$ 65
 of $MHLN^2(\mathbf{u}_{(i)}, \bar{\mathbf{u}}, \mathbf{S})$ 66
 of $\boldsymbol{\Sigma}_{yx}\boldsymbol{\Sigma}_{xx}^{-1}(\mathbf{x} - \boldsymbol{\mu}_x)$ 65
 of $\mathbf{AB}^{-1}\mathbf{C}$ 283
 of $\mathbf{BB}^{-1}\mathbf{C}$ 283
 of $\mathbf{K}'\hat{\boldsymbol{\beta}}$ 284
 of $\mathbf{X}'(\mathbf{V} + \mathbf{XUX}')^{-1}\mathbf{X}$ 286
 of $\mathbf{X}(\mathbf{X}'\mathbf{V}^{-1}\mathbf{X})^{-1}\mathbf{X}'\mathbf{V}^{-1}\mathbf{y}$ 149
 of $\mathbf{X}(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$ 284
 of the F -test 342
 of the Watson efficiency 239
 under orthogonal transformation 411
 inverse
 of $\mathbf{E} + \alpha\mathbf{ff}'$ 301
 of $\mathbf{E} + \mathbf{FBF}'$ 301
 of $\mathbf{I}_n + \alpha\mathbf{A}$ 304
 of $\mathbf{A} = \{a_{ij}\} = \{\min(i, j)\}$ 252
 of $\mathbf{X}'\mathbf{X}$ 295
 of a partitioned matrix 300
 of a partitioned nonnegative definite matrix 294
 of autocorrelation matrix 209
 of correlation matrix 172
 of sum of matrices 300
 involutory matrix 355, 390

K

Kantorovich inequality 235, 418
 matrix version 237, 424, 426
 Kantorovich ratio 418
 Kronecker product vi, 21, 32, 230, 263, 268, 280
 properties 52

L

Lagrange multipliers vi
 Lagrangian multiplier 366
 Laplace expansion of determinant 429
 $L^T E X$ x
 Latin square 67
 Latin square, philatelic 68
 Law of Eponymy, Stigler's 306
 least squares *see* OLSE
 least-squares g-inverse 117
 linear completeness 258
 linear expectation 198
 linear hypothesis *see* hypothesis
 linear model
 preliminaries 28–33
 centered 225

consistent 43, 125
 extended, \mathcal{M}_Z 180, 338
 mixed 253
 multivariate 32, 229
 reduced, $\mathcal{M}_{12 \cdot 1}$ 163, 328
 simple basic, \mathcal{M}_0 99, 168
 simple: $y = \beta_0 + \beta_1 x + \varepsilon$ 97–100, 166
 weakly singular 149, 236, 242
 without the i th observation, $\mathcal{M}_{(i)}$ 180, 338
 linear prediction sufficiency 252, 258
 linear sufficiency 257–259, 265
 linear unbiased estimator 37
 lower triangular matrix 207
 Löwner ordering *see* minimizing
 definition 12
 $\mathcal{C}(\mathbf{A}) \subset \mathcal{C}(\mathbf{B})$ & $\text{ch}_1(\mathbf{AB}^+) \leq 1$ 311
 implications 12, 40

