TILTA1B Matemaattisen tilastotieteen perusteet
Ratkaisut harjoitus 4
48. viikko 2008

1. a) $f_X(x) = \sum_{y=1}^{4} = \frac{x+y}{32} = \frac{2x+5}{16}$, $x = 1, 2$
   $f_Y(y) = \sum_{x=1}^{2} = \frac{x+y}{32} = \frac{3+2y}{32}$, $y = 1, 2, 3, 4$

   b) $E(X) = \frac{25}{16}$, $E(Y) = \frac{45}{16}$, $E(XY) = \frac{35}{8}$
   $Cov(X,Y) = E(XY) - E(X)E(Y) = -\frac{5}{256}$
   $Var(X) = \frac{63}{256}$, $Var(Y) = \frac{295}{256}$
   $Cor(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}} \approx -0.0367$

2. a) $f(y|x) = \frac{f(x,y)}{f_X(x)}$, $y = x, x+1, \ldots, 9$, joten
   $f(x,y) = \frac{1}{10(10-x)}$, $x = 0, \ldots, 9$ ja $y = x, x+1, \ldots, 9$

   b) $f_Y(y) = \frac{1}{10}\sum_{x=0}^{y} \frac{1}{10-x}$, $y = 0, \ldots, 9$
   $E(Y|x) = \sum_{y=x}^{9} = yf(y|x) = \ldots = \frac{x+9}{2}$

3. $X \sim \text{Bin}(3, \frac{1}{6})$ ja $Y \sim \text{Bin}(3, \frac{1}{2})$, joten
   $E(X) = \frac{1}{2}$, $E(Y) = \frac{3}{2}$,
   $Var(X) = \frac{5}{12}$, $Var(Y) = \frac{3}{4}$
   $Cov(X,Y) = -np_ip_j = -\frac{1}{4}$

   $Cor(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}} = -\frac{1}{\sqrt{5}}$

4. a) $f_X(-1) = 2a + b, f_X(0) = 2b, f_X(1) = 2a + b$
   $f_Y(-1) = 2a + b, f_Y(0) = 2b, f_Y(1) = 2a + b$

   b) E(X) = E(Y) = E(XY) = 0, joten
   $Cov(X,Y) = 0$, mutta X ja Y eivät ole riippumattomia, sillä esimerkiksi
   $f(0,0) = 0$, mutta $f_X(0)f_Y(0) = 4b^2 \neq 0$

5. $P(X < Y) = P(0 < X < Y) = \int_0^1 \int_0^y (x+y)\text{dxdy} = \ldots = \frac{1}{2}$

6. $P(X < 1) = \int_0^1 \int_0^\infty (2e^{-x}e^{-2y})\text{dydx} = \ldots = 1 - e^{-1}$

8. a) $f_X(x) = xe^{-x}$, $x > 0$
   $f_Y(y) = e^{-y}$

   b) $f(y|x) = e^{-y}$ $x > 0, y > 0$

   c) $P(X > \ln 4) = \int_{\ln 4}^\infty \int_0^\infty xe^{-x}\text{dydx} = \ldots = \frac{1+\ln 4}{4}$