Session on the occasion of

the 60th birthday of

Jerzy K. Baksalary

Będlewo, August 17, 2004

CURRICULUM VITAE

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Education:

M.Sc. in Mathematics - Adam Mickiewicz University, 1969

Ph.D. in Mathematical Sciences - Adam Mickiewicz University, 1975

Habilitated Doctor Degree in Mathematical Sciences - Adam Mickiewicz University, 1984

Title of Professor in Mathematical Sciences - President of Poland, 1990

Professional career:

1969-1975 assistant, Department of Mathematical and Statistical Methods, Academy of Agriculture in Poznań

1975-1987 assistant professor (adjunct), Department of Mathematical and Statistical Methods, Academy of Agriculture in Poznań

1987-1988 associate professor, Department of Mathematical and Statistical Methods, Academy of Agriculture in Poznań

1988-1991 associate professor, Institute of Mathematics, Tadeusz Kotarbiński Pedagogical University

1991- full professor, Institute of Mathematics, Tadeusz Kotarbiński Pedagogical University (now, Faculty of Mathematics, Informatics, and Econometrics, Zielona Góra University)

1989-1990 professor of the Finnish Academy of Sciences, Department of Mathematical Sciences, University of Tampere

1990-1996 Rector of Tadeusz Kotarbiński Pedagogical University

1996-1999 Dean of the Faculty of Mathematics, Physics, and Technics of Tadeusz Kotarbiński Pedagogical University

PROGRAM

13:00– Lunch

14:00–14:30 *T. Caliński:* On some Baksalary's contributions to the theory of block designs
14:30–15:00 *R. Kala:* On some seemingly uncorrelated results
15:00–15:30 *A. Markiewicz:* Admissible linear estimation in linear models

15:30–17:00 Wine break

17:00–17:30 *G.P.H. Styan:* Some remarks on the publications by Jerzy K. Baksalary
17:30–18:00 *S. Puntanen:* JKB through my camera
18:00–18:30 *G. Trenkler:* On a generalization of rotation matrices

Abstracts

On some Baksalary's contributions to the theory of block designs

Tadeusz Caliński

Agricultural University of Poznań, Poland

Abstract

A review of some results obtained by Jerzy Baksalary with regard to the theory of block designs is given. Particularly, attention is drawn to his results concerning various concepts of balance, some methods of constructing block designs, the connectedness of PBIB designs, conditions for a kind of robustness of block designs, and certain criteria concerning Fisher's condition for block designs. The importance of his results is stressed. References to other relevant works in this field are also made. There is no doubt that Baksalary's contributions to experimental design are important both from theoretical and practical point of view.

The full text of the review constitutes the final part of this booklet.

On some seemingly uncorrelated results

Radosław Kala

Agricultural University of Poznań, Poland

Abstract

A short nonlinear history following from the early joint papers with Jerzy K. Baksalary on linear models and linear algebra will be presented.

Admissible linear estimation in linear models

Augustyn Markiewicz

Agricultural University of Poznań, Poland

Abstract

A history of solving the problem of admissible linear estimation in a singular Gauss-Markov model is presented. A review of some additional results inspired by Rao's (1976) paper is given. In particular, it covers results related to restricted Gauss-Markov model, estimation with respect to the matrix risk, as well as the solution to the problem of "natural restrictions" in singular Gauss-Markov model.

References

Rao, C.R. (1976). Estimation of parameters in a linear model. *Ann. Statist. 4*, 1023-1037.

JKB through my camera

Simo Puntanen

University of Tampere, Tampere, Finland

Some remarks on the publications by Jerzy K. Baksalary

George P. H. Styan[1] & Simo Puntanen[2]

[1] McGill University, Montreal, Canada
[2] University of Tampere, Tampere, Finland

Abstract

In this talk we comment on the research publications by Jerzy K. Baksalary in statistics and in matrix theory, and in particular, we will describe some of our personal experiences in preparing joint work with him for publication.

On a generalization of rotation matrices

Götz Trenkler

Dortmund University, Germany

Abstract

Starting from the concept of the vector cross product and its corresponding skew-symmetric transformation matrix, a bigger class of matrices is considered containing the rotations of the three-dimensional euclidean space. Special attention is paid to Moore-Penrose inverse, determinant, eigenvalues and eigenspace of members of this class. Some emphasis is on teaching of modern matrix theory.

JKB's Passions

1. Family

Jerzy K. Baksalary and Mirosława Baksalary
(married since 1964)

Katarzyna Baksalary–Iżycka

Natalia
Dominika
Marianna
Iga

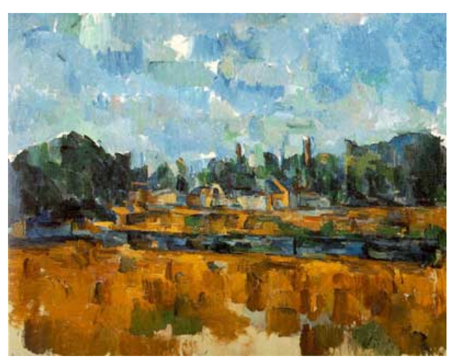
Oskar Maria Baksalary

JKB's Granddaughters

2. Paintings

The great favorites:

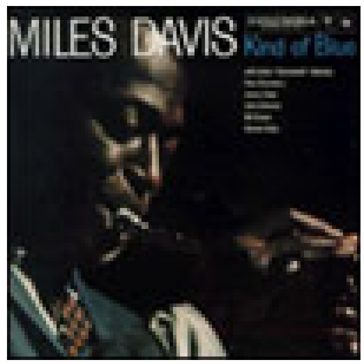
Johannes Vermeer
Rembrandt van Rijn
Paul Cézanne

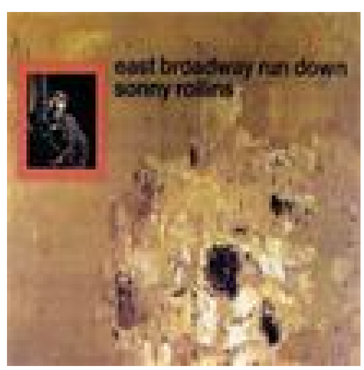


3. Jazz

The great favorites:

Miles Davis
John Coltrane
Sonny Rollins





4. New York

Lincoln Center
Empire State Building
Woodlawn Cemetery

JKB's Main Passion – Mathematics

LIST OF PUBLICATIONS

(compiled on August 2, 2004)

1. **Jerzy K. Baksalary, Radosław Kala**: Metody analizy doświadczeń nieortogonalnych. In: *Czwarte Colloquium Metodologiczne z Agro-Biometrii* (Eugeniusz Bilski, Tadeusz Caliński, Wiktor Oktaba, Witold Klonecki, Eds.), Polish Academy of Sciences and Polish Biometrical Society, Warszawa 1974, pp. 201-258.

2. **Jerzy K. Baksalary, Radosław Kala**: Procedura obliczania uogólnionej odwrotności macierzy. *Algorytmy Biometryczne i Statystyczne* **3** (1974) 157-165.

3. **Jerzy K. Baksalary, Anita Dobek, Radosław Kala**: Rozwiązywanie równań liniowych z nieujemnie określoną symetryczną macierzą układu. *Algorytmy Biometryczne i Statystyczne* **4** (1975) 243-260.

4. **Jerzy K. Baksalary, Anita Dobek, Radosław Kala**: A method for computing projectors. *Zhurnal Vychislitel'noĭ Matematiki i Matematicheskoĭ Fiziki* **16** (1976) 1038-1040.

5. **Jerzy K. Baksalary, Anita Dobek, Radosław Kala**: Wyznaczanie operatorów rzutowania ortogonalnego. *Algorytmy Biometryczne i Statystyczne* **5** (1976) 187-194.

6. **Jerzy K. Baksalary, Radosław Kala**: Criteria for estimability in multivariate linear models. *Mathematische Operationsforschung und Statistik* **7** (1976) 5-9.

7. **Jerzy K. Baksalary, Radosław Kala**: Extensions of Milliken's estimability criterion. *The Annals of Statistics* **4** (1976) 639-641.

8. **Jerzy K. Baksalary, Anita Dobek, Radosław Kala**: Wyznaczanie operatorów rzutowania. *Algorytmy Biometryczne i Statystyczne* **6** (1977) 175-183.

9. **Jerzy K. Baksalary, Radosław Kala**: A method of finding bases of a matrix. *Commentationes Mathematicae* **20** (1977) 1-5.