M

magic square 53–55
 Mahalanobis distance 157, 421
 $MHLN^2(\bar{\mathbf{y}}_1, \bar{\mathbf{y}}_2, \mathbf{S}_{\#})$ 233, 374
 $MHLN^2(\mathbf{u}_{(i)}, \bar{\mathbf{u}}, \mathbf{S})$ 25
 $MHLN^2(\mathbf{x}, \boldsymbol{\mu}, \boldsymbol{\Sigma})$ 25, 364
 $MHLN^2(\mathbf{x}_*, \bar{\mathbf{x}}, \mathbf{S}_{xx})$ 168
 data points having the same MHLN 25
 invariance 65
 maximizing $\frac{[\mathbf{a}'(\mathbf{u}_{(i)} - \bar{\mathbf{u}})]^2}{\mathbf{a}'\mathbf{S}\mathbf{a}}$ 373
 matrix $\ddot{\mathbf{M}}$
 $\mathbf{P}_V \ddot{\mathbf{M}} \mathbf{P}_V$ 317
 matrix $\dot{\mathbf{M}}$
 $\mathbf{M}(\mathbf{MVM})^{-1}\mathbf{M}$ 45, 317
 matrix derivative vi
 matrix inequality
 Cauchy–Schwarz 423
 Kantorovich 237, 424, 426
 Wielandt 425, 426
 maximizing
 $(\mathbf{a}'\mathbf{x})^2 / \mathbf{x}'\mathbf{B}\mathbf{x}$ 369
 $\text{cor}(y, \mathbf{f}'\mathbf{x})$ 417
 $\text{cor}_d(\mathbf{Xr}, \mathbf{Ys})$ 409
 $\text{cor}_d(\mathbf{y}, \mathbf{X}_0\mathbf{b})$ 79, 173
 $\|\mathbf{HV} - \mathbf{VH}\|_F^2$ 240
 $\frac{(\boldsymbol{\alpha}'\mathbf{A}'\mathbf{B}\boldsymbol{\beta})^2}{\boldsymbol{\alpha}'\mathbf{A}'\mathbf{A}\boldsymbol{\alpha} \cdot \boldsymbol{\beta}'\mathbf{B}'\mathbf{B}\boldsymbol{\beta}}$ 78
 $\frac{[\mathbf{a}'(\mathbf{u}_{(i)} - \bar{\mathbf{u}})]^2}{\mathbf{a}'\mathbf{S}\mathbf{a}}$ 373
 \mathbf{U} subject to $\mathbf{U} \leq_L \mathbf{V}$, $\mathcal{C}(\mathbf{U}) \subset \mathcal{C}(\mathbf{X})$ 312
 $\boldsymbol{\alpha}'\mathbf{A}'\mathbf{P}_B\mathbf{A}\boldsymbol{\alpha} / \boldsymbol{\alpha}'\mathbf{A}'\mathbf{A}\boldsymbol{\alpha}$ 78
 $\mathbf{a}'\boldsymbol{\Sigma}^2\mathbf{a} / \mathbf{a}'\boldsymbol{\Sigma}\mathbf{a}$ 204
 $\mathbf{u}'\mathbf{u}$ subject to $\mathbf{u}'\boldsymbol{\Sigma}^{-1}\mathbf{u} = c^2$ 365

