Notation

$\mathbb{R}$	real numbers
$\mathbb{R}^{n\times m}$	set of $n \times m$ real matrices
$\mathbb{R}^{n\times m}_r$	subset of $\mathbb{R}^{n\times m}$ consisting of matrices with rank r
$\mathbb{R}^n_s$	subset of $\mathbb{R}^{n\times n}$ consisting of symmetric matrices
NND_n	subset of $\mathbb{R}^n_s$ consisting of nonnegative definite (nnd) matrices: $\mathbf{A} \in \mathrm{NND}_n \iff \mathbf{A} = \mathbf{LL}'$ for some $\mathbf{L}$; instead of nnd, the term positive semidefinite is often used
PD_n	subset of NND_n consisting of positive definite (pd) matrices: $\mathbf{A} = \mathbf{LL}'$ for some nonsingular $\mathbf{L}$
$\mathbf{0}$	null vector, null matrix; denoted also as $\mathbf{0}_n$ or $\mathbf{0}_{n\times m}$
$\mathbf{1}_n$	column vector of ones, shortened $\mathbf{1}$
$\mathbf{I}_n$	identity matrix, shortened $\mathbf{I}$
$\mathbf{i}_j$	the jth column of $\mathbf{I}$; the jth standard basis vector
$\mathbf{A} = \{a_{ij}\}$	matrix $\mathbf{A}$ with its elements a_{ij}
$\mathbf{A}_{n\times m}$	$n \times m$ matrix $\mathbf{A}$
$\mathbf{a}$	column vector $\mathbf{a} \in \mathbb{R}^n$
$\mathbf{A}'$	transpose of the matrix $\mathbf{A}$
$(\mathbf{A} : \mathbf{B})$	partitioned (augmented) matrix
$\mathbf{A} = (\mathbf{a}_1 : \ldots : \mathbf{a}_m)$	$\mathbf{A}_{n\times m}$ represented columnwise
$\mathbf{A} = \begin{pmatrix} \mathbf{a}'_{(1)} \\ \vdots \\ \mathbf{a}'_{(n)} \end{pmatrix}$	$\mathbf{A}_{n\times m}$ represented row-wise
$\mathbf{A}^{-1}$	inverse of the matrix $\mathbf{A}$
$\mathbf{A}^{-}$	generalized inverse of the matrix $\mathbf{A}$: $\mathbf{AA}^{-}\mathbf{A} = \mathbf{A}$, also called $\{1\}$-inverse, or inner inverse
$\{\mathbf{A}^{-}\}$	the set of generalized inverses of $\mathbf{A}$

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$\mathbf{A}_{12}^{-}$	reflexive generalized inverse of $\mathbf{A}$: $\mathbf{A}\mathbf{A}^{-}\mathbf{A} = \mathbf{A}$, $\mathbf{A}^{-}\mathbf{A}\mathbf{A}^{-} = \mathbf{A}^{-}$, also called $\{12\}$-inverse
$\mathbf{A}^{+}$	the Moore–Penrose inverse of $\mathbf{A}$: the unique matrix satisfying the four Moore–Penrose conditions:

$$\text{(mp1)}\ \mathbf{A}\mathbf{A}^{-}\mathbf{A} = \mathbf{A}, \qquad \text{(mp2)}\ \mathbf{A}^{-}\mathbf{A}\mathbf{A}^{-} = \mathbf{A}^{-},$$
$$\text{(mp3)}\ (\mathbf{A}\mathbf{A}^{-})' = \mathbf{A}\mathbf{A}^{-}, \qquad \text{(mp4)}\ (\mathbf{A}^{-}\mathbf{A})' = \mathbf{A}^{-}\mathbf{A}$$

$\mathbf{A}_{ij}^{-}$	generalized inverse of $\mathbf{A}$ satisfying the Moore–Penrose conditions (mp$i$) and (mp$j$)
$\mathbf{A}^{1/2}$	symmetric nnd square root of $\mathbf{A} \in \mathrm{NND}_n$: $\mathbf{A}^{1/2} = \mathbf{T}\mathbf{\Lambda}^{1/2}\mathbf{T}'$, where $\mathbf{A} = \mathbf{T}\mathbf{\Lambda}\mathbf{T}'$ is the eigenvalue decomposition of $\mathbf{A}$
$\mathbf{A}^{+1/2}$	$(\mathbf{A}^{+})^{1/2}$
$\mathrm{In}(\mathbf{A}) = (\pi, \nu, \delta)$	inertia of the square matrix $\mathbf{A}$: π, ν, and δ are the number of positive, negative, and zero eigenvalues of $\mathbf{A}$, respectively, all counting multiplicities
$\langle \mathbf{a}, \mathbf{b} \rangle$	standard inner product in $\mathbb{R}^n$: $\langle \mathbf{a}, \mathbf{b} \rangle = \mathbf{a}'\mathbf{b}$; can denote also a general inner product in a vector space
$\langle \mathbf{a}, \mathbf{b} \rangle_{\mathbf{V}}$	inner product $\mathbf{a}'\mathbf{V}\mathbf{b}$; $\mathbf{V}$ is the inner product matrix (ipm)
$\mathbf{a} \perp \mathbf{b}$	vectors $\mathbf{a}$ and $\mathbf{b}$ are orthogonal with respect to a given inner product
$\Vert\mathbf{a}\Vert$	Euclidean norm (standard norm, 2-norm) of vector $\mathbf{a}$, also denoted $\Vert\mathbf{a}\Vert_2$: $\Vert\mathbf{a}\Vert^2 = \mathbf{a}'\mathbf{a}$; can denote also a general vector norm in a vector space
$\Vert\mathbf{a}\Vert_{\mathbf{V}}$	$\Vert\mathbf{a}\Vert_{\mathbf{V}}^2 = \mathbf{a}'\mathbf{V}\mathbf{a}$, norm when the ipm is $\mathbf{V}$ (ellipsoidal norm)
$\langle \mathbf{A}, \mathbf{B} \rangle$	standard matrix inner product between $\mathbf{A}, \mathbf{B} \in \mathbb{R}^{n \times m}$: $\langle \mathbf{A}, \mathbf{B} \rangle = \mathrm{tr}(\mathbf{A}'\mathbf{B}) = \sum_{i,j} a_{ij} b_{ij}$
$\Vert\mathbf{A}\Vert_F$	Euclidean (Frobenius) norm of the matrix $\mathbf{A}$: $\Vert\mathbf{A}\Vert_F^2 = \mathrm{tr}(\mathbf{A}'\mathbf{A}) = \sum_{i,j} a_{ij}^2$
$\Vert\mathbf{A}\Vert_2$	matrix 2-norm of the matrix $\mathbf{A}$ (spectral norm):

$$\Vert\mathbf{A}\Vert_2 = \max_{\Vert\mathbf{x}\Vert_2 = 1} \Vert\mathbf{A}\mathbf{x}\Vert_2 = \mathrm{sg}_1(\mathbf{A}) = +\sqrt{\mathrm{ch}_1(\mathbf{A}'\mathbf{A})}$$

