TILTA1B Matemaattisen tilastotieteen perusteet
Ratkaisut harjoitus 1
45. viikko 2008

1.

$$
\begin{aligned}
M(t) &= (q + pe^t)^n \\
M'(t) &= npe^t(q + pe^t)^{n-1} \\
M''(t) &= npe^t(q + pe^t)^{n-1} + n(n-1)(pe^t)^2(q + pe^t)^{n-2}, \quad n \geq 2 \\
E(X) &= M'(0) = np \\
Var(X) &= E(X^2) - (E(X))^2 = M''(0) - (M'(0))^2 = npq
\end{aligned}
$$

2.
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dpois(0:10, 1)
dbinom(0:10, 100, 0.01)
dpois(0:10, 1)/dbinom(0:10, 100, 0.01)
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3. (a) $X \sim Geo(0.1)$
   $E(X) = \frac{1}{p} = 10$
   (b) Y='Onnistuneiden puheluiden lkm'
   $Y \sim Bin(10, 0.1)$
   $P(Y \geq 2) = 1 - P(Y < 2) = 1 - P(Y = 0) - P(Y = 1) = 0.2639$
   (c) Z='Kolmeen onnistumiseen tarvittavien puheluiden lkm'
   $Z \sim NBin(3, 0.1)$
   $P(Z > 4) = 1 - P(Z \leq 4) = 0.9963$

4. X = 'Viiteen onnistuneeseen vastaukseen tarvittavien kysymysten lkm'
   $X \sim NBin(5, 0.5)$
   (a) $f(6) = 0.078125$
   (b) $P(5 \leq X \leq 8) \approx 0.363$
   (c) $E(X) = \sum_{x=5}^{7} x\binom{x-1}{4}(\frac{1}{2})^x + 8(1 - \sum_{x=5}^{7}\binom{x-1}{4}(\frac{1}{2})^x) \approx 7.633$

5. $X \sim Poi(1.5)$
   (a) $P(X = 0) \approx 0.223$
   (b) $Z = X + Y \sim Poi(3)$
   $P(Z = 4) \approx 0.168$
   (c) X_i='Onnettomuuksien lkm kuukautena i', $i = 1, \ldots, 12$
   Riippumattomuudesta seuraa, että
   $P(X_1 \geq 1, \ldots, X_{12} \geq 1) = P(X_1 \geq 1) \times \ldots \times P(X_{12} \geq 1) = (1 - e^{-1.5})^{12} \approx 0.048$

6. a) $X|X + Y = 10 \sim Bin(10, 0.25)$, $E(X) = 2.5$
   b) $Y|X + Y = 10 \sim Bin(10, 0.75)$
   $P(Y > 5|X + Y = 10)$
   ```
   = 1- pbinom(5, 10, 0.75)
   = sum(dbinom(6:10, 10, 0.75))
   ```

7. $\lambda = 9$ (vrt. esim. 5.11)

8. a) $X \sim Ber(\frac{2}{3})$

   b) $E(X) = p, Var(X) = p(1-p)$

   c) $Y_i \sim Ber(\frac{2}{3}), Y_1 + Y_2 + Y_3$
   $M_Y(t) = E(e^{Yt}) = E(e^{Y_1t+Y_2t+Y_3t}) = E(e^{Y_1t})E(e^{Y_2t})E(e^{Y_3t}) = (\frac{1}{3} + \frac{2}{3}e^t)^3$
   $Y \sim Bin(3, \frac{2}{3})$