- $\mathbf{x}'\mathbf{A}\mathbf{x}/\mathbf{x}'\mathbf{B}\mathbf{x}$ 369
 $\mathbf{x}'\mathbf{A}\mathbf{x}$ subject to $\mathbf{x}'\mathbf{x} = 1$ 360
 $\mathbf{x}'\mathbf{A}\mathbf{y}$ subject to $\mathbf{x}'\mathbf{x} = \mathbf{y}'\mathbf{y} = 1$ 395
 $\cos(\mathbf{y}, \mathbf{A}\mathbf{b})$ 77
 $\cos^2(\mathbf{u}, \mathbf{v}), \mathbf{u} \in \mathcal{A}, \mathbf{v} \in \mathcal{B}$ 78
 $\text{tr}(\mathbf{G}'\mathbf{A}\mathbf{G})$ 397
 $\text{tr}[\text{cov}(\mathbf{X}\hat{\beta}) - \text{cov}(\mathbf{X}\tilde{\beta})]$ 240
 $\text{var}(\mathbf{b}'\mathbf{x})$ subject to $\mathbf{b}'\mathbf{b} = 1$ 362
 mean squared error 194
 mean squared error matrix 194, 196, 259
 mean vector of x -variables
 $\bar{\mathbf{x}} = \mathbf{X}'_0 \mathbf{1} \frac{1}{n}$ 157
Melencolia I by Dürer 54
 minimizing
 $(\mathbf{Y} - \mathbf{X}\mathbf{B})'(\mathbf{Y} - \mathbf{X}\mathbf{B})$ 80, 231, 399
 $(\mathbf{y} - \mathbf{X}\beta)' \mathbf{W}^{-1}(\mathbf{y} - \mathbf{X}\beta)$ 44
 $(y_1 - \alpha)^2 + \dots + (y_n - \alpha)^2$ 92
 $E[\mathbf{y} - (\mathbf{A}\mathbf{x} + \mathbf{b})][\mathbf{y} - (\mathbf{A}\mathbf{x} + \mathbf{b})]'$ 196
 $\text{MSEM}(\mathbf{A}\mathbf{x} + \mathbf{b}; \mathbf{y})$ 196
 $\text{cov}(\mathbf{A}\mathbf{y} - \mathbf{y}_f)$ subject to $\mathbf{A}\mathbf{X} = \mathbf{X}_f$
 246
 $\text{cov}(\mathbf{B}\mathbf{y})$ subject to $\mathbf{B}\mathbf{X} = \mathbf{I}_p$ 41
 $\text{cov}(\mathbf{G}\mathbf{y})$ subject to $\mathbf{G}\mathbf{X} = \mathbf{X}$ 39, 41,
 216
 $\text{cov}(\mathbf{H}\mathbf{y} - \mathbf{F}\mathbf{M}\mathbf{y})$ 210
 $\text{cov}(\mathbf{y} - \mathbf{F}\mathbf{x})$ 191
 $\text{eff}(\hat{\beta}) = |\text{cov}(\hat{\beta})|/|\text{cov}(\hat{\beta})|$ 238
 $\|\tilde{\mathbf{X}} - \tilde{\mathbf{X}}\mathbf{P}_{\mathbf{G}}\|$ 404
 $\|\tilde{\mathbf{X}} - \mathbf{B}\|$ 406
 $\|\mathbf{A} - \mathbf{B}\mathbf{Z}\|_F$ subject to \mathbf{Z} orthogonal
 401
 $\|\mathbf{A} - \mathbf{B}\|$ subject to $\text{rk}(\mathbf{B}) = k$ 399,
 400
 $\|\Sigma - \Sigma\mathbf{A}(\mathbf{A}'\Sigma\mathbf{A})^{-1}\mathbf{A}'\Sigma\|$ 205
 $\|\mathbf{y} - \mathbf{A}\mathbf{x}\|_{\mathbf{V}}$ 14, 81
 $\|\mathbf{y} - \mathbf{A}\mathbf{x}\|_{\mathbf{V}}$, singular \mathbf{V} 87
 $\|\mathbf{y} - \mathbf{A}\mathbf{x}\|$ 71, 76
 $\|\mathbf{y} - \mathbf{X}\beta\|$ 33
 $\|\mathbf{y}_1 - \alpha_1 \mathbf{1}\| + \dots + \|\mathbf{y}_g - \alpha_g \mathbf{1}\|$ 36
 $\Sigma - \Sigma\mathbf{a}(\mathbf{a}'\Sigma\mathbf{a})^{-1}\mathbf{a}'\Sigma$ 204
 $\mathbf{b}'\mathbf{N}\mathbf{b}$ subject to $\mathbf{A}\mathbf{b} = \mathbf{y}$ 116
 $\mathbf{b}'\mathbf{b}$ subject to $\mathbf{A}\mathbf{b} = \mathbf{y}$ 115
 $\mathbf{x}'\mathbf{A}\mathbf{x}$ subject to $\mathbf{x}'\mathbf{x} = 1$ 360
 $\mathbf{x}'\mathbf{A}\mathbf{y}$ subject to $\mathbf{x}'\mathbf{x} = \mathbf{y}'\mathbf{y} = 1$ 395
 $\cos^2(\mathbf{V}\mathbf{z}, \mathbf{z})$ 237
 $\text{tr}(\mathbf{P}_{\Sigma^{1/2}\mathbf{A}}\Sigma)$ 205
 $\text{var}(\mathbf{a}'\mathbf{y})$ subject to $\mathbf{X}'\mathbf{a} = \mathbf{k}$ 42, 117
 $\text{var}(\mathbf{g}'\mathbf{y})$ subject to $\mathbf{1}'\mathbf{g} = 1$ 223
 $\text{var}(y - \mathbf{f}'\mathbf{x})$ 193
 $\text{var}_d(\mathbf{y} - \mathbf{X}_0\mathbf{b})$ 79, 174
 $f[\text{cov}(\mathbf{y} - \mathbf{A}\mathbf{x})]$ 412
 angle 77
 mean squared error 195
 mean squared error matrix 196
 orthogonal distances 403
 minimum norm g-inverse 116
 minimum seminorm g-inverse 116, 315
 minus ordering 218, 315
 mixed linear model 253
 $\mathbf{A}(\mathbf{X} : \Sigma\mathbf{X}^{\perp}) = (\mathbf{0} : \mathbf{D}\mathbf{Z}'\mathbf{X}^{\perp})$,
 $\mathbf{A}\mathbf{y} = \text{BLUP}(\gamma)$ 254
 $\mathbf{B}(\mathbf{X} : \Sigma\mathbf{X}^{\perp}) = (\mathbf{X} : \mathbf{0})$,
 $\mathbf{B}\mathbf{y} = \text{BLUE}(\mathbf{X}\beta)$ 254
 and stochastic restrictions 273
 as a model with a new future
 observation 254
 BLUE of $\mathbf{X}\beta$ 254
 BLUP of γ 254
 Henderson's mixed model equation
 255
 Pandora's Box 254
 two mixed models 277
 model matrix
 $\mathbf{X} = (\mathbf{1} : \mathbf{X}_0)$ 30, 127
 \mathbf{X}_{bad} 235–237, 239
 extended, $(\mathbf{X} : \mathbf{i}_n)$ 180, 338
 extended, $[\mathbf{X} : (\mathbf{0} : \mathbf{I}_q)']$ 188
 rank 128
 Montréal Canadiens 29
 Moore–Penrose inverse, \mathbf{A}^+
 four conditions 4, 107
 $(\mathbf{A}^+)' = (\mathbf{A}')^+$ 113
 $\mathcal{C}(\mathbf{A}^+)$ 110
 $\mathbf{A}\mathbf{A}^+ = \mathbf{P}_{\mathbf{A}}$ 113
 $\mathbf{A}^+ = (\mathbf{A}'\mathbf{A})^+\mathbf{A}' = \mathbf{A}'(\mathbf{A}\mathbf{A}')^+$ 4
 $\mathbf{A}^+\mathbf{A} = \mathbf{P}_{\mathbf{A}'}$ 113
 \mathbf{a}^+ 109
 through FRD 107
 through SVD 109
 multinomial distribution *see* distribu-
 tion
 multinormal distribution
 definition 18
 conditional 291
 conditional covariance $\text{cov}(\mathbf{y} | \mathbf{x})$ 194,
 197
 conditional mean $E(\mathbf{y} | \mathbf{x})$ 194, 197
 conditional variance $\text{var}(y | \mathbf{x})$ 197
 density function 18, 365
 $E(\mathbf{y} | \mathbf{x}) = \text{BLP}(\mathbf{y} | \mathbf{x}) = \text{BP}(\mathbf{y} | \mathbf{x})$
 198
 multiple correlation
 R_{3-12} 309
 $R_{y-\mathbf{x}}$ 80, 169, 296
 x_k explained by other x_i 's 129, 162
 as a max correlation 172
 how to end up to R^2 ? 99
 intraclass correlation 310