$\Vert\mathbf{A}^{-1}\Vert_2$	matrix 2-norm of nonsingular $\mathbf{A}_{n \times n}$: $\Vert\mathbf{A}^{-1}\Vert_2 = 1/\mathrm{sg}_n(\mathbf{A})$
$\mathrm{cond}(\mathbf{A})$	condition number of nonsingular $\mathbf{A}_{n \times n}$: $\mathrm{cond}(\mathbf{A}) = \Vert\mathbf{A}\Vert_2 \Vert\mathbf{A}^{-1}\Vert_2 = \mathrm{sg}_1(\mathbf{A})/\mathrm{sg}_n(\mathbf{A})$
$\cos(\mathbf{a}, \mathbf{b})$	$\cos\angle(\mathbf{a}, \mathbf{b})$, the cosine of the angle, θ, between the nonzero vectors $\mathbf{a}$ and $\mathbf{b}$: $\cos(\mathbf{a}, \mathbf{b}) = \cos\theta = \cos\angle(\mathbf{a}, \mathbf{b}) = \frac{\langle \mathbf{a}, \mathbf{b} \rangle}{\Vert\mathbf{a}\Vert \Vert\mathbf{b}\Vert}$
$\angle(\mathbf{a}, \mathbf{b})$	the angle, θ, $0 \leq \theta \leq \pi$, between the nonzero vectors $\mathbf{a}$ and $\mathbf{b}$: $\theta = \angle(\mathbf{a}, \mathbf{b}) = \cos^{-1}(\mathbf{a}, \mathbf{b})$
$\mathbf{A}[\alpha, \beta]$	submatrix of $\mathbf{A}_{n \times n}$, obtained by choosing the elements of $\mathbf{A}$ which lie in rows α and columns β; α and β are index sets of the rows and the columns of $\mathbf{A}$, respectively
$\mathbf{A}[\alpha]$	$\mathbf{A}[\alpha, \alpha]$, principal submatrix; same rows and columns chosen

$\mathbf{A}_i^{\mathrm{L}}$	ith leading principal submatrix of $\mathbf{A}_{n\times n}$: $\mathbf{A}_i^{\mathrm{L}} = \mathbf{A}[\alpha,\alpha]$, where $\alpha = \{1,\dots,i\}$
$\mathbf{A}(\alpha,\beta)$	submatrix of $\mathbf{A}$, obtained by choosing the elements of $\mathbf{A}$ which do not lie in rows α and columns β
$\mathbf{A}(i,j)$	submatrix of $\mathbf{A}$, obtained by deleting row i and column j from $\mathbf{A}$
$\mathrm{minor}(a_{ij})$	ijth minor of $\mathbf{A}$ corresponding to a_{ij}: $\mathrm{minor}(a_{ij}) = \det(\mathbf{A}(i,j))$, $i,j \in \{1,\dots,n\}$
$\mathrm{cof}(a_{ij})$	ijth cofactor of $\mathbf{A}$: $\mathrm{cof}(a_{ij}) = (-1)^{i+j}\,\mathrm{minor}(a_{ij})$
$\det(\mathbf{A})$	determinant of the matrix $\mathbf{A}_{n\times n}$: $\det(a) = a$, $a \in \mathbb{R}$, $\det(\mathbf{A}) = \sum_{j=1}^n a_{ij}\,\mathrm{cof}(a_{ij})$, $i \in \{1,\dots,n\}$: the Laplace expansion by minors along the ith row
$\det(\mathbf{A}[\alpha])$	principal minor
$\det(\mathbf{A}_i^{\mathrm{L}})$	leading principal minor of order i
$\lvert\mathbf{A}\rvert$	determinant of the matrix $\mathbf{A}_{n\times n}$
$\mathrm{diag}(\mathbf{A})$	diagonal matrix formed by the diagonal entries of $\mathbf{A}_{n\times n}$
$\mathrm{diag}(d_1,\dots,d_n)$	$n\times n$ diagonal matrix with listed diagonal entries
$\mathrm{diag}(\mathbf{d})$	$n\times n$ diagonal matrix whose ith diagonal element is d_i
$\mathbf{A}_\delta$	diagonal matrix formed by the diagonal entries of $\mathbf{A}_{n\times n}$
$\mathrm{rk}(\mathbf{A})$	rank of the matrix $\mathbf{A}$
$\mathrm{rank}(\mathbf{A})$	rank of the matrix $\mathbf{A}$
$\mathrm{tr}(\mathbf{A})$	trace of the matrix $\mathbf{A}_{n\times n}$: $\mathrm{tr}(\mathbf{A}) = \sum_{i=1}^n a_{ii}$
$\mathrm{trace}(\mathbf{A})$	trace of the matrix $\mathbf{A}_{n\times n}$
$\mathrm{vec}(\mathbf{A})$	vectoring operation: the vector formed by placing the columns of $\mathbf{A}$ under one another successively
$\mathbf{A}\otimes\mathbf{B}$	Kronecker product of $\mathbf{A}_{n\times m}$ and $\mathbf{B}_{p\times q}$: $\mathbf{A}\otimes\mathbf{B} = \begin{pmatrix} a_{11}\mathbf{B} & \dots & a_{1m}\mathbf{B} \\ \vdots & \vdots & \vdots \\ a_{n1}\mathbf{B} & \dots & a_{nm}\mathbf{B} \end{pmatrix} \in \mathbb{R}^{np\times mq}$
$\mathbf{A}/\mathbf{A}_{11}$	Schur complement of $\mathbf{A}_{11}$ in $\mathbf{A} = \begin{pmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{pmatrix}$: $\mathbf{A}/\mathbf{A}_{11} = \mathbf{A}_{22} - \mathbf{A}_{21}\mathbf{A}_{11}^{-}\mathbf{A}_{12}$
$\mathbf{A}_{22\cdot 1}$	$\mathbf{A}_{22} - \mathbf{A}_{21}\mathbf{A}_{11}^{-}\mathbf{A}_{12}$
$\mathbf{A} \geq_{\mathsf{L}} \mathbf{0}$	$\mathbf{A}$ is nonnegative definite: $\mathbf{A} = \mathbf{L}\mathbf{L}'$ for some $\mathbf{L}$; $\mathbf{A} \in \mathrm{NND}_n$
$\mathbf{A} >_{\mathsf{L}} \mathbf{0}$	$\mathbf{A}$ is positive definite: $\mathbf{A} = \mathbf{L}\mathbf{L}'$ for some invertible $\mathbf{L}$; $\mathbf{A} \in \mathrm{PD}_n$
$\mathbf{A} \leq_{\mathsf{L}} \mathbf{B}$	$\mathbf{B} - \mathbf{A}$ is nonnegative definite; $\mathbf{B} - \mathbf{A} \in \mathrm{NND}_n$; $\mathbf{A}$ lies below $\mathbf{B}$ with respect to the Löwner ordering
$\mathbf{A} <_{\mathsf{L}} \mathbf{B}$	$\mathbf{B} - \mathbf{A}$ is positive definite; $\mathbf{B} - \mathbf{A} \in \mathrm{PD}_n$
$\mathbf{A} \leq_{\mathsf{rs}} \mathbf{B}$	$\mathbf{A}$ and $\mathbf{B}$ are rank-subtractive; $\mathrm{rk}(\mathbf{B}-\mathbf{A}) = \mathrm{rk}(\mathbf{B}) - \mathrm{rk}(\mathbf{A})$; $\mathbf{A}$ lies below $\mathbf{B}$ with respect to the minus ordering