10. **Jerzy K. Baksalary, Radosław Kala**: An extension of a rank criterion for the least squares estimator to be the best linear unbiased estimator. *Journal of Statistical Planning and Inference* **1** (1977) 309-312.

11. **Jerzy K. Baksalary, Radosław Kala**: Sums of squares and products matrices for a non-full ranks hypothesis in the model of Potthoff and

Roy. *Mathematische Operationsforschung und Statistik, Series Statistics* **8** (1977) 459-465.

12. **Jerzy K. Baksalary, Radosław Kala, Krystyna Katulska**: Analiza wariancji dla klasyfikacji krzyżowych metodą Bocka. *Algorytmy Biometryczne i Statystyczne* **6** (1977) 3-32.

13. **Jerzy K. Baksalary, L. C. A. Corsten, Radosław Kala**: Reconciliation of two different views on estimation of growth curve parameters. *Biometrika* **65** (1978) 662-665.

14. **Jerzy K. Baksalary, Anita Dobek, Radosław Kala**: Analiza statystyczna w ogólnym modelu liniowym. *Algorytmy Biometryczne i Statystyczne* **7** (1978) 3-23.

15. **Jerzy K. Baksalary, Anita Dobek, Radosław Kala**: Estymacja krzywych wzrostu. *Algorytmy Biometryczne i Statystyczne* **7** (1978) 81-113.

16. **Jerzy K. Baksalary, Anita Dobek, Radosław Kala**: Rozkład macierzy rzeczywistej na czynniki pełnych rzędów. *Algorytmy Biometryczne i Statystyczne* **7** (1978) 179-183.

17. **Jerzy K. Baksalary, Radosław Kala**: A bound for the Euclidean norm of the difference between the least squares and the best linear unbiased estimators. *The Annals of Statistics* **6** (1978) 1390-1393.

18. **Jerzy K. Baksalary, Radosław Kala**: Estymowalność liniowych funkcji parametrycznych w jednowymiarowym modelu liniowym. *Matematyka Stosowana* **12** (1978) 133-144.

19. **Jerzy K. Baksalary, Radosław Kala**: Estymowalność liniowych funkcji parametrycznych w jednowymiarowym modelu liniowym z restrykcjami. *Matematyka Stosowana* **12** (1978) 145-151.

20. **Jerzy K. Baksalary, Radosław Kala**: Relationships between some representations of the best linear unbiased estimator in the general Gauss-Markoff model. *SIAM Journal on Applied Mathematics* **35** (1978) 515-520.

21. **Jerzy K. Baksalary, Radosław Kala**: Best linear unbiased estimation in the restricted general linear model. *Mathematische Operationsforschung und Statistik, Series Statistics* **10** (1979) 27-35.

22. **Jerzy K. Baksalary, Radosław Kala**: Covariance adjustment when a vector of parameters is restricted to a given subspace. *SIAM Journal on Applied Mathematics* **37** (1979) 20-21.

23. **Jerzy K. Baksalary, Radosław Kala**: On the prediction problem in the seemingly unrelated regression equations model. *Mathematische Operationsforschung und Statistik, Series Statistics* **10** (1979) 203-208.

24. **Jerzy K. Baksalary, Radosław Kala**: The matrix equation $\mathbf{AX} - \mathbf{YB} = \mathbf{C}$. *Linear Algebra and Its Applications* **25** (1979) 41-43.

25. **Jerzy K. Baksalary, Radosław Kala**: Two relations between oblique and Λ-orthogonal projectors. *Linear Algebra and Its Applications* **24** (1979) 99-103.

26. **Jerzy K. Baksalary, Tadeusz Caliński, Radosław Kala**: Estymacja krzywych wzrostu i jej zastosowanie do oceny odmian gatunków o plonowaniu wielokrotnym. *Biuletyn Oceny Odmian* **2** (1980) 167-182.

27. **Jerzy K. Baksalary, Anita Dobek, Radosław Kala**: A necessary condition for balance of a block design. *Biometrical Journal* **22** (1980) 47-50.

28. **Jerzy K. Baksalary, Anita Dobek, Radosław Kala**: Calculation of projections. *Zastosowania Matematyki* **17** (1980) 209-215.

29. **Jerzy K. Baksalary, Anita Dobek, Radosław Kala**: Some methods for constructing efficiency-balanced block designs. *Journal of Statistical Planning and Inference* **4** (1980) 25-32.

30. **Jerzy K. Baksalary, Jan Hauke, Radosław Kala**: Nonnegative definite solutions to some matrix equations occurring in distribution theory of quadratic forms. *Sankhyā, Series A* **42** (1980) 283-291.

31. **Jerzy K. Baksalary, Radosław Kala**: A new bound for the Euclidean norm of the difference between the least squares and the best linear unbiased estimators. *The Annals of Statistics* **8** (1980) 679-681.

32. **Jerzy K. Baksalary, Radosław Kala**: A note on Ahlers and Lewis representation of the best linear unbiased estimator in the general Gauss-Markoff model. *Banach Center Publications* **6** (1980) 17-21.

33. **Jerzy K. Baksalary, Radosław Kala**: On connectedness of ordinary two-way elimination of heterogeneity designs. *Biometrical Journal* **22** (1980) 105-109.

34. **Jerzy K. Baksalary, Radosław Kala**: On estimation problems in a general Gauss-Markov model. In: *Data Analysis and Informatics* (E. Diday, L. Lebart, I. P. Pages, R. Tomassone, Eds.), North-Holland, Amsterdam 1980, pp. 163-167.

35. **Jerzy K. Baksalary, Radosław Kala**: On the difference between two second degree polynomials, each following a chi-square distribution. *Sankhyā, Series A* **42** (1980) 123-127.

36. **Jerzy K. Baksalary, Radosław Kala**: The matrix equation $\mathbf{AXB} + \mathbf{CYD} = \mathbf{E}$. *Linear Algebra and Its Applications* **30** (1980) 141-147.

37. **Jerzy K. Baksalary, Radosław Kala**: Two properties of a nonnegative definite matrix. *Bulletin de l'Academie Polonaise des Sciences, Serie des Sciences Mathematiques* **28** (1980) 233-235.

38. **Jerzy K. Baksalary, Radosław Kala**: Linear transformations preserving best linear unbiased estimators in a general Gauss-Markoff model. *The Annals of Statistics* **9** (1981) 913-916.

39. **Jerzy K. Baksalary, Radosław Kala**: Simple least squares estimation versus best linear unbiased prediction. *Journal of Statistical Planning and Inference* **5** (1981) 147-151.

40. **Jerzy K. Baksalary, Radosław Kala**: Symmetrizers of matrices. *Linear Algebra and Its Applications* **35** (1981) 51-62.

41. **Jerzy K. Baksalary**: The pair of matrix equations $\mathbf{AX} = \mathbf{B}$ and $\mathbf{AY} + \mathbf{CX} = \mathbf{D}$. *Atti della Accademia Nazionale dei Lincei, Rendiconti della Classe di Scienze Fisiche, Matematiche e Naturali* **73** (1982) 81-88.

42. **Jerzy K. Baksalary, Radosław Kala**: Admissible estimation by covariance adjustment technique. *Sankhyā, Series A* **44** (1982) 281-285.

43. **Jerzy K. Baksalary**: An invariance property of Farebrother's procedure for estimation with aggregated data. *Journal of Econometrics* **22** (1983) 317-322.

44. **Jerzy K. Baksalary, Tadeusz Caliński, Radosław Kala**: Estymacja krzywych wzrostu w układzie bloków kompletnych. *Biuletyn Oceny Odmian* **10** (1983) 105-117.

45. **Jerzy K. Baksalary, Radosław Kala**: Estimation via linearly combining two given statistics. *The Annals of Statistics* **11** (1983) 691-696.

46. **Jerzy K. Baksalary, Radosław Kala**: On equalities between BLUEs, WLSEs, and SLSEs. *The Canadian Journal of Statistics* **11** (1983) 119-123.

47. **Jerzy K. Baksalary, Radosław Kala**: On the distribution of a nonnegative difference between two χ^2-distributed second degree polynomial statistics. *Zastosowania Matematyki* **18** (1983) 55-59.

48. **Jerzy K. Baksalary, Radosław Kala**: Partial orderings between matrices one of which is of rank one. *Bulletin of the Polish Academy of Sciences, Series Mathematics* **31** (1983) 5-7.