population, $\varrho_{y \cdot \mathbf{x}}$ 193, 207, 315, 418
 representations for $R_{y \cdot \mathbf{x}}^2$ 171
 multivariate linear model 32, 229
 Mustonen's measure of multivariate dispersion, Mvar 212

N

natural restrictions 39
 Nobel Prize
 Jan Tinbergen 414
 John William Strutt 414
 Leonid Vitaliyevich Kantorovich 154
 Paul Anthony Samuelson 414
 Ragnar Frisch 414
 nonnegative definite matrix 11
 nonnegative definiteness
 of \mathbf{R} 308
 of partitioned matrix 305
 nonzero eigenvalues
 of \mathbf{UV} and \mathbf{VU} 299
 norm 7
 ellipsoidal 13
 Euclidean 7
 Frobenius 9
 matrix 9
 matrix 2-norm 9
 semi 8
 spectral 9
 standard 7
 normal distribution *see* multinormal distribution
 normal equations 34, 76
 general solution 34
 generalized 44, 82, 288
 null space 3

O

observation space 20, 91, 157, 403
 OLSE
 definition 37
 of β_0 166
 of β_k 162
 of β 34
 of $\beta_1(\mathcal{M}_{12})$ 161
 of β_2 159
 of $\beta_{\mathbf{x}}$ 166
 of $\mathbf{K}'\beta$ 37
 of $\mathbf{M}_1\mathbf{X}_2\beta_2$ 159
 of $\mathbf{X}\beta$ 33
 restricted 177, 264
 ordinary least squares *see* OLSE
 orthocomplement