$\mathrm{Sh}(\mathbf{V} \mid \mathbf{X})$	the shorted matrix of $\mathbf{V} \in \mathrm{NND}_n$ with respect to $\mathbf{X}_{n\times p}$, $\mathrm{Sh}(\mathbf{V} \mid \mathbf{X})$ is the maximal element $\mathbf{U}$ (in the Löwner ordering) in the set $\mathcal{U} = \{\, \mathbf{U} : \mathbf{0} \leq_{\mathsf{L}} \mathbf{U} \leq_{\mathsf{L}} \mathbf{V},\ \mathscr{C}(\mathbf{U}) \subset \mathscr{C}(\mathbf{X}) \,\}$
$\mathbf{P}_{\mathbf{A}}$	orthogonal projector onto $\mathscr{C}(\mathbf{A})$ (w.r.t. $\mathbf{I}$): $\mathbf{P}_{\mathbf{A}} = \mathbf{A}(\mathbf{A}'\mathbf{A})^{-}\mathbf{A}' = \mathbf{A}\mathbf{A}^{+}$
$\mathbf{P}_{\mathbf{A};\mathbf{V}}$	orthogonal projector onto $\mathscr{C}(\mathbf{A})$ w.r.t. $\mathbf{V} \in \mathrm{PD}_n$: $\mathbf{P}_{\mathbf{A};\mathbf{V}} = \mathbf{A}(\mathbf{A}'\mathbf{V}\mathbf{A})^{-}\mathbf{A}'\mathbf{V}$
$\mathbf{P}_{\mathbf{A};\mathbf{V}}$	generalized orthogonal projector onto $\mathscr{C}(\mathbf{A})$ w.r.t. $\mathbf{V} \in \mathrm{NND}_n$: $\mathbf{P}_{\mathbf{A};\mathbf{V}} = \mathbf{A}(\mathbf{A}'\mathbf{V}\mathbf{A})^{-}\mathbf{A}'\mathbf{V} + \mathbf{A}[\mathbf{I} - (\mathbf{A}'\mathbf{V}\mathbf{A})^{-}\mathbf{A}'\mathbf{V}\mathbf{A}]\mathbf{U}$, where $\mathbf{U}$ is arbitrary
$\mathbf{P}_{\mathbf{A}\mid\mathbf{B}}$	projector onto $\mathscr{C}(\mathbf{A})$ along $\mathscr{C}(\mathbf{B})$: $\mathbf{P}_{\mathbf{A}\mid\mathbf{B}}(\mathbf{A} : \mathbf{B}) = (\mathbf{A} : \mathbf{0})$
$\{\mathbf{P}_{\mathbf{A}\mid\mathbf{B}}\}$	set of matrices satisfying: $\mathbf{P}_{\mathbf{A}\mid\mathbf{B}}(\mathbf{A} : \mathbf{B}) = (\mathbf{A} : \mathbf{0})$
$\mathbf{P}_{\mathcal{U}}$	orthogonal projector onto the vector space $\mathcal{U}$ (w.r.t. a given inner product)
$\mathscr{C}(\mathbf{A})$	column space of the matrix $\mathbf{A}_{n\times p}$: $\mathscr{C}(\mathbf{A}) = \{\, \mathbf{y} \in \mathbb{R}^n : \mathbf{y} = \mathbf{A}\mathbf{x} \text{ for some } \mathbf{x} \in \mathbb{R}^p \,\}$
$\mathscr{N}(\mathbf{A})$	null space of the matrix $\mathbf{A}_{n\times p}$: $\mathscr{N}(\mathbf{A}) = \{\, \mathbf{x} \in \mathbb{R}^p : \mathbf{A}\mathbf{x} = \mathbf{0} \,\}$
$\mathscr{C}(\mathbf{A})^{\perp}$	orthocomplement of $\mathscr{C}(\mathbf{A})$ w.r.t. $\mathbf{I}$: $\mathscr{C}(\mathbf{A})^{\perp} = \{\, \mathbf{z} \in \mathbb{R}^n : \mathbf{z}'\mathbf{A}\mathbf{x} = \mathbf{0}\ \forall\, \mathbf{x} \in \mathbb{R}^p \,\} = \mathscr{N}(\mathbf{A}')$
$\mathbf{A}^{\perp}$	matrix whose column space is $\mathscr{C}(\mathbf{A}^{\perp}) = \mathscr{C}(\mathbf{A})^{\perp}$
$\mathscr{C}(\mathbf{A})^{\perp}_{\mathbf{V}}$	orthocomplement of $\mathscr{C}(\mathbf{A})$ w.r.t. $\mathbf{V}$: $\mathscr{C}(\mathbf{A})^{\perp}_{\mathbf{V}} = \{\, \mathbf{z} \in \mathbb{R}^n : \mathbf{z}'\mathbf{V}\mathbf{A}\mathbf{x} = \mathbf{0}\ \forall\, \mathbf{x} \in \mathbb{R}^p \,\} = \mathscr{N}(\mathbf{A}'\mathbf{V})$
$\mathbf{A}^{\perp}_{\mathbf{V}}$	matrix whose column space is $\mathscr{C}(\mathbf{A})^{\perp}_{\mathbf{V}}$
$p_{\mathbf{A}}(x)$	the characteristic polynomial of $\mathbf{A}$: $p_{\mathbf{A}}(x) = \det(\mathbf{A} - x\mathbf{I})$
$\mathcal{U} \subset \mathcal{V}$	$\mathcal{U}$ is a subset of $\mathcal{V}$; possibly $\mathcal{U} = \mathcal{V}$
$\mathcal{U} + \mathcal{V}$	sum of the vector spaces $\mathcal{U}$ and $\mathcal{V}$
$\mathcal{U} \oplus \mathcal{V}$	direct sum of the vector spaces $\mathcal{U}$ and $\mathcal{V}$
$\mathcal{U} \boxplus \mathcal{V}$	direct sum of the orthogonal vector spaces $\mathcal{U}$ and $\mathcal{V}$
$\mathcal{U} \cap \mathcal{V}$	intersection of the vector spaces $\mathcal{U}$ and $\mathcal{V}$
$\mathrm{ch}_i(\mathbf{A}) = \lambda_i$	the ith largest eigenvalue of $\mathbf{A}_{n\times n}$ (all eigenvalues being real)
$\mathrm{ch}(\mathbf{A})$	set of all n eigenvalues of $\mathbf{A}_{n\times n}$, including multiplicities, called also the spectrum of $\mathbf{A}$: $\mathrm{ch}(\mathbf{A}) = \{\mathrm{ch}_1(\mathbf{A}), \dots, \mathrm{ch}_n(\mathbf{A})\}$
$\mathrm{ch}(\mathbf{A}, \mathbf{B})$	set of proper eigenvalues of symmetric $\mathbf{A}_{n\times n}$ with respect to $\mathbf{B} \in \mathrm{NND}_n$; $\lambda \in \mathrm{ch}(\mathbf{A}, \mathbf{B})$ if $\mathbf{A}\mathbf{w} = \lambda\mathbf{B}\mathbf{w}, \mathbf{B}\mathbf{w} \neq \mathbf{0}$
$\mathrm{nzch}(\mathbf{A})$	set of the nonzero eigenvalues of $\mathbf{A}_{n\times n}$: $\mathrm{nzch}(\mathbf{A}) = \{\mathrm{ch}_1(\mathbf{A}), \dots, \mathrm{ch}_r(\mathbf{A})\}$, $r = \mathrm{rank}(\mathbf{A})$
$\mathrm{chv}_i(\mathbf{A})$	eigenvector of $\mathbf{A}_{n\times n}$ with respect to $\lambda_i = \mathrm{ch}_i(\mathbf{A})$: a nonzero vector $\mathbf{t}_i$ satisfying the equation $\mathbf{A}\mathbf{t}_i = \lambda_i\mathbf{t}_i$
$\mathrm{sg}_i(\mathbf{A}) = \delta_i$	the ith largest singular value of $\mathbf{A}_{n\times m}$: $\mathrm{sg}_i(\mathbf{A}) = +\sqrt{\mathrm{ch}_i(\mathbf{A}'\mathbf{A})} = +\sqrt{\mathrm{ch}_i(\mathbf{A}\mathbf{A}')}$