49. **Jerzy K. Baksalary, Radosław Kala**: Range invariance of certain matrix products. *Linear and Multilinear Algebra* **14** (1983) 89-96.

50. **Jerzy K. Baksalary, Radosław Kala, Krzysztof Kłaczyński**: The matrix inequality $\mathbf{M} \geq \mathbf{B}^*\mathbf{MB}$. *Linear Algebra and Its Applications* **54** (1983) 77-86.

51. **Jerzy K. Baksalary**: A study of the equivalence between a Gauss-Markoff model and its augmentation by nuisance parameters. *Mathematische Operationsforschung und Statistik, Series Statistics* **15** (1984) 3-35.

52. **Jerzy K. Baksalary**: Comparing stochastically restricted estimators in a linear regression model. *Biometrical Journal* **26** (1984) 555-557.

53. **Jerzy K. Baksalary**: Nonnegative definite and positive definite solutions to the matrix equation $\mathbf{AXA} = \mathbf{B}$. *Linear and Multilinear Algebra* **16** (1984) 133-139.

54. **Jerzy K. Baksalary, Jan Hauke**: Inheriting independence and chi-squaredness under certain matrix orderings. *Statistics & Probability Letters* **2** (1984) 35-38.

55. **Jerzy K. Baksalary, Anna Molińska**: Nonnegative unbiased estimability of linear combinations of variance components. *Journal of Statistical Planning and Inference* **10** (1984) 1-8.

56. **Jerzy K. Baksalary**: Milliken's estimability criterion. In: *Encyclopedia of Statistical Sciences*, vol. 5 (Samuel Kotz, Norman L. Johnson, Eds.), John Wiley and Sons, New York 1985, pp. 503-504.

57. **Jerzy K. Baksalary**: Strong unified-least-squares matrices for a general linear model. *Linear Algebra and Its Applications* **70** (1985) 61-65.

58. **Jerzy K. Baksalary, Augustyn Markiewicz**: Admissible linear estimators in restricted linear models. *Linear Algebra and Its Applications* **70** (1985) 9-19.

59. **Jerzy K. Baksalary, Paweł R. Pordzik**: A note on using linear restrictions in a Gauss-Markov model. *Statistica (Bologna)* **45** (1985) 209-212.

60. **Jerzy K. Baksalary, Friedrich Pukelsheim**: A note on the matrix ordering of special C-matrices. *Linear Algebra and Its Applications* **70** (1985) 263-267.

61. **Jerzy K. Baksalary, Zenon Tabis**: Existence and constructions of connected block designs with given vectors of treatment replications and block sizes. *Journal of Statistical Planning and Inference* **12** (1985) 285-293.

62. **Jerzy K. Baksalary**: A relationship between the star and minus orderings. *Linear Algebra and Its Applications* **82** (1986) 163-167.

63. **Jerzy K. Baksalary, Radosław Kala**: Linear sufficiency with respect to a given vector of parametric functions. *Journal of Statistical Planning and Inference* **14** (1986) 331-338.

64. **Jerzy K. Baksalary, Augustyn Markiewicz**: Characterizations of admissible linear estimators in restricted linear models. *Journal of Statistical Planning and Inference* **13** (1986) 395-398.

65. **Jerzy K. Baksalary, Thomas Mathew**: Linear sufficiency and completeness in an incorrectly specified general Gauss-Markov model. *Sankhyā, Series A* **48** (1986) 169-180.

66. **Jerzy K. Baksalary**: Algebraic characterizations and statistical implications of the commutativity of orthogonal projectors. In: *Proceedings of the Second International Tampere Conference in Statistics* (T. Pukkila, S. Puntanen, Eds.), Department of Mathematical Sciences, University of Tampere, Tampere 1987, pp. 113-142.

67. **Jerzy K. Baksalary, Jan Hauke**: Partial orderings of matrices referring to singular values or eigenvalues. *Linear Algebra and Its Applications* **96** (1987) 17-26.

68. **Jerzy K. Baksalary, P. D. Puri**: On bounds for the parameters of binary block designs. *Journal of Statistical Planning and Inference (Statistical Discussion Forum)* **16** (1987) 134-135.

68. **Jerzy K. Baksalary, Zenon Tabis**: Conditions for the robustness of block designs against the unavailability of data. *Journal of Statistical Planning and Inference* **16** (1987) 49-54.

70. **Jerzy K. Baksalary, Zenon Tabis**: Connectedness of PBIB designs. *The Canadian Journal of Statistics* **15** (1987) 147-150.

71. **Jerzy K. Baksalary**: A comment on an admissibility criterion. *Journal of Statistical Computation and Simulation (Comments, Conjectures and Conclusions)* **28** (1988) 345-347.

72. **Jerzy K. Baksalary**: Criteria for the equality between ordinary least squares and best linear unbiased estimators under certain linear models. *The Canadian Journal of Statistics* **16** (1988) 97-102.

73. **Jerzy K. Baksalary**: Solution to Problem 213. *Statistica Neerlandica* **42** (1988) 150-151.

74. **Jerzy K. Baksalary, Augustyn Markiewicz**: Admissible linear estimators in the general Gauss-Markov model. *Journal of Statistical Planning and Inference* **19** (1988) 349-359.

75. **Jerzy K. Baksalary, Thomas Mathew**: Admissible linear estimation in a general Gauss-Markov model with an incorrectly specified dispersion matrix. *Journal of Multivariate Analysis* **27** (1988) 53-67. [Reprinted in: *Multivariate Statistics and Probability. Essays in Memory of Parachuri R. Krishnaiah* (C. R. Rao, M. M. Rao, Eds.). Academic Press, Boston 1989, pp. 53-67].

76. **Jerzy K. Baksalary, P. D. Puri**: Criteria for the validity of Fisher's condition for balanced block designs. *Journal of Statistical Planning and Inference* **18** (1988) 119-123.

77. **Jerzy K. Baksalary, Adrian C. van Eijnsbergen**: A comparison of two criteria for ordinary least squares estimators to be best linear unbiased estimators. *The American Statistician* **42** (1988) 205-208.

78. **Jerzy K. Baksalary**: A rank characterization of linear models with nuisance parameters and its application to block designs. *Journal of Statistical Planning and Inference* **22** (1989) 173-179.

79. **Jerzy K. Baksalary, Erkki P. Liski, Götz Trenkler**: Mean square error matrix improvements and admissibility of linear estimators. *Journal of Statistical Planning and Inference* **23** (1989) 313-325.

80. **Jerzy K. Baksalary, Augustyn Markiewicz**: A matrix inequality and admissibility of linear estimators with respect to the mean square error matrix criterion. *Linear Algebra and Its Applications* **112** (1989) 9-18.

81. **Jerzy K. Baksalary, Paweł R. Pordzik**: Inverse-partitioned-matrix method for the general Gauss-Markov model with linear restrictions. *Journal of Statistical Planning and Inference* **23** (1989) 133-143.

82. **Jerzy K. Baksalary, Friedrich Pukelsheim, George P. H. Styan**: Some properties of matrix partial orderings. *Linear Algebra and Its Applications* **119** (1989) 57-85.

83. **Jerzy K. Baksalary, Simo Puntanen**: Weighted-least-squares estimation in the general Gauss-Markov model. In: *Statistical Data Analysis and Inference* (Yadolah Dodge, Ed.), North-Holland, Amsterdam 1989, pp. 355-368.

84. **Jerzy K. Baksalary, Götz Trenkler**: Solution to and comments on Problem 88.3.4. *Econometric Theory* **5** (1989) 463-465.

85. **Jerzy K. Baksalary**: Solution to and comments on Problem 89-7. *The Institute of Mathematical Statistics Bulletin* **19** (1990) 213-214.

86. **Jerzy K. Baksalary, Anita Dobek, Stanislaw Gnot**: Characterizations of two-way layouts from the point of view of variance component estimation in the corresponding mixed linear models. *Journal of Statistical Planning and Inference* **26** (1990), 35-45.

87. **Jerzy K. Baksalary, Jan Hauke**: A further algebraic version of Cochran's theorem and matrix partial orderings. *Linear Algebra and Its Applications* **127** (1990) 157-169.

88. **Jerzy K. Baksalary, Augustyn Markiewicz**: Admissible linear estimators of an arbitrary vector of parametric functions in the general

Gauss-Markov model. *Journal of Statistical Planning and Inference* **26** (1990) 161-171.

89. **Jerzy K. Baksalary, Thomas Mathew**: Rank invariance criterion and its application to the unified theory of least squares. *Linear Algebra and Its Applications* **127** (1990) 393-401.

