Matemaattinen tilastotiede
2. välikoe, 13.12.2004 klo 9-11.30 ls Pinni A1081
Ratkaisut

1. Harjoitus 7.1

2. (a) Jos $f(y)$ on tiheysfunktio, niin

$$a\int_{-\infty}^{\infty} f(y)\,\mathrm{d}y = a\int_{0}^{1}(y^2-y+1)\,\mathrm{d}y = 1.$$

   Silloin $a=\frac{6}{5}$.

   (b)

$$P(X>2)=1-P(X\le 2)=1-F(x)=e^{-4}=0.0183.$$

   (c) Mediaani toteuttaa ehdon

$$F(\pi_{0.5})=1-e^{-\pi_{0.5}^2}=0.5,$$

   joten $\pi_{0.5}=\sqrt{-\ln 0.5}=0.8325$.

3. (a) $P(-1/8\le X\le 1/8)=\frac{1}{2}\int_{-1/8}^{1/8}\mathrm{d}x=\frac{1}{2}(\frac{1}{8}-(-\frac{1}{8}))=\frac{1}{8}$.

   (b)

$$F(x)=\begin{cases}0, & x\le -1\\ (x+1)/2, & -1<x<x\\ 1, & x\ge 1\end{cases}$$

   (c)

$$\begin{aligned}P(-1/8\le Y\le 1/8) &= P(-1/8\le X^3\le 1/8)\\ &= P(-\sqrt[3]{1/8}\le X\le \sqrt[3]{1/8})\\ &= P(-1/2\le X\le 1/2)=1/2.\end{aligned}$$

   (d)

$$\begin{aligned}F_Y(y) &= P(Y\le y)=P(X^3\le y)\\ &= P(X\le \sqrt[3]{y})=F_X(\sqrt[3]{y})\\ &= (\sqrt[3]{y}+1)/2, \quad \text{kun} \quad -1<y<1.\end{aligned}$$

   Siis

$$F_Y(y)=\begin{cases}0, & y\le -1\\ (\sqrt[3]{y}+1)/2, & -1<y<1\\ 1, & y\ge 1\end{cases}$$

(e)

$$f_Y(y) = F'(y) = \begin{cases} \frac{1}{6\sqrt[3]{y^2}}, & -1 < y < 1 \\ 0, & \text{muualla} \end{cases}$$

Huomaa, että $f_Y(y) \to \infty$, kun $y \to 0$.

4. a)

$$\begin{aligned} E(Y \mid X = 37.25) &= \mu_Y + \frac{\rho\sigma_Y}{\sigma_X}(x - \mu_X) \\ &= 33.38 + \frac{0.829 \cdot 7.711}{6.769}(37.25 - 31.25) = 39.4962 \end{aligned}$$

b)

$$\begin{aligned} E(X \mid Y = 39.38) &= \mu_X + \frac{\rho\sigma_X}{\sigma_Y}(y - \mu_Y) \\ &= 31.25 + \frac{0.829 \cdot 6.769}{7.711}(39.38 - 33.83) = 35.2889 \end{aligned}$$

c)

$$\sigma^2_{X|Y} = \sigma^2_X\left(1 - \rho^2\right) = 6.769^2(1 - 0.829^2) = 14.3304$$

$X_y = X \mid Y = 39.38$ noudattaa $N(35.2889, 14.3304)$, joten

$$Z = \frac{X_y - 35.2889}{\sqrt{14.3304}} \sim N(0,1).$$

Silloin

$$P(X_y > 37.25) = P\left(Z > \frac{37.25 - 35.2889}{\sqrt{14.3304}}\right) = 1 - \Phi(0.5180) = 0.3015.$$