$\mathrm{sg}(\mathbf{A})$ set of the singular values of $\mathbf{A}_{n\times m}$ $(m \le n)$: $\mathrm{sg}(\mathbf{A}) = \{\mathrm{sg}_1(\mathbf{A}), \dots, \mathrm{sg}_m(\mathbf{A})\}$

$\mathrm{nzsg}(\mathbf{A})$ set of the nonzero singular values of $\mathbf{A}_{n\times m}$: $\mathrm{nzsg}(\mathbf{A}) = \{\mathrm{sg}_1(\mathbf{A}), \dots, \mathrm{sg}_r(\mathbf{A})\}$, $r = \mathrm{rank}(\mathbf{A})$

$\rho(\mathbf{A})$ the spectral radius of $\mathbf{A}_{n\times n}$: the maximum of the absolute values of the eigenvalues of $\mathbf{A}_{n\times n}$

$\mathrm{var}_{\mathrm{s}}(y)$ sample variance of the variable y

$\mathrm{var}_{\mathrm{d}}(\mathbf{y}) = s_y^2$ sample variance: argument is the variable vector $\mathbf{y} \in \mathbb{R}^n$: $\mathrm{var}_{\mathrm{d}}(\mathbf{y}) = \frac{1}{n-1}\mathbf{y}'\mathbf{C}\mathbf{y} = \frac{1}{n-1}\sum_{i=1}^n (y_i - \bar{y})^2$

$\mathrm{cov}_{\mathrm{s}}(x, y)$ sample covariance between the variables x and y

$\mathrm{cov}_{\mathrm{d}}(\mathbf{x}, \mathbf{y}) = s_{xy}$ sample covariance: arguments are variable vectors $\in \mathbb{R}^n$: $\mathrm{cov}_{\mathrm{d}}(\mathbf{x}, \mathbf{y}) = \frac{1}{n-1}\mathbf{x}'\mathbf{C}\mathbf{y} = \frac{1}{n-1}\sum_{i=1}^n (x_i - \bar{x}_i)(y_i - \bar{y})$

$\mathrm{cor}_{\mathrm{d}}(\mathbf{x}, \mathbf{y}) = r_{xy}$ sample correlation: $r_{xy} = \mathbf{x}'\mathbf{C}\mathbf{y}/\sqrt{\mathbf{x}'\mathbf{C}\mathbf{x} \cdot \mathbf{y}'\mathbf{C}\mathbf{y}} = \cos(\mathbf{C}\mathbf{x}, \mathbf{C}\mathbf{y})$

$\bar{\bar{\mathbf{x}}}$ projection of $\mathbf{x}$ onto $\mathscr{C}(\mathbf{1}_n)$: $\bar{\bar{\mathbf{x}}} = \mathbf{J}\mathbf{x} = \bar{x}\mathbf{1}_n$

$\tilde{\mathbf{x}}$ centered $\mathbf{x}$: $\tilde{\mathbf{x}} = \mathbf{C}\mathbf{x} = \mathbf{x} - \mathbf{J}\mathbf{x} = \mathbf{x} - \bar{x}\mathbf{1}_n$