90. **Jerzy K. Baksalary, Kenneth Nordström, George P. H. Styan**: Löwner ordering antitonicity of generalized inverses of Hermitian matrices. *Linear Algebra and Its Applications* **127** (1990) 171-182.

91. **Jerzy K. Baksalary, Paweł R. Pordzik**: A note on comparing the unrestricted and restricted least-squares estimators. *Linear Algebra and Its Applications* **127** (1990) 371-378.

92. **Jerzy K. Baksalary, Paweł R. Pordzik, Götz Trenkler**: A note on generalized ridge estimators. *Communications in Statistics, Series A - Theory and Methods* **19** (1990) 2871-2877.

93. **Jerzy K. Baksalary, Simo Puntanen**: A complete solution to the problem of robustness of Grubbs's test. *The Canadian Journal of Statistics* **18** (1990) 285-287.

94. **Jerzy K. Baksalary, Simo Puntanen**: Characterizations of the best linear unbiased estimator in the general Gauss-Markov model with the use of matrix partial orderings. *Linear Algebra and Its Applications* **127** (1990) 363-370.

95. **Jerzy K. Baksalary, Simo Puntanen**: Spectrum and trace invariance criterion and its statistical applications. *Linear Algebra and Its Applications* **142** (1990) 121-128.

96. **Jerzy K. Baksalary, Simo Puntanen, George P. H. Styan**: A property of the dispersion matrix of the best linear unbiased estimator in the general Gauss-Markov model. *Sankhyā, Series A* **52** (1990) 279-296.

97. **Jerzy K. Baksalary, Simo Puntanen, George P. H. Styan**: On T. W. Anderson's contributions to solving the problem of when the ordinary least-squares estimator is best linear unbiased and to characterizing the rank additivity of matrices. In: *The Collected Papers of T. W. Anderson: 1943-1985* (George P. H. Styan, Ed.), John Wiley and Sons, New York 1990, pp.1579-1591.

98. **Jerzy K. Baksalary, P. D. Puri**: Pairwise-balanced, variance-balanced and resistant incomplete block designs revisited. *Annals of the Institute of Statistical Mathematics* **42** (1990) 163-171.

99. **Jerzy K. Baksalary, K. R. Shah**: Some properties of two-way elimination of heterogeneity designs. In: *Probability, Statistics and Design of*

Experiment (R. R. Bahadur, Ed.), Wiley Eastern, New Delhi 1990, pp. 75-85.

100. **Jerzy K. Baksalary, Sujit Kumar Mitra**: Left-star and right-star partial orderings. *Linear Algebra and Its Applications* **149** (1991) 73-89.

101. **Jerzy K. Baksalary, Friedrich Pukelsheim**: On the Löwner, minus, and star partial orderings of nonnegative definite matrices and their squares. *Linear Algebra and Its Applications* **151** (1991) 135-141.

102. **Jerzy K. Baksalary, Simo Puntanen**: A counterexample to a conjecture of Magness and McGuire. In: *A Spectrum of Statistical Thought. Essays in Statistical Theory, Economics and Population Genetics in Honour of Johan Fellman* (Gunnar Rosenqvist, Katarina Juselius, Kenneth Nordström, Juni Palmgren, Eds.), Swedish School of Economics and Business Administration, Helsinki 1991, pp. 15-18.

103. **Jerzy K. Baksalary, Simo Puntanen**: Generalized matrix versions of the Cauchy-Schwarz and Kantorovich inequalities. *Aequationes Mathematicae* **41** (1991) 103-110.

104. **Jerzy K. Baksalary, Götz Trenkler**: Covariance adjustment in biased estimation. *Computational Statistics, Data Analysis* **12** (1991) 221-230.

105. **Jerzy K. Baksalary, Götz Trenkler**: Nonnegative and positive definiteness of matrices modified by two matrices of rank one. *Linear Algebra and Its Applications* **151** (1991) 169-184.

106. **Jerzy K. Baksalary, Jan Hauke**: Minimum number of experimental units in connected block designs with certain additional properties. *Journal of Statistical Planning and Inference* **30** (1992) 173-183.

107. **Jerzy K. Baksalary, Markku Nurhonen, Simo Puntanen**: Effect of correlations and unequal variances in testing for outliers in linear regression. *Scandinavian Journal of Statistics. Theory and Applications* **19** (1992) 91-95.

108. **Jerzy K. Baksalary, Paweł R. Pordzik**: Implied linear restrictions in the general Gauss-Markov model. *Journal of Statistical Planning and Inference* **30** (1992) 237-248.

109. **Jerzy K. Baksalary, Friedrich Pukelsheim**: Adjusted orthogonality properties in multi-way block designs. In: *Data Analysis and Statistical Inference. Festschrift in Honour of Friedhelm Eicker* (Siegfried Schach, Götz Trenkler, Eds.), Verlag Josef Eul, Bergisch Gladbach 1992, pp. 413-420.

110. **Jerzy K. Baksalary, Tarmo Pukkila**: A note on invariance of the eigenvalues, singular values, and norms of matrix products involving

generalized inverses. *Linear Algebra and Its Applications* **165** (1992) 125-130.

111. **Jerzy K. Baksalary, Simo Puntanen**: An inequality for the trace of matrix product. *IEEE Transactions on Automatic Control* **37** (1992) 239-240.

112. **Jerzy K. Baksalary, Simo Puntanen, Haruo Yanai**: Canonical correlations associated with symmetric reflexive generalized inverses of the dispersion matrix. *Linear Algebra and Its Applications* **176** (1992) 61- 74.

113. **Jerzy K. Baksalary, C. Radhakrishna Rao, Augustyn Markiewicz**: A study of the influence of the 'natural restrictions' on estimation problems in the singular Gauss-Markov model. *Journal of Statistical Planning and Inference* **31** (1992) 335-351.

114. **Jerzy K. Baksalary, Bernhard Schipp, Götz Trenkler**: Some further results on Hermitian matrix inequalities. *Linear Algebra and Its Applications* **160** (1992) 119-129.

115. **Jerzy K. Baksalary, Paweł R. Pordzik**: Preliminary test estimation of a vector of parametric functions in the general Gauss-Markov model. *Journal of Statistical Planning and Inference* **36** (1993) 227-239.

116. **Jerzy K. Baksalary, Idzi Siatkowski**: Decomposability of the C-matrix of a two-way elimination of heterogeneity design. *Journal of Statistical Planning and Inference* **36** (1993) 301-309.

117. **Jerzy K. Baksalary, George P. H. Styan**: Around a formula for the rank of a matrix product with some statistical applications. In: *Graphs, Matrices and Designs* (Rolf S. Rees, Ed.), Marcel Dekker, New York 1993, 1-18.

118. **Jerzy K. Baksalary, Jan Hauke, George P. H. Styan**: On some distributional properties of quadratic forms in normal variables and on some associated matrix partial orderings. In: *Multivariate Analysis and Its Applications.* (T. W. Anderson, K. T. Fang and I. Olkin, Eds.), Institute of Mathematical Statistics. Lecture Notes-Monograph Series **24** (1994) 111-121.

119. **Jerzy K. Baksalary, Stanisław Gnot, Sanpei Kageyama**: Best estimation of variance components with arbitrary kurtosis in two-way layouts mixed models. *Journal of Statistical Planning and Inference* **44** (1995), 65-75.

120. **Jerzy K. Baksalary, Augustyn Markiewicz, C. Radhakrishna Rao**: Admissible linear estimation in the general Gauss-Markov model

with respect to an arbitrary quadratic risk function. *Journal of Statistical Planning and Inference* **44** (1995) 341-347.

121. **Jerzy K. Baksalary, Augustyn Markiewicz**: Further results on invariance of the eigenvalues of matrix products involving generalized inverses. *Linear Algebra and Its Applications* **237**/**238** (1996) 115-121.

122. **Jerzy K. Baksalary, Peter Šemrl, George P. H. Styan**: A note on rank additivity and range additivity. *Linear Algebra and Its Applications* **237**/**238** (1996) 489-498.

123. **Jerzy K. Baksalary, Oskar Maria Baksalary**: Idempotency of linear combinations of two idempotent matrices. *Linear Algebra and Its Applications* **321** (2000), 3-7.