$(\mathbf{M} : -\mathbf{MZ})' \in \{\mathbf{X}_*^\perp\}$ 274, 280
 $[(\mathbf{B}' : \mathbf{I})']^\perp$ 62
 $\mathcal{C}(\mathbf{V}\mathbf{X}^\perp)$ 140
 $\mathcal{C}(\mathbf{V}\mathbf{X}^\perp) = \mathcal{C}(\mathbf{W}^- \mathbf{X} : \mathbf{I} - \mathbf{W}^- \mathbf{W})^\perp$
 140
 $\mathcal{C}(\mathbf{V}^{-1}\mathbf{X})^\perp$ 140
 $\mathcal{C}(\mathbf{X})_{\mathbf{V}^{-1}}^\perp$ 140
 \mathcal{U}^\perp 10
 \mathbf{A}^\perp 10, 11, 71, 113
 $\mathbf{A}_{\mathbf{V}}^\perp$ 13, 81
 $\mathbf{A}_{\mathbf{V}}^\perp, \mathbf{V}$ singular 85
 orthogonal matrix 10
 orthogonal projector *see* projector
 $\mathbf{C} = \mathbf{I} - \mathbf{J}$ 36, 91, 172
 $\mathbf{H} = \mathbf{P}_{\mathbf{X}}$ 36, 172
 $\mathbf{J} = \mathbf{P}_{\mathbf{1}} = \frac{1}{n}\mathbf{1}\mathbf{1}'$ 36, 91, 172
 $\mathbf{P}_{\mathcal{C}(\mathbf{A}) \cap \mathcal{C}(\mathbf{B})}$ 190
 $\mathbf{P}_{\mathcal{C}(\mathbf{A}) \cap \mathcal{C}(\mathbf{B}^\perp)}$ 152
 $\mathbf{P}_{\mathbf{A}; \mathbf{V}}$ 81
 $\mathbf{P}_{\mathbf{A}; \mathbf{V}}, \mathbf{V}$ singular 87
 $\mathbf{P}_{\mathbf{X}; \mathbf{V}^{-1}}$ 43
 $\mathbf{P}_{\mathbf{X}; \mathbf{V}^{-1}} = \mathbf{I} - \mathbf{P}'_{\mathbf{M}; \mathbf{V}}$ 82, 89, 319
 $\mathbf{P}_{\mathbf{X}; \mathbf{W}^-}$ 88
 $\mathbf{P}_{\mathbf{A}}$ 72
 commuting $\mathbf{P}_{\mathbf{A}}\mathbf{P}_{\mathbf{B}} = \mathbf{P}_{\mathbf{B}}\mathbf{P}_{\mathbf{A}}$ 152, 190
 decomposition $\mathbf{P}_{(\mathbf{A} : \mathbf{B})} = \mathbf{P}_{\mathbf{A}} + \mathbf{P}_{\mathbf{Q}_{\mathbf{A}}\mathbf{B}}$
 155
 difference $\mathbf{P}_{\mathbf{A}} - \mathbf{P}_{\mathbf{B}}$ 152
 generalized 87
 onto $\mathcal{C}(\mathbf{P})$ 11
 Schur complement, $\mathbf{P}_{22 \cdot 1}$ 303
 sum $\mathbf{P}_{\mathbf{A}} + \mathbf{P}_{\mathbf{B}}$ 151
 orthogonality and uncorrelatedness
 92–95
 orthonormal
 columns 10
 over-speeding 45

P

Pandora's Box
 BLUE 223
 BLUP 248
 mixed model 254
 properties 264
 when $\mathbf{X} = \mathbf{1}$ 223
 parallel summable matrices 289
 parallelepiped 132
 parallelogram 131
 partial correlation 178, 179, 309
 partial correlation geometrically
 101–102
 partitioned matrix
 g-inverse 294