$\mathbf{U}$ $n \times d$ data matrix of the u-variables:

$$\mathbf{U} = (\mathbf{u}_1 : \dots : \mathbf{u}_d) = \begin{pmatrix} \mathbf{u}'_{(1)} \\ \vdots \\ \mathbf{u}'_{(n)} \end{pmatrix}$$

$\mathbf{u}_1, \dots, \mathbf{u}_d$ "variable vectors" in "variable space" $\mathbb{R}^n$

$\mathbf{u}_{(1)}, \dots, \mathbf{u}_{(n)}$ "observation vectors" in "observation space" $\mathbb{R}^d$

$\bar{\mathbf{u}}$ vector of means of the variables $u_1, \dots, u_d$: $\bar{\mathbf{u}} = (\bar{u}_1, \dots, \bar{u}_d)'$

$\widetilde{\mathbf{U}}$ centered $\mathbf{U}$: $\widetilde{\mathbf{U}} = \mathbf{C}\mathbf{U}$, $\mathbf{C}$ is the centering matrix

$\tilde{\mathbf{u}}_1, \dots, \tilde{\mathbf{u}}_d$ centered variable vectors

$\tilde{\mathbf{u}}_{(1)}, \dots, \tilde{\mathbf{u}}_{(n)}$ centered observation vectors

$\mathrm{var}_{\mathrm{d}}(\mathbf{u}_i) = s_i^2$ sample variance: argument is the variable vector $\mathbf{u}_i \in \mathbb{R}^n$: $\mathrm{var}_{\mathrm{d}}(\mathbf{u}_i) = \frac{1}{n-1}\mathbf{u}_i'\mathbf{C}\mathbf{u}_i = \frac{1}{n-1}\sum_{\ell=1}^n (u_{\ell i} - \bar{u}_i)^2$

$\mathrm{cov}_{\mathrm{d}}(\mathbf{u}_i, \mathbf{u}_j) = s_{ij}$ sample covariance: arguments are variable vectors $\in \mathbb{R}^n$: $s_{ij} = \frac{1}{n-1}\mathbf{u}_i'\mathbf{C}\mathbf{u}_j = \frac{1}{n-1}\sum_{\ell=1}^n (u_{\ell i} - \bar{u}_i)(u_{\ell j} - \bar{u}_j)$

$\mathrm{ssp}(\mathbf{U}) = \{t_{ij}\}$ matrix $\mathbf{T}$ $(d \times d)$ of the sums of squares and products of deviations about the mean: $\mathbf{T} = \mathbf{U}'\mathbf{C}\mathbf{U} = \sum_{i=1}^n (\mathbf{u}_{(i)} - \bar{\mathbf{u}})(\mathbf{u}_{(i)} - \bar{\mathbf{u}})'$

$\mathrm{cov}_{\mathrm{d}}(\mathbf{U}) = \{s_{ij}\}$ sample covariance matrix $\mathbf{S}$ $(d \times d)$ of the data matrix $\mathbf{U}$: $\mathbf{S} = \frac{1}{n-1}\mathbf{T} = \frac{1}{n-1}\sum_{i=1}^n (\mathbf{u}_{(i)} - \bar{\mathbf{u}})(\mathbf{u}_{(i)} - \bar{\mathbf{u}})'$

$\mathrm{cor}_{\mathrm{d}}(\mathbf{u}_i, \mathbf{u}_j) = r_{ij}$ sample correlation: arguments are variable vectors $\in \mathbb{R}^n$

$\mathrm{cor}_{\mathrm{d}}(\mathbf{U}) = \{r_{ij}\}$ sample correlation matrix $\mathbf{R}$ $(d \times d)$ of the data matrix $\mathbf{U}$: $\mathbf{R} = \mathrm{cor}_{\mathrm{d}}(\mathbf{U}) = (\mathrm{diag}\,\mathbf{S})^{-1/2}\mathbf{S}(\mathrm{diag}\,\mathbf{S})^{-1/2}$

$\mathrm{MHLN}^2(\mathbf{u}_{(i)}, \bar{\mathbf{u}}, \mathbf{S})$ sample Mahalanobis distance (squared) of the ith observation from the mean: $\mathrm{MHLN}^2(\mathbf{u}_{(i)}, \bar{\mathbf{u}}, \mathbf{S}) = (\mathbf{u}_{(i)} - \bar{\mathbf{u}})'\mathbf{S}^{-1}(\mathbf{u}_{(i)} - \bar{\mathbf{u}})$

$\mathrm{MHLN}^2(\bar{\mathbf{u}}_i, \bar{\mathbf{u}}_j, \mathbf{S}_*)$	sample Mahalanobis distance (squared) between two mean vectors: $\mathrm{MHLN}^2(\bar{\mathbf{u}}_i, \bar{\mathbf{u}}_j, \mathbf{S}_*) = (\bar{\mathbf{u}}_i - \bar{\mathbf{u}}_j)'\mathbf{S}_*^{-1}(\bar{\mathbf{u}}_i - \bar{\mathbf{u}}_j)$, where $\mathbf{S}_* = \frac{1}{n_1+n_2-2}(\mathbf{U}_1'\mathbf{C}_{n_1}\mathbf{U}_1 + \mathbf{U}_2'\mathbf{C}_{n_2}\mathbf{U}_2)$
$\mathrm{MHLN}^2(\mathbf{u}, \boldsymbol{\mu}, \boldsymbol{\Sigma})$	population Mahalanobis distance squared: $\mathrm{MHLN}^2(\mathbf{u}, \boldsymbol{\mu}, \boldsymbol{\Sigma}) = (\mathbf{u} - \boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\mathbf{u} - \boldsymbol{\mu})$
$\mathrm{E}(\cdot)$	expectation of a random argument: $\mathrm{E}(x) = p_1x_1 + \cdots + p_kx_k$ if x is a discrete random variable whose values are $x_1, \dots, x_k$ with corresponding probabilities $p_1, \dots, p_k$
$\mathrm{var}(x) = \sigma_x^2$	variance of the random variable x: $\sigma_x^2 = \mathrm{E}(x - \mu_x)^2$, $\mu_x = \mathrm{E}(x)$
$\mathrm{cov}(x, y) = \sigma_{xy}$	covariance between the random variables x and y: $\sigma_{xy} = \mathrm{E}(x - \mu_x)(y - \mu_y)$, $\mu_x = \mathrm{E}(x)$, $\mu_y = \mathrm{E}(y)$
$\mathrm{cor}(x, y) = \varrho_{xy}$	correlation between the random variables x and y: $\varrho_{xy} = \dfrac{\sigma_{xy}}{\sigma_x\sigma_y}$
$\mathrm{cov}(\mathbf{x})$	covariance matrix $(d \times d)$ of a d-dimensional random vector $\mathbf{x}$: $\mathrm{cov}(\mathbf{x}) = \boldsymbol{\Sigma} = \mathrm{E}(\mathbf{x} - \boldsymbol{\mu}_{\mathbf{x}})(\mathbf{x} - \boldsymbol{\mu}_{\mathbf{x}})'$
$\mathrm{cor}(\mathbf{x})$	correlation matrix $(d \times d)$ of the random vector $\mathbf{x}$: $\mathrm{cor}(\mathbf{x}) = \boldsymbol{\rho} = (\mathrm{diag}\,\boldsymbol{\Sigma})^{-1/2}\boldsymbol{\Sigma}(\mathrm{diag}\,\boldsymbol{\Sigma})^{-1/2}$
$\mathrm{cov}(\mathbf{x}, \mathbf{y})$	(cross-)covariance matrix between the random vectors $\mathbf{x}$ and $\mathbf{y}$: $\mathrm{cov}(\mathbf{x}, \mathbf{y}) = \mathrm{E}(\mathbf{x} - \boldsymbol{\mu}_{\mathbf{x}})(\mathbf{y} - \boldsymbol{\mu}_{\mathbf{y}})' = \boldsymbol{\Sigma}_{\mathbf{xy}}$
$\mathrm{cov}(\mathbf{x}, \mathbf{x})$	$\mathrm{cov}(\mathbf{x}, \mathbf{x}) = \mathrm{cov}(\mathbf{x})$
$\mathrm{cor}(\mathbf{x}, \mathbf{y})$	(cross-)correlation matrix between the random vectors $\mathbf{x}$ and $\mathbf{y}$
$\mathrm{cov}\binom{\mathbf{x}}{y}$	partitioned covariance matrix of the random vector $\binom{\mathbf{x}}{y}$:

