# TILTA1B Matemaattisen tilastotieteen perusteet
# Ratkaisut harjoitus 3
# 47. viikko 2008

1. a) Ratkaisemalla $\int_0^1 \frac{x^2-x+1}{c} \mathrm{dx} = 1$ saadaan, että $c = \frac{5}{6}$

   b) $P(X > \frac{3}{4}) = 1 - \int_0^{\frac{3}{4}} \frac{6}{5}(x^2 - x + 1)\mathrm{dx} \approx 0.269$

   c) $\mathrm{E(X)} = \int_0^1 x\frac{6}{5}(x^2 - x + 1)\mathrm{dx} = \ldots = 0.5$

   $E(X^2) = \int_0^1 x^2\frac{6}{5}(x^2 - x + 1)\mathrm{dx} = \ldots = \frac{17}{50}$

   $\mathrm{Var(X)} = E(X^2) - (E(X))^2 = 0.09$

2. $Y = X^3$; $X = y^{1/3} = g(y)$; $g'(y) = \frac{1}{3}y^{-\frac{2}{3}}$, joten

   $f_Y(y) = f_X[g(y)]\,|g'(y)| = \frac{1}{27}, 0 < y < 27.$

3. $f_Y(y) = f_X[g(y)]\,|g'(y)| = 1 \times \left|-\frac{1}{2}e^{-\frac{y}{2}}\right| = \frac{1}{2}e^{-\frac{y}{2}}$, $y > 0$.

   Eli $Y \sim$ Khi2(2).

4. $g(y) = y^2$, $g'(y) = 2y$

   $f_Y(y) = f_X[g(y)]\,|g'(y)| = \frac{1}{8}y^5e^{-\frac{y^2}{2}}$, $0 < y < \infty$

5. $F_Y(Y) = P(Y \leq y) = P(e^X \leq y) = P(X \leq \ln y) = F_X(\ln y)$

   $f_Y(y) = F_Y'(y) = F_X'(\ln y)\frac{1}{y} = f_X(\ln y)\frac{1}{y} = \frac{1}{y\sigma\sqrt{2\pi}}e^{-\frac{1}{2\sigma^2}(\ln y-\mu)^2}$

6. a) $f_Y(y) = \frac{1}{2r}y^{\frac{1}{r}-1}$, $-1 \leq y \leq 1$

   b) $P(Y \leq y) = P(X^r \leq y) = P(0 \leq X^r \leq y) = P(-y^{\frac{1}{r}} \leq X \leq y^{\frac{1}{r}}) = F_X(y^{\frac{1}{r}}) - F_X(-y^{\frac{1}{r}}) = y^{\frac{1}{r}}$

   $f_Y(y) = F_Y'(y) = \frac{1}{r}y^{\frac{1}{r}-1}$, $0 \leq y \leq 1$

7. $M_X(t) = M_Y(t) = \frac{1}{1-t}, t < 1$

   a) $M_U(t) = E(e^{tU}) = E(e^{t(X+Y)}) = E(e^{tX})E(e^{tY}) = (\frac{1}{1-t})^2$

   $U \sim \mathrm{Gamma}(2, 1)$

   b) $f_U(u) = xe^{-x}$

   c) E(U) = Var(X) =2

8. a) $X \sim N(\dfrac{5\mu - 160}{9}, \dfrac{25\sigma^2}{81})$

   b) $a \approx 32.22$ ja $b \approx 35$.