124. **Jerzy K. Baksalary, Oskar Maria Baksalary**: Solution 25-1.1 (to Problem 25-1 "Moore-Penrose inverse of a skew-symmetric matrix" proposed by Jürgen Groß, Sven-Oliver Troshke, and Götz Trenkler) *IMAGE* **26** (2001), 2.

125. **Jerzy K. Baksalary, Oskar Maria Baksalary**: Solution 25-4.1 (to Problem 25-4 "Two rank equalities associated with blocks of an orthogonal projector" proposed by Yongge Tian). *IMAGE* **26** (2001), 6-7.

126. **Jerzy K. Baksalary, Oskar Maria Baksalary**: Solution 25-5.1 (to Problem 25-5 "Three inequalities involving Moore-Penrose inverses" proposed by Yongge Tian). *IMAGE* **26** (2001), 9-10.

127. **Jerzy K. Baksalary, Oskar Maria Baksalary**: Solution 25-6.1 (to Problem 25-6 "Generalized inverse of a matrix product" proposed by Yongge Tian). *IMAGE* **26** (2001), 10-11.

128. **Jerzy K. Baksalary, Oskar Maria Baksalary**: Solution 26-4.1 (to Problem 26-4 "Commutativity of EP matrices" proposed by Yongge Tian). *IMAGE* **27** (2001), 30.

129. **Jerzy K. Baksalary, Oskar Maria Baksalary**: Solution 26-5.1 (to Problem 26-5 "Convex matrix inequalities" proposed by Bao-Xue Zhang). *IMAGE* **27** (2001), 33-34.

130. **Jerzy K. Baksalary, Oskar Maria Baksalary**: Commutativity of projectors. *Linear Algebra and Its Applications* **341** (2002), 129-142.

131. **Jerzy K. Baksalary, Oskar Maria Baksalary**: Solution 27-2.1 (to Problem 27-2 "Specific generalized inverses" proposed by Jürgen Groß, Götz Trenkler). *IMAGE* **28** (2002), 29.

132. **Jerzy K. Baksalary, Oskar Maria Baksalary, George P. H. Styan**: Idempotency of linear combinations of an idempotent matrix

and a tripotent matrix. *Linear Algebra and Its Applications* **354** (2002), 21-34.

133. **Jerzy K. Baksalary, Oskar Maria Baksalary, Tomasz Szulc**: A property of orthogonal projectors. *Linear Algebra and Its Applications* **354** (2002), 35-39.

134. **Jerzy K. Baksalary, Richard William Farebrother**: Solution 27-1.1 (to Problem 27-1 "A class of square roots of involutory matrices" proposed by Richard William Farebrother). *IMAGE* **28** (2002), 26-28.

135. **Jerzy K. Baksalary, Jan Hauke**: Solution 27-6.1 (to Problem 27-6 "Inequalities of Hadamard products of nonnegative definite matrices" proposed by Xingzhi Zhan). *IMAGE* **28** (2002), 33.

136. **Jerzy K. Baksalary, George P. H. Styan**: Generalized inverses of partitioned matrices in Banachiewicz-Schur form. *Linear Algebra and Its Applications* **354** (2002), 41-47.

137. **Jerzy K. Baksalary**: Solution 29-10.1 (to Problem 29-10 "Equivalence of three reverse-order laws" proposed by Yongge Tian). *IMAGE* **30** (2003) 31.

138. **Jerzy K. Baksalary, Oskar Maria Baksalary**: Solution 29-5.1 (to Problem 29-5 "Product of two Hermitian nonnegative definite matrices" proposed by Jürgen Groß and Götz Trenkler). *IMAGE* **30** (2003) 24-25.

139. **Jerzy K. Baksalary, Oskar Maria Baksalary**: Solution 30-5.1 (to Problem 30-5 "A range equality for the difference or orthogonal projectors" proposed by Yongge Tian). *IMAGE* **31** (2003) 36-37.

140. **Jerzy K. Baksalary, Oskar Maria Baksalary**: Solution 30-6.1 (to Problem 30-6 "A matrix related to an idempotent matrix" proposed by Götz Trenkler). *IMAGE* **31** (2003) 39.

141. **Jerzy K. Baksalary, Oskar Maria Baksalary**: Solution 30-7.1 (to Problem 30-7 "A condition for an idempotent matrix to be Hermitian" proposed by Götz Trenkler). *IMAGE* **31** (2003) 41.

142. **Jerzy K. Baksalary, Oskar Maria Baksalary, Xiaoji Liu**: Further properties of the star, left-star, right-star, and minus partial orderings. *Linear Algebra and Its Applications* **375** (2003) 83-94.

143. **Jerzy K. Baksalary, Oskar Maria Baksalary, Xiaoji Liu**: Further relationships between certain partial orders of matrices and their squares. *Linear Algebra and Its Applications* **375** (2003) 171-180.

144. **Jerzy K. Baksalary, Oskar Maria Baksalary, Xiaoji Liu**: Problem 30-1 "Star partial ordering, left-star partial ordering, and commutativity". *IMAGE* **30** (2003) 36.

145. **Jerzy K. Baksalary, Oskar Maria Baksalary, Xiaoji Liu**: Solution 30-1.1 (to Problem 30-1 "Star partial ordering, left-star partial ordering, and commutativity" proposed by Jerzy K. Baksalary, Oskar Maria Baksalary, Xiaoji Liu). *IMAGE* **31** (2003) 30-31.

146. **Jerzy K. Baksalary, Oskar Maria Baksalary, Götz Trenkler**: A revisitation of formulae for the Moore-Penrose inverse of modified matrices. *Linear Algebra and Its Applications* **372** (2003) 207-224.

147. **Jerzy K. Baksalary, Jan Hauke**: Solution 29-9.1 (to Problem 29-9 "Equality of two nonnegative definite matrices" proposed by Yongge Tian). *IMAGE* **30** (2003) 29-30.

148. **Jerzy K. Baksalary, Roger A. Horn**: Solution 29-7.1 (to Problem 29-7 "Complementary principal submatrices and their eigenvalues" proposed by Chi-Kwong Li). *IMAGE* **30** (2003) 26-27.

149. **Jerzy K. Baksalary, Xiaoji Liu**: Solution 29-8.1 (to Problem 29-8 "A range equality involving an idempotent matrix" proposed by Yongge Tian). *IMAGE* **30** (2003) 27.

150. **William F. Trench, Jerzy K. Baksalary, Oskar Maria Baksalary**: Solution 29-1.2 (to Problem 29-1 "A condition for an EP matrix to be Hermitian" proposed by Jerzy K. Baksalary, Oskar Maria Baksalary). *IMAGE* **30** (2003) 22.

151. **Jerzy K. Baksalary**: Solution 31-3.1 (to Problem 31-3 "A range equality for block matrices" proposed by Yongge Tian). *IMAGE* **32** (2004) 23-24.

152. **Jerzy K. Baksalary, Oskar Maria Baksalary**: Solution 31-7.1 (to Problem 31-7 "On the product of orthogonal projectors" proposed by Götz Trenkler). *IMAGE* **32** (2004) 30-31.

153. **Jerzy K. Baksalary, Oskar Maria Baksalary, Xiaoji Liu**: Solution 31-2.1 (to Problem 31-2 "Matrices commuting with all nilpotent matrices" proposed by Henry Ricardo). *IMAGE* **32** (2004) 21-22.

154. **Jerzy K. Baksalary, Oskar Maria Baksalary, Tomasz Szulc**: Properties of Schur complements in partitioned idempotent matrices. *Linear Algebra and Its Applications* **379** (2004) 303-318.

155. **Jerzy K. Baksalary, Jan Hauke, Xiaoji Liu, Sanyang Liu**: Relationships between partial orders of matrices and their powers. *Linear Algebra and Its Applications* **379** (2004) 277-287.

156. **Jerzy K. Baksalary, Paulina Kik, Augustyn Markiewicz**: Solution 31-6.1 (to Problem 31-6 "A full rank factorization of a skew-symmetric matrix" proposed by Götz Trenkler). *IMAGE* **32** (2004) 27-28.

157. **Jerzy K. Baksalary, Anna Kuba**: Solution 31-7.2 (to Problem 31-7 "On the product of orthogonal projectors" proposed by Götz Trenkler). *IMAGE* **32** (2004) 31-34.

Papers being in press, expected to appear in 2004

158. **Jerzy K. Baksalary**: An elementary development of the equation characterizing best linear unbiased estimators. *Linear Algebra and Its Applications* (2004) (in press; LAA # 7695).

