TILTA1B Matemaattisen tilastotieteen perusteet
Syksy 2009
Ratkaisut harjoitus 1

1.

a) $Y = 6 - X$

$P(X = Y) = P(X = 3) = \binom{6}{3} p^3 (1-p)^3$

b) $P(X + Y = 6) = 1$

$f_{X+Y}(v) = \begin{cases} 1, v = 6 \\ 0, muulloin \end{cases}$

c) $X \sim Y$, jos $f_x(v) = f_y(v)$, $v = 0,1,\ldots,6$

$f_x(0) = f_y(0) \Leftrightarrow p^6 = (1-p)^6 \Leftrightarrow p = \frac{1}{2}$

2. Olk. X poikien lkm,
A= "6 ensimmäistä poikia, loput neljä tyttöjä"

$P(A|X = 6) = \frac{P(A \cap \{X = 6\})}{P(X = 6)} = \frac{P(A)}{P(X = 6)} = \frac{p^{10}}{\binom{10}{6} p^6 p^4} = \frac{1}{\binom{10}{6}} \approx 0{,}0048$

3.

a)
$X \sim Ber(\frac{2}{3})$
$E(X) = \frac{2}{3}$
$Var(X) = \frac{2}{3} \cdot (1 - \frac{2}{3}) = \frac{2}{9}$

b)
$X \sim Bin(12, \frac{3}{4})$
$E(X) = 12 \cdot \frac{3}{4} = 9$
$Var(X) = 12 \cdot \frac{3}{4} \cdot (1 - \frac{3}{4}) = \cdot \frac{9}{4}$

4.

$f(x) = \begin{cases} \frac{2}{5}, x = 1,3 \\ \frac{1}{5}, x = 2 \\ 0, muulloin \end{cases}$

$E(X) = 1 \cdot \frac{2}{5} + 2 \cdot \frac{1}{5} + 3 \cdot \frac{2}{5} = 2$

$Var(X) = E(X^2) - E(X)^2 = 1^2 \cdot \frac{2}{5} + 2^2 \cdot \frac{1}{5} + 3^2 \cdot \frac{2}{5} - 2^2 = \frac{4}{5}$

5.

$X \sim Tasd(1,10)$

$f(x) = \begin{cases} \frac{1}{10}, x = 1,2,\ldots,10 \\ 0, muulloin \end{cases}$

$P(1 \le x \le 2) = P(X = 1) + P(x = 2) = \frac{1}{10} + \frac{1}{10} = \frac{1}{5}$

6.

$X \sim Bin(400000, \frac{3}{4})$

$P(299000 \le X \le 301000) = P(X \le 301000) - P(X < 299000) = F_x(301000) - F_x(299000-1)$

R:llä:
```
> pbinom(301000,400000,3/4)-pbinom(299000-1,400000,3/4)
[1] 0.9997411
```

Tai:

$P(299000 \le X \le 301000) = \sum_{i=299000}^{301000} f_x(i)$

R:llä:
```
> sum(dbinom(299000:301000,400000,3/4))
[1] 0.9997411
```

7. $Z_i = \begin{cases} 1, \; jos X_i = X_{i+1} = 1 \\ 0, muulloin \end{cases}$

$P(Z_i = 1) = p^2$

$P(Z_i = 0) = 1 - p^2$

$E(Z_i) = p^2$

a) $E(Z) = E(Z_1) + E(Z_2) + \cdots + E(Z_{n-1}) = (n-1)p^2$

b) $E(Z) = 199 \cdot \frac{1}{4} = 49{,}75$

E(Z) on odotusarvo peräkkäisten kruunujen (RR-toistojen) lukumäärälle 200 heitossa.

8.

$P(S_3 = 2) = P(S_2 = 2, X_3 = 0) + P(S_2 = 1, X_3 = 1)$

$= P(S_2 = 2) \cdot P(X_3 = 0) + P(S_2 = 1) \cdot P(X_3 = 1)$

$= [P(S_1 = 2, X_2 = 0) + P(S_1 = 1, X_2 = 1)] \cdot P(X_3 = 0) + [P(S_1 = 1, X_2 = 0) + P(S_1 = 0, X_2 = 1)] \cdot P(X_3 = 1)$

$= [P(X_1 = 2) \cdot P(X_2 = 0) + P(X_1 = 1) \cdot P(X_2 = 1)] \cdot P(X_3 = 0) + [P(X_1 = 1) \cdot P(X_2 = 0) + P(X_1 = 0) \cdot P(X_2 = 1)] \cdot P(X_3 = 1)$

$= p^2(1-p) + [p(1-p) + (1-p)p]p$

$= 3p^2(1-p)$