- inverse 294
 - MP-inverse 294
 - nonnegative definiteness 305
 - rank 121
 - pencil 368, 375
 - permutation matrix 55
 - philatelic Latin square 68
 - photo of
 - Anderson, Theodore W. xi, 19
 - Baksalary, Jerzy K. 39
 - Ben-Israel, Adi 5
 - Bhimasankaram, Pochiraju 350
 - Cook, R. Dennis 199
 - Flury, Bernard 198
 - Golub, Gene H. 393
 - Gustafson, Karl 225
 - Harville, David A. 273
 - Haslett, Stephen J. 256
 - Isotalo, Jarkko xi, 252
 - Kala, Radosław 257
 - Liu, Shuangzhe 425
 - Markiewicz, Augustyn 260
 - Markov, Andrei Viktorovich 29
 - Markov, Andrey (Andrei) Andreyevich 29
 - Mathew, Thomas 227
 - Mitra, Sujit Kumar 44
 - Olkin, Ingram 13
 - Pukelsheim, Friedrich 312
 - Puntanen, Simo xi
 - Rao, C. Radhakrishna 6
 - Scott, Alastair J. 398
 - Searle, Shayle R. 34
 - Seber, George A. F. 16
 - Styan, George P. H. xi
 - Tappara defenceman 252
 - Tian, Yongge 87
 - Trenkler, Götz 112
 - Watson, Geoffrey S. 241
 - Werner, Hans Joachim 216
 - Yanai, Haruo 307
 - Zmysłony, Roman 225
 - Poincaré separation theorem 398
 - positive definite matrix 11
 - predicted value \hat{y}_* 167
 - prediction error 246
 - $\mathbf{y} - \text{BLP}(\mathbf{y}; \mathbf{x})$ 196, 198
 - $e_{i \cdot 1 \dots i-1}$ 207
 - autocorrelation structure 209
 - Mahalanobis distance 167
 - with a given \mathbf{x}_* 167
 - prediction interval for y_* 168
 - principal components 362, 402
 - and Hotelling 206
 - from the SVD of $\tilde{\mathbf{X}}$ 407
 - predictive approach 203
 - principal minors 12, 308, 429
 - leading 12, 429
 - projector
 - $\mathbf{P}_{\mathbf{A}|\mathbf{B}}$ 87
 - $\mathbf{P}_{\mathbf{X}|\mathbf{V}\mathbf{X}^\perp}$ 43, 88
 - oblique 11
 - onto $\mathcal{C}(\mathbf{A})$ along $\mathcal{C}(\mathbf{B})$ 6
 - proper eigenvalues
 - and canonical correlations 383
 - PSTricks x
 - psychometrics viii, 401
- Q**
- quadratic risk
 - risk($\mathbf{F}\mathbf{y}; \mathbf{K}'\beta$) 259
- R**
- random effects 253
 - random sample without replacement 53, 234
 - rank
 - $\text{rk}(\mathbf{AB}) = \text{rk}(\mathbf{A}) - \dim \mathcal{C}(\mathbf{A}') \cap \mathcal{C}(\mathbf{B}^\perp)$ 121
 - $\text{rk}(\mathbf{A} : \mathbf{B}) = \text{rk}(\mathbf{A}) + \text{rk}(\mathbf{Q}_\mathbf{A}\mathbf{B})$ 121
 - $\text{rk}(\mathbf{A}\mathbf{P}_\mathbf{B}) = \text{rk}(\mathbf{AB})$ 127
 - $\text{rk}(\mathbf{P}_\mathbf{A}\mathbf{P}_\mathbf{B}\mathbf{Q}_\mathbf{A})$ 144
 - additive on the Schur complement 126, 299, 303
 - of $\text{cov}(\hat{\boldsymbol{\epsilon}})$ 137
 - of $\text{cov}(\hat{\boldsymbol{\beta}})$ 137
 - of $\text{cov}(\mathbf{X}\hat{\boldsymbol{\beta}})$ 137
 - of $\mathbf{A} + \mathbf{B}$ 351
 - of $\mathbf{C}\mathbf{X}_0$ 128
 - of $\mathbf{H}\mathbf{P}_\mathbf{V}\mathbf{M}$ 135, 321
 - of $\mathbf{T}_{\mathbf{xx}} = \mathbf{X}'_0\mathbf{C}\mathbf{X}_0$ 128
 - of \mathbf{X} 128
 - of $\mathbf{X}'(\mathbf{V} + \mathbf{X}\mathbf{U}\mathbf{X}')^{-1}\mathbf{X}$ 286
 - of $\mathbf{X}'\mathbf{V}\mathbf{X}$ 132, 147
 - of $\mathbf{X}'\mathbf{V}^+\mathbf{X}$ 132
 - of correlation matrix 127
 - of the extended model matrix 130
 - of the model matrix 127
 - of the partitioned matrix 297
 - rank additivity
 - $\text{rk}(\mathbf{A}) = \text{rk}(\mathbf{A}_{11}) + \text{rk}(\mathbf{A}_{22 \cdot 1})$ 126, 299, 303
 - $\text{rk}(\mathbf{A} + \mathbf{B}) = \text{rk}(\mathbf{A}) + \text{rk}(\mathbf{B})$ 351
 - rank cancellation rule, RCR 145
 - rank-subtractivity *see* minus ordering
 - Rayleigh ratio (quotient) 361

