TILTA1B Matemaattisen tilastotieteen perusteet
Syksy 2009
Ratkaisut harjoitus 4

1. Olk. X tuotto prosentteina.

$X \sim N(10{,}12^2)$

$P(X \geq 100 \cdot \frac{750}{6000}) = 1 - \Phi(\frac{12{,}5-10}{12}) \approx 0{,}417$

2.

$Y = |X - \mu|$

$G_Y(y) = P(Y \leq y) = P(-y \leq X - \mu \leq y) = P(-\frac{y}{\sigma} \leq Z \leq \frac{y}{\sigma}) = 2\Phi(\frac{y}{\sigma}) - 1, y \geq 0$

$g_Y(y) = G_Y'(y) = 2 \cdot \frac{1}{\sigma} \cdot \Phi'(\frac{y}{\sigma}) = \begin{cases} \frac{2}{\sigma}\frac{1}{\sqrt{2\pi}} e^{-y^2/(2\sigma^2)}, y \geq 0 \\ 0, muualla \end{cases}$

$E(Y) = \int_0^\infty y \cdot \frac{2}{\sigma}\frac{1}{\sqrt{2\pi}} e^{-y^2/(2\sigma^2)} dy = \sigma \cdot \sqrt{\frac{2}{\pi}} \int_0^\infty \frac{y}{\sigma^2} e^{-y^2/(2\sigma^2)} dy = \sigma\sqrt{\frac{2}{\pi}} \Big/_0^\infty e^{-y^2/(2\sigma^2)} = \sigma\sqrt{\frac{2}{\pi}}$

3. $X \sim N(-6{,}8^2)$

a) $P(-4 \leq X \leq 16) = \Phi(\frac{22}{8}) - \Phi(\frac{1}{4}) \approx 0{,}398$

b) $P(-10 \leq X \leq 0) = \Phi(\frac{3}{4}) - \Phi(-\frac{1}{2}) \approx 0{,}464$

c) $P(|X - 8| \leq c) = P(-c \leq X - 8 \leq c) = \Phi(\frac{-c+14}{8}) - \Phi(\frac{c+14}{8}) = 0{,}95$

$\Leftrightarrow c \approx 27{,}16$

c ratkeaa kokeilemalla, esim. R:llä:

```
c<-seq(25,30,by=0.01)
a<-pnorm(c/8+14/8)-pnorm(-c/8+14/8)
cbind(c,a)
           c         a
  [1,] 25.00 0.9154337
  [2,] 25.01 0.9156273
…
[216,] 27.15 0.9498859
[217,] 27.16 0.9500150
[218,] 27.17 0.9501437
…
[500,] 29.99 0.9771823
[501,] 30.00 0.9772498
```

4. Kaavan 6.5.4 (luontomonisteen s. 164) mukaan

a) $f_U(u) = f_x(\sqrt{u})\frac{1}{2\sqrt{u}} + f_x(-\sqrt{u})\frac{1}{2\sqrt{u}}$

$= \frac{1}{\sigma\sqrt{2\pi}} e^{-(\sqrt{u}-\mu)^2/(2\sigma^2)} + \frac{1}{\sigma\sqrt{2\pi}} e^{-(-\sqrt{u}-\mu)^2/(2\sigma^2)} = \ldots = \frac{1}{\sqrt{\pi} \cdot 8^{1/2}} \cdot u^{1/2} e^{-u/8}$

$\Rightarrow U \sim Gamma(\frac{1}{2}, 8)$

b) $X - 7 \sim N(0{,}4)$, joten a-kohdan perusteella

$(X-7)^2 \sim Gamma(\frac{1}{2}, 8)$

R:llä:

```
> pgamma(20.096,shape=0.5,scale=8)-pgamma(15.364,shape=0.5,scale=8)
[1] 0.02501533
```

5. $W \sim Gamma(31, \frac{1}{12})$

$E(W) = 31 \cdot \frac{1}{12} \approx 2{,}583$ (tuntia)

6.

a) $f(x)=\frac{1}{\Gamma(\frac{r}{2})2^{r/2}}x^{r/2-1}e^{-x/2}$

$f'(x)=\frac{1}{\Gamma(\frac{r}{2})2^{r/2}}\left[(\frac{r}{2}-1)e^{-x/2}-\frac{1}{2}x^{r/2-2}e^{-x/2}\right]=0,$

kun $x=0$ tai $x=r-2$

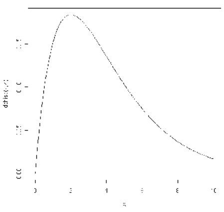
$f'(x)>0, x<r-2$

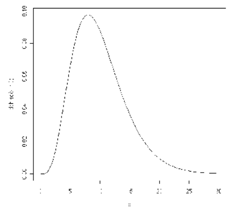
$f'(x)<0, x>r-2$

Siis *r-2* on maksimi.

b)
```
> qchisq(0.5,4)
  [1] 3.356694
```

c)
```
> curve(dchisq(x,4),0,10)
> curve(dchisq(x,10),0,30)
```





7.

$G_Y(y)=P(Y\le y)=P(-2\ln X\le y)=1-P(X<e^{-\frac{1}{2}y})=1-e^{-\frac{1}{2}y}$

$g_Y(y)=G_Y'(y)=\begin{cases}\frac{1}{2}e^{-\frac{1}{2}y}, y>0\\ 0, muualla\end{cases}$

$X \sim Khi2(2)$

$f_X(x)=\frac{1}{\Gamma(\frac{2}{2})2}x^{2/2-1}e^{-x/2}=\frac{1}{2}e^{-\frac{1}{2}x}$

$\Rightarrow Y \sim X$

8.

a) $\int_{-\infty}^{\infty} f(x)dx=\int_{-\infty}^{0} k\cdot e^{x}dx+\int_{0}^{\infty} k\cdot e^{-x}dx=...=2k=1$

$\Rightarrow k=\frac{1}{2}$

b) *E(X)=0*, sillä tiheysfunktio f on origon suhteen symmetrinen: $f(x)=f(-x), \forall x\in\Re$.

$E(X^2)=\int_{-\infty}^{\infty} x^2 f(x)dx=2\cdot\int_0^{\infty}\frac{1}{2}x^2e^{-x}dx=\int_0^{\infty}x^2e^{-x}dx=\Gamma(3)=2$