159. **Jerzy K. Baksalary**: A new approach to the concept of a strong unified-least-squares matrix. *Linear Algebra and Its Applications* (2004) (in press; LAA # 8059).

160. **Jerzy K. Baksalary, Oskar Maria Baksalary**: Nonsingularity of linear combinations of idempotent matrices. *Linear Algebra and Its Applications* (2004) (in press; LAA # 8084).

161. **Jerzy K. Baksalary, Oskar Maria Baksalary**: On linear combinations of generalized projectors. *Linear Algebra and Its Applications* (2004) (in press; LAA # 7902).

162. **Jerzy K. Baksalary, Oskar Maria Baksalary**: Relationships between generalized inverses of a matrix and generalized inverses of its rank-one-modifications. *Linear Algebra and Its Applications* (2004) (in press; LAA # 8120).

163. **Jerzy K. Baksalary, Oskar Maria Baksalary, Xiaoji Liu**: Further properties of generalized and hypergeneralized projectors. *Linear Algebra and Its Applications* (2004) (in press; LAA # 8111).

164. **Jerzy K. Baksalary, Oskar Maria Baksalary, Halim Özdemir**: A note on linear combinations of commuting tripotent matrices. *Linear Algebra and Its Applications* (2004) (in press; LAA # 8034).

165. **Jerzy K. Baksalary, Jan Hauke**: Characterizations of minus and star orders between the squares of Hermitian matrices. *Linear Algebra and Its Applications* (2004) (in press; LAA # 7763).

166. **Jerzy K. Baksalary, Xiaoji Liu**: An alternative characterization of generalized projectors. *Linear Algebra and Its Applications* (2004) (in press; LAA # 8031).

On some Baksalary's contributions to the theory of block designs

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Abstract

A review of some results obtained by Jerzy Baksalary with regard to the theory of block designs is given. Particularly, attention is drawn to his results concerning various concepts of balance, some methods of constructing block designs, the connectedness of PBIB designs, conditions for a kind of robustness of block designs, and certain criteria concerning Fisher's condition for block designs. The importance of his results is stressed. References to other relevant works in this field are also made. There is no doubt that Baksalary's contributions to experimental design are important both from theoretical and practical point of view.

0. Introduction and preliminaries

Jerzy Baksalary became interested in the theory of block designs in the late seventies, when the Poznań school of mathematical statistics and biometry was already quite advanced in this field. He was trying to investigate the mathematical background of the various concepts related to the theory of experimental designs, particularly of block designs, a subject of intensive study in Poznań at that time.

It will be helpful first to recall that any block design can be described by its $v \times b$ incidence matrix $\boldsymbol{N} = [n_{ij}]$, with a row for each treatment and a column for each block, where n_{ij} is the number of experimental units in the jth block receiving the ith treatment ($i = 1, 2, ..., v;\ j = 1, 2, ..., b$). This matrix, together with the vector of block sizes, $\boldsymbol{k} = [k_1, k_2, ..., k_b]' = \boldsymbol{N}'\mathbf{1}_v$, the vector of treatment replications, $\boldsymbol{r} = [r_1, r_2, ..., r_v]' = \boldsymbol{N}\mathbf{1}_b$, and the total number of units, $n = \mathbf{1}_b'\boldsymbol{k} = \mathbf{1}_v'\boldsymbol{r} = \mathbf{1}_v'\boldsymbol{N}\mathbf{1}_b$, where $\mathbf{1}_a$ is an $a \times 1$ vector of ones, is used in defining various matrices that help to understand the statistical properties of the design. In particular, an important role in studying these properties is played by the $v \times v$ matrix

$$\boldsymbol{C} = \boldsymbol{r}^{\delta} - \boldsymbol{N}\boldsymbol{k}^{-\delta}\boldsymbol{N}',$$

where $\boldsymbol{r}^\delta = \text{diag}[r_1, r_2, ..., r_v]$, $\boldsymbol{k}^\delta = \text{diag}[k_1, k_2, ..., k_b]$ and $\boldsymbol{k}^{-\delta} = (\boldsymbol{k}^\delta)^{-1}$. On it, the so-called intra-block analysis of the experimental data is based [see Caliński and Kageyama (2000, Section 3.2.1)]. The interest of Baksalary was at that time confined to this type of analysis.

1. Concepts of balance

In one of his earliest papers in this field [Baksalary, Dobek, and Kala (1980a)], the concept of balance of a block design is considered. Two notions of balance are defined there, for connected and disconnected block designs. But first it is noted that the rank of $\boldsymbol{C}$ is strictly related to the concept of connectedness.

Definition 1 (1980a). A block design is said to be connected if $\text{rank}(\boldsymbol{C}) = v-1$, and is said to be disconnected of degree $g-1$, $g \geq 2$, if $\text{rank}(\boldsymbol{C}) = v-g$.

Definition 2 (1980a). A connected (disconnected of degree $g-1$) block design is said to be V-balanced if all the nonzero eigenvalues of its matrix $\boldsymbol{C}$, $v-1$ $(v-g)$ in number, are equal.

Definition 3 (1980a). A connected (disconnected of degree $g-1$) block design is said to be J-balanced if all the nonzero eigenvalues of its matrix $\boldsymbol{C}$ with respect to the matrix $\boldsymbol{r}^\delta$, $v-1$ $(v-g)$ in number, are equal.

The notion of V-balance can be traced back to Vartak (1963). Now, it is more commonly termed "variance-balance (VB)" [cf., e.g., Raghavarao (1971, p. 54)]. The notion of J-balance goes back to the concept of balance introduced by Jones (1959), though implicitly already used by Nair and Rao (1948). Graf-Jaccottet (1977) introduced the term J-balanced, or "balanced in the Jones sense". More frequently, this type of balance is called M'efficiency-balance (EB)", due to Williams (1975) and Puri and Nigam (1975a, 1975b). However. it can be shown that the introduction of the terms VB and EB has been to some extent arbitrary [cf., e.g., Caliński and Kageyama (2000, Section 4.1)]. An extreme case of J-balance is the orthogonality of a block design.

Definition 4 (1980a). A connected (disconnected of degree $g-1$) block design is said to be orthogonal if all the nonzero eigenvalues of its matrix $\boldsymbol{C}$ with respect to the matrix $\boldsymbol{r}^\delta$, $v-1$ $(v-g)$ in number, are equal to 1.

See also Corollary 2.3.3 and Remark 2.4.2 in Caliński and Kageyama (2000).

An equivalent condition is given in the following theorem.

Theorem 2 (1980a). *If a block design is orthogonal, then the rank of its incidence matrix $\boldsymbol{N}$ is equal to 1 when the design is connected, and is equal to g when the design is disconnected of degree $g-1$.*

2. Constructional methods

Other characterizations of EB and VB designs are given in Baksalary, Dobek, and Kala (1980b), as follows.

Lemma 1 (1980b). *A block design is connected and EB if and only if, for some positive scalar p, $\boldsymbol{N}\boldsymbol{k}^{-\delta}\boldsymbol{N}' - p\boldsymbol{r}\boldsymbol{r}'$ is a diagonal matrix. If this is the case, the efficiency factor of the design equals $\varepsilon = np$.*

Lemma 2 (1980b). *A block design is connected and VB if and only if, for some positive scalar q, $\boldsymbol{N}\boldsymbol{k}^{-\delta}\boldsymbol{N}' - q\mathbf{1}_v\mathbf{1}_v'$ is a diagonal matrix.*

It may be noted that this way of defining balance is related to the early definitions based on the off-diagonal elements of the matrix $\boldsymbol{N}\boldsymbol{k}^{-\delta}\boldsymbol{N}'$, called "weighted concurrences" by Pearce (1976). Thus, Lemma 2 (1980b) is equivalent to the concept of total balance (Type T_0) introduced by Pearce (1976, Section 4.A) for the case when the weighted concurrences are all equal. On the other hand, Lemma 1 (1980b) is equivalent to the concept of total balance in the sense of Jones (1959), introduced for the case when the weighted concurrences are equally proportional to the products of the relevant treatment replications [cf. Definitions 2.4.3 and 2.4.5 in Caliński and Kageyama (2000)].

Using these characterizations of balance, Baksalary et al. (1980b) gave several theorems useful for constructing connected EB designs (Theorems 1, 4, and 5) and connected VB designs (Theorem 2 and 3). Of particular interst is a corollary following from their Theorem 4, which can be written as follows.