reduced model
 $\mathcal{M}_{12.1}$ 163, 328
 \mathcal{M}_{1H} 333
reflection 52
regression
 what is it? 198
regression coefficients
 $\hat{\beta}_0 = \bar{y} - \bar{\mathbf{x}}' \hat{\beta}_{\mathbf{x}}$ 166
 $\hat{\beta}_{\mathbf{x}} = \mathbf{T}_{\mathbf{xx}}^{-1} \mathbf{t}_{\mathbf{xy}} = \mathbf{S}_{\mathbf{xx}}^{-1} \mathbf{s}_{\mathbf{xy}}$ 166
 correlation between $\hat{\beta}_1$ and $\hat{\beta}_2$ 167
 correlation between $\hat{\beta}_i$ and $\hat{\beta}_j$ 178
 standardized 166, 171
regression fallacy 200
regression line and conditional means
 36, 198
regression towards mean 199
relative reduction in SSE 169
residual
 $\mathbf{e}_{(i)} = (\mathbf{I}_m - \mathbf{P}_{\mathbf{G}}) \tilde{\mathbf{x}}_{(i)}$ 404
 $\mathbf{e}_{i.1\dots i-1}$ 208
 $\mathbf{y} - \text{BLP}(\mathbf{y}; \mathbf{x})$ 196, 198
 $e_{i.1\dots i-1}$ 207
 externally Studentized 182, 186, 341
 internally Studentized 181
 of BLUE, $\tilde{\varepsilon} = \mathbf{V} \mathbf{M} \mathbf{y}$ 44, 325, 339
 of OLSE, $\hat{\varepsilon} = \mathbf{M} \mathbf{y}$ 33
residual sum of squares, SSE 99, 168
restricted BLUE 188
restricted OLSE 177, 264
rotation 52, 392, 404, 407, 409

S

Samuelson's inequality 420
scalar-potent matrix 355
Schmidt's approximation theorem 401
Schur complement vii, 291
 $\Sigma / \mathbf{Z}' \mathbf{V} \mathbf{Z}$ 135
 Banachiewicz–Schur form 295
 BLUE's covariance matrix 211
 determinant 294
 in the projector 303
 inverse of a pd matrix 294
 rank-additivity 299, 303
seminorm 87
Sherman & Morrison formula 301
shorted matrix 312
shrinking 392
similarity 299, 360
simple basic model, \mathcal{M}_0 99, 168
simultaneous diagonalization
 by an orthogonal matrix 370
 of commuting matrices 370
single regression coefficient 161

singular value decomposition 391–413
 honorary sv's 395
 in psychometrics 401
 reduced 394
 thin 394
 uniqueness 395
singular vectors: left, right 395
skew-symmetric 50
solution
 to $\mathbf{A} \mathbf{Y} \mathbf{B} = \mathbf{C}$ 267–282
space
 observation 20, 91, 157, 403
 variable 20, 91, 157, 403
spectral radius 389
spectrum *see* eigenvalues
square root of
 \mathbf{A} 15, 236, 360, 416
 \mathbf{A}^+ 416
 Σ 302
stamp
 al-Khwārizmī, Muḥammad 154
 Banachiewicz, Tadeusz 290
 Carroll, Lewis 356
 Cauchy, Augustin-Louis 290
 Dodgson, Charles 356
 Dürer's *Melencolia I* 54
 Einstein, Albert 55
 Euler, Leonhard Paul 55
 Galilei, Galileo 55
 Gauss, Johann Carl Friedrich 214
 Gödel, Kurt 282
 Japan 1992 356
 Kantorovich, Leonid Vitaliyevich 154
 Kolmogorov, Andrei Nikolaevich 282
 Laplace, Pierre-Simon de 154
 Leibniz, Gottfried Wilhelm von 356
 Llull, Ramon 68
 Mahalanobis, Prasanta Chandra 90
 Neumann, John von 282
 Newton, Isaac 55
 Pakistan 2004 70
 Pitcairn Islands 2007 69
 Poincaré, Henri 282
 Rao, C. Radhakrishna (special cover)
 150
 Samuelson, Paul Anthony 414
 Seki Kōwa, Takakazu 356
 St. Vincent 1991 356
 Strutt, John William (3rd Baron
 Rayleigh) 414
 Tinbergen, Jan 414
 Tristan da Cunha 1981 356
stamps
 stochastic vi