$$\mathrm{cov}\begin{pmatrix}\mathbf{x}\\ y\end{pmatrix} = \begin{pmatrix}\boldsymbol{\Sigma}_{\mathbf{xx}} & \boldsymbol{\sigma}_{\mathbf{x}y}\\ \boldsymbol{\sigma}_{\mathbf{x}y}' & \sigma_y^2\end{pmatrix} = \begin{pmatrix}\mathrm{cov}(\mathbf{x}, \mathbf{x}) & \mathrm{cov}(\mathbf{x}, y)\\ \mathrm{cov}(\mathbf{x}, y)' & \mathrm{var}(y)\end{pmatrix}$$

$\mathbf{x} \sim (\boldsymbol{\mu}, \boldsymbol{\Sigma})$	$\mathrm{E}(\mathbf{x}) = \boldsymbol{\mu}$, $\mathrm{cov}(\mathbf{x}) = \boldsymbol{\Sigma}$
$\mathbf{x} \sim \mathrm{N}_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$	$\mathbf{x}$ follows the p-dimensional normal distribution $\mathrm{N}_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$
$n(\mathbf{x}; \boldsymbol{\mu}, \boldsymbol{\Sigma})$	density for $\mathbf{x} \sim \mathrm{N}_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, $\boldsymbol{\Sigma}$ pd:

$$n(\mathbf{x}; \boldsymbol{\mu}, \boldsymbol{\Sigma}) = \frac{1}{(2\pi)^{p/2}|\boldsymbol{\Sigma}|^{1/2}}\mathrm{e}^{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})'\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})}$$

$\mathrm{cc}_i(\mathbf{x}, \mathbf{y})$	ith largest canonical correlation between the random vectors $\mathbf{x}$ and $\mathbf{y}$
$\mathrm{cc}(\mathbf{x}, \mathbf{y})$	set of the canonical correlations between the random vectors $\mathbf{x}$ and $\mathbf{y}$
$\mathrm{cc}_+(\mathbf{x}, \mathbf{y})$	set of the nonzero (necessarily positive) canonical correlations between the random vectors $\mathbf{x}$ and $\mathbf{y}$; square roots of the nonzero eigenvalues of $\mathbf{P_A P_B}$:

$$\mathrm{cc}_+(\mathbf{x}, \mathbf{y}) = \mathrm{nzch}^{1/2}(\mathbf{P_A P_B}) = \mathrm{nzsg}[(\mathbf{A'A})^{+1/2}\mathbf{A'B}(\mathbf{B'B})^{+1/2}],$$

$$\mathrm{cov}\begin{pmatrix}\mathbf{x}\\ \mathbf{y}\end{pmatrix} = \begin{pmatrix}\mathbf{A'A} & \mathbf{A'B}\\ \mathbf{B'A} & \mathbf{B'B}\end{pmatrix}$$

$\mathbf{X} = (\mathbf{1} : \mathbf{X}_0)$	in regression context often the model matrix

$\mathbf{X}_0$ — $n \times k$ data matrix of the x-variables:

$$\mathbf{X}_0 = (\mathbf{x}_1 : \ldots : \mathbf{x}_k) = \begin{pmatrix} \mathbf{x}'_{(1)} \\ \vdots \\ \mathbf{x}'_{(n)} \end{pmatrix}$$

$\mathbf{x}_1, \ldots, \mathbf{x}_k$ — variable vectors in the variable space $\mathbb{R}^n$

$\mathbf{x}_{(1)}, \ldots, \mathbf{x}_{(n)}$ — observation vectors in the observation space $\mathbb{R}^k$

$\mathrm{ssp}(\mathbf{X}_0 : \mathbf{y})$ — partitioned matrix of the sums of squares and products of deviations about the mean of data $(\mathbf{X}_0 : \mathbf{y})$:

$$\mathrm{ssp}(\mathbf{X}_0 : \mathbf{y}) = \begin{pmatrix} \mathbf{T}_{\mathbf{xx}} & \mathbf{t}_{\mathbf{x}y} \\ \mathbf{t}'_{\mathbf{x}y} & t_{yy} \end{pmatrix} = (\mathbf{X}_0 : \mathbf{y})'\mathbf{C}(\mathbf{X}_0 : \mathbf{y})$$