Corollary (1980b). *If $\boldsymbol{N}_h$, $h = 1, 2, ..., a$, are the incidence matrices of connected EB designs with a common number of treatments and with the replications of treatments mutually proportional among the designs, then their juxtaposition (assemblage) $[\boldsymbol{N}_1 : \boldsymbol{N}_2 : \cdots : \boldsymbol{N}_a]$ is the incidence matrix of a connected EB design, with its efficiency factor equal to the weighted average of the efficiency factors of the initial designs.*

For some applications of this result see, e.g., Caliński and Kageyama (2003, Section 8.2.2).

Further characterizations of connected designs as well as some constructions of these designs are considered in another Baksalary's paper [Baksalary and Tabis (1985)]. In particular, of interest is the following result.

Lemma 2 (1985). *A block design is connected if and only if it is not isomorphic, with respect to permutations of blocks and/or treatments, to a design with the incidence matrix of the form* $\mathrm{diag}[\boldsymbol{N}_1 : \boldsymbol{N}_2 : \cdots : \boldsymbol{N}_g]$, *where* $2 \leq g \leq v$ *and* $\boldsymbol{N}_\ell$, $\ell = 1, 2, ..., g$, *are all incidence matrices of connected block designs.*

This result rephrases Theorem 3.1 of Eccleston and Hedayat (1974). Evidently, if the design is not connected (in the above sense), it is disconnected of degree $g - 1$ [cf. Definition 2.2.6a in Caliński and Kageyama (2000)].

From both the theoretical and practical point of view, connectedness is a desirable property of a block design. In fact, the most frequent block designs used in practice are binary (i.e., with $n_{ij} = 0$ or $n_{ij} = 1$ for every $i = 1, 2, ..., v$ and $j = 1, 2, ..., b$) and connected designs.

When designing an experiment, the research project and the experimental material available determine the treatment replications and the block sizes, i.e., the vectors $\boldsymbol{r}$ and $\boldsymbol{k}$, of a block design to be used. In Baksalary and Tabis (1985), three theorems are proved that allow to construct binary and connected block designs for given $\boldsymbol{r}$ and $\boldsymbol{k}$, starting from a known binary block design, not necessarily connected. The first two theorems show that although disconnected designs are not desirable in general, under certain conditions they can be transformed into connected binary block designs with desired treatment replications and block sizes. The third theorem provides a sequential procedure for transforming a connected binary block design with the minimal number of experimental units into a connected binary block design with desired vectors $\boldsymbol{r}$ and $\boldsymbol{k}$, preserving in each step the property of connectedness.

3. Connectedness of PBIB designs

Another paper written by Baksalary and Tabis (1987a) concerns the connectedness of partially balanced incomplete block (PBIB) designs. These binary designs are often used when balanced incomplete block (BIB) designs with required treatment replications and block sizes are not available. The properties of a PBIB design are determined by a relevant so-called association scheme with m classes [see, e.g., Raghavarao (1971, Chapter 8); Caliński and Kageyama (2003, Section 6.0.2)]. Usually, the association schemes provide connected PBIB designs, but there may be cases where the connectedness is not preserved. In this paper a theorem is proved which gives a suitable criterion for examining the connectedness of PBIB designs based on various association schemes. Its applicabilty is shown in the context of the group-divisible m-associate-class PBIB designs introduced by Roy (1953-1954). In such a design there are $v = s_1 s_2 \cdots s_m$ treatments, each denoted by m indices $(i_1, i_2, ..., i_m)$, where $i_1 = 1, 2, ..., s_1$, $i_2 = 1, 2, ..., s_2$, ..., $i_m = 1, 2, ..., s_m$. Two treatments $(i_1, i_2, ..., i_m)$ and $(j_1, j_2, ..., j_m)$ are the uth associates if only their first $m - u$ indices are the same. They, then, occur together in exactly λ_u blocks, this number being independent of the particular pair of uth associates chosen, $u = 1, 2, ..., m$. [See also Raghavarao (1971, Section 8.12.6).] In practice, PBIB designs of this type of association scheme are used mainly for $m = 2$ or $m = 3$. But the established criterion (their Corollary 1, below) can be applied for any m, so extending the previously known results of Kageyama (1982) and Ogawa, Ikeda, and Kageyama (1984).

Corollary 1 (1987a). *A group-divisible m-associate-class PBIB design is connected if and only if $\lambda_m > 0$ (where λ_m is the number of blocks in which any two treatments being the mth associates occur together).*

Let this be illustrated by an example [Example 6.0.7 in Caliński and Kageyama (2003)].

The following incidence matrix shows a group-divisible 3-associate-class PBIB design with parameters $v = b = 8$, $r = k = 4$, $s_1 = s_2 = s_3 = 2$, $\lambda_1 = 2$, $\lambda_2 = 1$, $\lambda_3 = 2$, by taking the eight treatments as (1, 1, 1), (1, 1, 2), (1, 2, 1), (1, 2, 2), (2, 1, 1), (2, 1, 2), (2, 2, 1), (2, 2, 2):

$$\begin{array}{c} (1,1,1) \\ (1,1,2) \\ (1,2,1) \\ (1,2,2) \\ (2,1,1) \\ (2,1,2) \\ (2,2,1) \\ (2,2,2) \end{array} \left[\begin{array}{cccc|cccc} 1&1&0&0&1&0&1&0 \\ 1&1&0&0&0&1&0&1 \\ 0&0&1&1&1&0&0&1 \\ 0&0&1&1&0&1&1&0 \\ 1&0&0&1&0&0&1&1 \\ 0&1&1&0&0&0&1&1 \\ 1&0&1&0&1&1&0&0 \\ 0&1&0&1&1&1&0&0 \end{array}\right]$$

(which is a 2-resolvable design). Evidently, this design could well be used for a 2^3 factorial experiment, allowing the contrast between main effects of one of the factors to be estimated in the intra-block analysis with full efficiency.

4. Robustness of block designs

Another subject of interest studied by Baksalary was related to the robustness of block designs against the unavailability of data. Three sufficient conditions for a block design to be maximally robust have been derived by Baksalary and Tabis (1987b). They have used the following definition.

Definition (1987b). Let a block design $\mathcal{D}$ be binary and connected, let $r_{[v]}$ denote the smallest treatment replication of $\mathcal{D}$, and let $\mathcal{D}_{\#}$ denote a design obtained from $\mathcal{D}$ by deleting any $r_{[v]} - 1$ blocks. Then $\mathcal{D}$ is said to be maximally robust against the unavailability of data and with respect to the estimability of treatment contrasts if $\mathcal{D}_{\#}$ is connected irrespective of the choice of the blocks deleted.

Their main results are as follows.

Theorem 1 (1987b). *Let a block design $\mathcal{D}$ be binary and connected, and let $r_{[1]} \geq r_{[2]} \geq \cdots \geq r_{[v]}$ and $k_{[1]} \geq k_{[2]} \geq \cdots \geq k_{[b]}$ be its treatment replications and block sizes. Then the condition*

$$k_{[r_{[v]}]} + k_{[b]} > v$$

is sufficient for $\mathcal{D}$ to be maximally robust against the unavailability of data and with respect to the estimability of treatment contrasts.

Theorem 2 (1987b). *Let a block design $\mathcal{D}$ be binary and connected, and let $r_{[1]} \geq r_{[2]} \geq \cdots \geq r_{[v]}$ and $k_{[1]} \geq k_{[2]} \geq \cdots \geq k_{[b]}$ be its treatment replications and block sizes. Further, let κ_* and λ_* denote the smallest off-diagonal elements of $\boldsymbol{N}\boldsymbol{k}^{-\delta}\boldsymbol{N}'$ and $\boldsymbol{N}\boldsymbol{N}'$, respectively, and let*

$$K = \sum_{j=1}^{r_{[v]}-1} k_{[j]} \quad \text{and} \quad L = \sum_{j=1}^{r_{[v]}-1} k_{[j]}^2.$$

Then each of the conditions

$$\kappa_* > K/[4k_{[b]}(v - k_{[b]})]$$

and

$$\lambda_* > L/[4k_{[b]}(v - k_{[b]})]$$

is sufficient for $\mathcal{D}$ to be maximally robust against the unavailability of data and with respect to the estimability of treatment contrasts.

An immediate consequence of Theorem 2 (1987b) is the following result.