- Stigler's Law of Eponymy 306
- stochastic matrix 224, 389
- doubly stochastic 224, 389
 - superstochastic 224
- stochastic restrictions 273
- stochastic stamps vi
- strictly increasing function 411
- Studentized residual 181, 182, 186, 341
- generalized 341
 - multiple analogue 189
- subvector
- $\hat{\beta}_0 = \bar{y} - \bar{x}'\hat{\beta}_x$ 166
 - $\hat{\beta}_x = \mathbf{T}_{xx}^{-1}\mathbf{t}_{xy} = \mathbf{S}_{xx}^{-1}\mathbf{s}_{xy}$ 166
 - efficiency of $\hat{\beta}_2$ 332
 - of $\hat{\beta}$ 159
 - of $\hat{\beta}$ 327, 330
- sufficiency
- linear 257–259, 265
 - linear minimal 258, 266
 - linear prediction 252, 258
- sufficient dimension reduction 199
- sum of squares
- $SS_{\text{Between}} = n_1(\bar{y}_1 - \bar{y})^2 + \cdots + n_g(\bar{y}_g - \bar{y})^2$ 35, 187
 - $SS_{\text{Within}} = SS_1 + \cdots + SS_g$ 187
 - $\mathbf{E}_{\text{Within}}$ 233
 - due to regression, SSR 99, 169
 - of errors, SSE 99, 168
 - of errors, weighted 45, 339, 342
 - total, SST 99, 168
- superstochastic matrix 224
- Survo x
- Sylvester's inequality 143
- T**
- Tappara, Tampere 252
- Theil–Goldberger mixed estimator 264
- theorem
- Albert 305
 - Cochran 352, 354, 390
 - Courant–Fischer 389
 - Craig–Sakamoto–Matusita 19
 - Eckart–Young 204, 400
 - Frisch–Waugh–Lovell 99, 163, 180, 328
 - Gauss–Markov 41, 216
 - Hogg–Craig 355
 - Poincaré 398
 - Schmidt 401
 - Wedderburn–Guttman 303
- total sum of squares *see* sum of squares
- trace
- $\|\mathbf{A}\|_F^2 = \text{tr}(\mathbf{A}'\mathbf{A})$ 9
 - $\mathbf{A} = \mathbf{A}^2 \implies \text{rk}(\mathbf{A}) = \text{tr}(\mathbf{A})$ 350
 - $\text{tr}(\mathbf{KL}) = \text{tr}(\mathbf{LK})$ 9
 - sum of the eigenvalues 358
- treatments
- three treatments A, B, C 30, 35, 188
 - two treatments, estimability 38
- triangular factorization 207
- tripotent matrix 390
- U**
- unbiased estimator of $\sigma^2, \bar{\sigma}^2$ 339
 - unbiasedly predictable 246
 - unit canonical correlations 136, 321
- V**
- variable space 20, 91, 157, 403
 - variance
 - $\text{var}(x) = \sigma_x^2 = E(x - \mu)^2$ 16
 - sample variance $\text{var}_s(x) = \text{var}_d(\mathbf{x}) = \frac{1}{n-1}\mathbf{x}'\mathbf{C}\mathbf{x}$ 21
 - $\text{var}(\hat{y}_*)$ 167
 - $\text{var}(e_*)$ 168
 - conditional: $\text{var}(y \mid \mathbf{x})$ under multinormality 197
 - generalized variance, $|\mathbf{S}|$ 131, 132
 - of $\hat{\beta}_k$ 162, 296
 - of prediction error with a given \mathbf{x}_* 168
- variance inflation factor, VIF 162, 172, 296
 - vec-operation 8, 21, 32, 52, 230, 263, 268, 280
 - vector space of linear combinations of random variables 212
 - volume of the matrix 131
- W**
- Watson efficiency 238
 - of $\hat{\beta}_2$ 332
 - weakly singular model 149, 236, 242
 - Wedderburn–Guttman theorem 303
 - weighted least squares estimator, WLSE 147
 - weighted sum of squares of errors 45, 339, 342
 - $E(\mathbf{y}'\mathbf{M}\mathbf{y}/f)$ 339
 - Wielandt inequality 419
 - matrix version 425, 426 - Wielandt ratio 418
 - Wishart-distribution *see* distribution
 - Woodbury formula 301