$\mathrm{cov_d}(\mathbf{X}_0 : \mathbf{y})$ — partitioned sample covariance matrix of data $(\mathbf{X}_0 : \mathbf{y})$:

$$\mathrm{cov_d}(\mathbf{X}_0 : \mathbf{y}) = \begin{pmatrix} \mathbf{S}_{\mathbf{xx}} & \mathbf{s}_{\mathbf{x}y} \\ \mathbf{s}'_{\mathbf{x}y} & s_y^2 \end{pmatrix}$$

$\mathrm{cor_d}(\mathbf{X}_0 : \mathbf{y})$ — partitioned sample correlation matrix of data $(\mathbf{X}_0 : \mathbf{y})$:

$$\mathrm{cor_d}(\mathbf{X}_0 : \mathbf{y}) = \begin{pmatrix} \mathbf{R}_{\mathbf{xx}} & \mathbf{r}_{\mathbf{x}y} \\ \mathbf{r}'_{\mathbf{x}y} & 1 \end{pmatrix}$$

$\mathbf{H}$ — orthogonal projector onto $\mathscr{C}(\mathbf{X})$, the hat matrix: $\mathbf{H} = \mathbf{X}(\mathbf{X}'\mathbf{X})^-\mathbf{X}' = \mathbf{X}\mathbf{X}^+ = \mathbf{P}_\mathbf{X}$

$\mathbf{M}$ — orthogonal projector onto $\mathscr{C}(\mathbf{X})^\perp$: $\mathbf{M} = \mathbf{I}_n - \mathbf{H}$

$\mathbf{J}$ — the orthogonal projector onto $\mathscr{C}(\mathbf{1}_n)$: $\mathbf{J} = \frac{1}{n}\mathbf{1}_n\mathbf{1}'_n = \mathbf{P}_{\mathbf{1}_n}$

$\mathbf{C}$ — centering matrix, the orthogonal projector onto $\mathscr{C}(\mathbf{1}_n)^\perp$: $\mathbf{C} = \mathbf{I}_n - \mathbf{J}$

$(\mathbf{X}_1 : \mathbf{X}_2)$ — partitioned model matrix $\mathbf{X}$

$\mathbf{M}_1$ — orthogonal projector onto $\mathscr{C}(\mathbf{X}_1)^\perp$: $\mathbf{M}_1 = \mathbf{I}_n - \mathbf{P}_{\mathbf{X}_1}$

$\hat{\boldsymbol\beta}$ — solution to normal equation $\mathbf{X}'\mathbf{X}\boldsymbol\beta = \mathbf{X}'\mathbf{y}$, $\mathrm{OLSE}(\boldsymbol\beta)$

$\mathbf{X}\hat{\boldsymbol\beta} = \hat{\mathbf{y}}$ — $\hat{\mathbf{y}} = \mathbf{H}\mathbf{y} =$ OLS fitted values, $\mathrm{OLSE}(\mathbf{X}\boldsymbol\beta)$, denoted also $\widehat{\mathbf{X}\boldsymbol\beta} = \hat{\boldsymbol\mu}$, when $\boldsymbol\mu = \mathbf{X}\boldsymbol\beta$

$\tilde{\boldsymbol\beta}$ — solution to generalized normal equation $\mathbf{X}'\mathbf{W}^-\mathbf{X}\boldsymbol\beta = \mathbf{X}'\mathbf{W}^-\mathbf{y}$, where $\mathbf{W} = \mathbf{V} + \mathbf{X}\mathbf{U}\mathbf{X}'$, $\mathscr{C}(\mathbf{W}) = \mathscr{C}(\mathbf{X} : \mathbf{V})$

$\tilde{\boldsymbol\beta}$ — if $\mathbf{V}$ is positive definite and $\mathbf{X}$ has full column rank, then $\tilde{\boldsymbol\beta} = \mathrm{BLUE}(\boldsymbol\beta) = (\mathbf{X}'\mathbf{V}^{-1}\mathbf{X})^{-1}\mathbf{X}'\mathbf{V}^{-1}\mathbf{y}$

$\mathbf{X}\tilde{\boldsymbol\beta}$ — $\mathrm{BLUE}(\mathbf{X}\boldsymbol\beta)$, denoted also $\widetilde{\mathbf{X}\boldsymbol\beta} = \tilde{\boldsymbol\mu}$

$\bar{y}$ — mean of the response variable y: $\bar{y} = (y_1 + \cdots + y_n)/n$

$\bar{\mathbf{x}}$ — vector of the means of k regressor variables $x_1, \ldots, x_k$: $\bar{\mathbf{x}} = (\bar{x}_1, \ldots, \bar{x}_k)' \in \mathbb{R}^k$

$\bar{\bar{\mathbf{y}}}$ — projection of $\mathbf{y}$ onto $\mathscr{C}(\mathbf{1}_n)$: $\bar{\bar{\mathbf{y}}} = \mathbf{J}\mathbf{y} = \bar{y}\mathbf{1}_n$

$\tilde{\mathbf{y}}$	centered $\mathbf{y}$, $\tilde{\mathbf{y}} = \mathbf{Cy} = \mathbf{y} - \bar{\bar{\mathbf{y}}}$
$\hat{\boldsymbol{\beta}}_{\mathbf{x}}$	$\hat{\boldsymbol{\beta}}_{\mathbf{x}} = \mathbf{T}_{\mathbf{xx}}^{-1}\mathbf{t}_{\mathbf{x}y} = \mathbf{S}_{\mathbf{xx}}^{-1}\mathbf{s}_{\mathbf{x}y}$: the OLS-regression coefficients of x-variables when $\mathbf{X} = (\mathbf{1} : \mathbf{X}_0)$
$\hat{\beta}_0$	$\hat{\beta}_0 = \bar{y} - \hat{\boldsymbol{\beta}}_{\mathbf{x}}'\bar{\mathbf{x}} = \bar{y} - (\hat{\beta}_1\bar{x}_1 + \cdots + \hat{\beta}_k\bar{x}_k)$: OLSE of the constant term (intercept) when $\mathbf{X} = (\mathbf{1} : \mathbf{X}_0)$
$\mathrm{BLP}(\mathbf{y};\mathbf{x})$	the best linear predictor of the random vector $\mathbf{y}$ on the basis of the random vector $\mathbf{x}$
$\mathrm{BLUE}(\mathbf{K}'\boldsymbol{\beta})$	the best linear unbiased estimator of estimable parametric function $\mathbf{K}'\boldsymbol{\beta}$, denoted as $\mathbf{K}'\tilde{\boldsymbol{\beta}}$ or $\widetilde{\mathbf{K}'\boldsymbol{\beta}}$
$\mathrm{BLUP}(\mathbf{y}_f;\mathbf{y})$	the best linear unbiased predictor of a new unobserved $\mathbf{y}_f$
$\mathrm{LE}(\mathbf{K}'\boldsymbol{\beta};\mathbf{y})$	(homogeneous) linear estimator of $\mathbf{K}'\boldsymbol{\beta}$, where $\mathbf{K} \in \mathbb{R}^{p\times q}$: $\{\mathrm{LE}(\mathbf{K}'\boldsymbol{\beta};\mathbf{y})\} = \{\, \mathbf{Ay} : \mathbf{A} \in \mathbb{R}^{q\times n}\,\}$
$\mathrm{LP}(\mathbf{y};\mathbf{x})$	(inhomogeneous) linear predictor of the p-dimensional random vector $\mathbf{y}$ on the basis of the q-dimensional random vector $\mathbf{x}$: $\{\mathrm{LP}(\mathbf{y};\mathbf{x})\} = \{\, f(\mathbf{x}) : f(\mathbf{x}) = \mathbf{Ax} + \mathbf{a},\ \mathbf{A} \in \mathbb{R}^{p\times q},\ \mathbf{a} \in \mathbb{R}^p\,\}$
$\mathrm{LUE}(\mathbf{K}'\boldsymbol{\beta};\mathbf{y})$	(homogeneous) linear unbiased estimator of $\mathbf{K}'\boldsymbol{\beta}$: $\{\mathrm{LUE}(\mathbf{K}'\boldsymbol{\beta};\mathbf{y})\} = \{\, \mathbf{Ay} : \mathrm{E}(\mathbf{Ay}) = \mathbf{K}'\boldsymbol{\beta}\,\}$
$\mathrm{LUP}(\mathbf{y}_f;\mathbf{y})$	linear unbiased predictor of a new unobserved $\mathbf{y}_f$: $\{\mathrm{LUP}(\mathbf{y}_f;\mathbf{y})\} = \{\, \mathbf{Ay} : \mathrm{E}(\mathbf{Ay} - \mathbf{y}_f) = \mathbf{0}\,\}$
$\mathrm{MSEM}(f(\mathbf{x});\mathbf{y})$	mean squared error matrix of $f(\mathbf{x})$ (= random vector, function of the random vector $\mathbf{x}$) with respect to $\mathbf{y}$ (= random vector or a given fixed vector): $\mathrm{MSEM}[f(\mathbf{x});\mathbf{y}] = \mathrm{E}[\mathbf{y} - f(\mathbf{x})][\mathbf{y} - f(\mathbf{x})]'$
$\mathrm{MSEM}(\mathbf{Fy};\mathbf{K}'\boldsymbol{\beta})$	mean squared error matrix of the linear estimator $\mathbf{Fy}$ under $\{\mathbf{y},\, \mathbf{X}\boldsymbol{\beta},\, \sigma^2\mathbf{V}\}$ with respect to $\mathbf{K}'\boldsymbol{\beta}$: $\mathrm{MSEM}(\mathbf{Fy};\mathbf{K}'\boldsymbol{\beta}) = \mathrm{E}(\mathbf{Fy} - \mathbf{K}'\boldsymbol{\beta})(\mathbf{Fy} - \mathbf{K}'\boldsymbol{\beta})'$
$\mathrm{OLSE}(\mathbf{K}'\boldsymbol{\beta})$	the ordinary least squares estimator of parametric function $\mathbf{K}'\boldsymbol{\beta}$, denoted as $\mathbf{K}'\hat{\boldsymbol{\beta}}$ or $\widehat{\mathbf{K}'\boldsymbol{\beta}}$; here $\hat{\boldsymbol{\beta}}$ is any solution to the normal equation $\mathbf{X}'\mathbf{X}\boldsymbol{\beta} = \mathbf{X}'\mathbf{y}$
$\mathrm{risk}(\mathbf{Fy};\mathbf{K}'\boldsymbol{\beta})$	quadratic risk of $\mathbf{Fy}$ under $\{\mathbf{y},\, \mathbf{X}\boldsymbol{\beta},\, \sigma^2\mathbf{V}\}$ with respect to $\mathbf{K}'\boldsymbol{\beta}$: $\mathrm{risk}(\mathbf{Fy};\mathbf{K}'\boldsymbol{\beta}) = \mathrm{tr}[\mathrm{MSEM}(\mathbf{Fy};\mathbf{K}'\boldsymbol{\beta})] = \mathrm{E}(\mathbf{Fy} - \mathbf{K}'\boldsymbol{\beta})'(\mathbf{Fy} - \mathbf{K}'\boldsymbol{\beta})$
$\mathscr{M}$	linear model: $\{\mathbf{y},\, \mathbf{X}\boldsymbol{\beta},\, \sigma^2\mathbf{V}\}$: $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$, $\mathrm{cov}(\mathbf{y}) = \mathrm{cov}(\boldsymbol{\varepsilon}) = \sigma^2\mathbf{V}$, $\mathrm{E}(\mathbf{y}) = \mathbf{X}\boldsymbol{\beta}$
$\mathscr{M}_{\mathrm{mix}}$	mixed linear model: $\mathscr{M}_{\mathrm{mix}} = \{\mathbf{y},\, \mathbf{X}\boldsymbol{\beta} + \mathbf{Z}\boldsymbol{\gamma},\, \mathbf{D},\, \mathbf{R}\}$: $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{Z}\boldsymbol{\gamma} + \boldsymbol{\varepsilon}$; $\boldsymbol{\gamma}$ is the vector of the random effects, $\mathrm{cov}(\boldsymbol{\gamma}) = \mathbf{D}$, $\mathrm{cov}(\boldsymbol{\varepsilon}) = \mathbf{R}$, $\mathrm{cov}(\boldsymbol{\gamma}, \boldsymbol{\varepsilon}) = \mathbf{0}$, $\mathrm{E}(\mathbf{y}) = \mathbf{X}\boldsymbol{\beta}$
$\mathscr{M}_f$	linear model with new future observations $\mathbf{y}_f$:

$$\mathscr{M}_f = \left\{ \begin{pmatrix} \mathbf{y} \\ \mathbf{y}_f \end{pmatrix}, \begin{pmatrix} \mathbf{X}\boldsymbol{\beta} \\ \mathbf{X}_f\boldsymbol{\beta} \end{pmatrix}, \sigma^2 \begin{pmatrix} \mathbf{V} & \mathbf{V}_{12} \\ \mathbf{V}_{21} & \mathbf{V}_{22} \end{pmatrix} \right\}$$