Corollary 2 (1987b). *Let a block design $\mathcal{D}$ be binary and connected, let $r_{[1]} \geq r_{[2]} \geq \cdots \geq r_{[v]}$ and $k_{[1]} \geq k_{[2]} \geq \cdots \geq k_{[b]}$ be its treatment replications and block sizes, and let K be as defined in Theorem 2 (1987b). If $\mathcal{D}$ is VB and*

$$\frac{n-b}{v(v-1)} > \frac{K}{4k_{[b]}(v - k_{[b]})},$$

or if $\mathcal{D}$ is EB and

$$\frac{(n-b)r_{[v-1]}r_{[v]}}{n^2 - \boldsymbol{r}'\boldsymbol{r}} > \frac{K}{4k_{[b]}(v - k_{[b]})},$$

then $\mathcal{D}$ is maximally robust against the unavailability of data and with respect to the estimability of treatment contrasts.

This is due to the fact that a connected and binary block design is VB if and only if

$$\boldsymbol{C} = \boldsymbol{r}^{\delta} - \boldsymbol{N}\boldsymbol{k}^{-\delta}\boldsymbol{N}' = \frac{n-b}{v-1}(\boldsymbol{I}_v - v^{-1}\boldsymbol{1}_v\boldsymbol{1}_v'),$$

and is EB if and only if

$$\boldsymbol{C} = \boldsymbol{r}^{\delta} - \boldsymbol{N}\boldsymbol{k}^{-\delta}\boldsymbol{N}' = \frac{n(n-b)}{n^2 - \boldsymbol{r}'\boldsymbol{r}}(\boldsymbol{r}^{\delta} - n^{-1}\boldsymbol{r}\boldsymbol{r}')$$

[cf., e.g., Caliński and Kageyama (2000, Section 2.4)].

Another consequence of Theorem 2 (1987b) is the following result originally given by Ghosh (1982).

Corollary 3 (1987b). *Every BIB design is maximally robust against the unavailability of data and with respect to the estimability of treatment contrasts.*

Further results on this topic are given by Kageyama and Saha (1987), Kageya- ma (1987), Baksalary and Puri (1990), and Baksalary and Hauke (1992). For other references see Caliński and Kageyama (2003, Section 10.2).

5. Fisher's condition

Attention should also be paid to an interesting paper by Baksalary and Puri (1988) concerning Fisher's (1940) condition for BIB designs. The paper extends some earlier result obtained by Baksalary et al. (1980a) with regard to a direct relationship between EB of a block design and the rank of its incidence matrix $\boldsymbol{N}$. They have replaced the so-called Fisher's inequality, $v \leq b$, by *Fisher's condition*, defined as follows.

Definition (1988). A block design is said to satisfy Fisher's condition if the rows of its incidence matrix are linearly independent.

Baksalary and Puri (1988) have obtained necessary and sufficient conditions that give complete characterizations of all combinatorially-balanced (also called pairwise-balanced) and VB designs which satisfy Fisher's condition (and, consequently, Fisher's inequality). Their main results are as follows.

Theorem 1 (1988). *A combinatorially-balanced (not necessarily binary) block design satisfies Fisher's condition if and only if*

$$r_1^* > \lambda - \frac{\lambda}{1 + \lambda\xi} \quad \textit{and} \quad r_2^* > \lambda,$$

where r_1^* and r_2^*, $r_1^* \leq r_2^*$, are the two smallest numbers among r_i^*, $i = 1, 2, ..., v$, the diagonal elements of the concurrence matrix, $\boldsymbol{N}\boldsymbol{N}'$, of the design, and where λ is the constant off-diagonal element of that matrix and $\xi = \sum_{i=2}^{v} 1/(r_i^* - \lambda)$. (Recall that a block design is combinatorially-balanced if the off-diagonal elements of its matrix $\boldsymbol{N}\boldsymbol{N}'$ are all equal.)

Theorem 2 (1988). *A connected VB (not necessarily binary) block design satisfies Fisher's condition if and only if*

$$r_1 > \theta - \frac{\theta}{v + \theta\zeta} \quad \textit{and} \quad r_2 > \theta,$$

where r_1 and r_2, $r_1 \leq r_2$, are the smallest treatment replications, and where $(v-1)\theta = n - tr(\boldsymbol{N}\boldsymbol{k}^{-\delta}\boldsymbol{N}')$ and $\zeta = \sum_{i=2}^{v} 1/(r_i - \theta)$.

These results strengthen those given by Kageyama and Tsuji (1980, 1984). Certainly, they also complete the result of Baksalary et al. (1980a) for EB designs, which now may be written as follows.

Theorem 1 (1980a). *An EB but not orthogonal block design satisfies Fisher's condition, irrespective of the connectedness or disconnectedness of the design.*

It may be mentioned here, that a more general result can be stated as follows.

A block design satisfies Fisher's condition if and only if the following two equivalent conditions hold:

(a) *the matrix $\boldsymbol{N}\boldsymbol{k}^{-\delta}\boldsymbol{N}'$ has no zero eigenvalues*;

(b) *the matrix $\boldsymbol{C} = \boldsymbol{r}^{\delta} - \boldsymbol{N}\boldsymbol{k}^{-\delta}\boldsymbol{N}'$ has no unit eigenvalue with respect to $\boldsymbol{r}^{\delta}$.*

For a proof see Corollary 2.3.1 in Caliński and Kageyama (2000).

Note, finally, that the latter result corresponds to the following result given in Baksalary (1989). It can be written as follows.

Corollary 2 (1989). *A block design with a $v \times b$ incidence matrix $\boldsymbol{N}$ satisfies the condition*

$$\operatorname{rank}(\boldsymbol{N}) = v - \rho$$

if and only if its matrix $\boldsymbol{C}$ has the unit eigenvalue with respect to $\boldsymbol{r}^{\delta}$ of multiplicity ρ. In particular, the design satisfies Fisher's condition if and only if all the eigenvalues of $\boldsymbol{C}$ with respect to $\boldsymbol{r}^{\delta}$ are strictly less than one, i.e., $\rho = 0$.

This result can also be expressed in terms of the intra-block estimation of some treatment contrasts, because the unit eigenvalue of $\boldsymbol{C}$ with respect to $\boldsymbol{r}^{\delta}$ implies that certain of these contrasts can be estimated intra-block with full efficiency. [For more on this see Caliński and Kageyama (2000, Sections 2.3 and 3.2).]

6. Conclusion

Concluding, it can be said that several results of Baksalary, obtained usually with some co-authors, have clarified certain important aspects of the theory of block designs, particularly those related to

(a) conditions for various concepts of balance,

(b) constructional methods for EB and VB block designs,

(c) conditions for constructing desirable connected designs, PBIB designs in particular,

(d) conditions for a kind of robustness of block designs,

(e) criteria concerning the validity of Fisher's condition for block designs.

Further results of Baksalary, useful for the theory of block designs, concern the estimation of variance components under a mixed model approach, as can be seen, e.g., in Baksalary, Dobek, and Gnot (1990), or in Baksalary, Gnot, and Kageyama (1995). This line of research is, however, beyond the scope of the present paper.

It should also be mentioned that Baksalary later extended his interest from block designs to two-way elimination of heterogeneity designs, giving further interesting results, e.g., in the papers Baksalary and Shah (1992) and Baksalary and Siatkowski (1993).

References

Baksalary, J. K. (1989). A rank characterization of linear models with nuisance parameters and its application to block designs. *J. Statist. Plann. Inference* **22**, 173-179.

Baksalary, J. K., Dobek, A., and Gnot, S. (1990). Characterizations of two-way layouts from the point of view of variance component estimation in the corresponding mixed linear models. *J. Statist. Plann. Inference* **26**, 35-45.

Baksalary, J. K., Dobek, A., and Kala, R. (1980a). A necessary condition for balance of a block design. *Biom. J.* **22**, 47-50.

Baksalary, J. K., Dobek, A., and Kala, R. (1980b). Some methods for constructing efficiency-balanced block designs. *J. Statist. Plann. Inference* **4**, 25-32.

Baksalary, J. K., Gnot, S., and Kageyama, S. (1995). Best estimation of variance components with arbitrary kurtosis in two-way layouts mixed models. *J. Statist. Plann. Inference* **44**, 65-75.

Baksalary, J. K. and Hauke, J. (1992). Minimum number of experimental units in connected block designs with certain additional properties. *J. Statist. Plann. Inference* **30**, 173-183.

Baksalary, J. K. and Puri, P. D. (1988). Criteria for the validity of Fisher's condition for balanced block designs. *J. Statist. Plann. Inference* **18**, 119-123.
Baksalary, J. K. and Puri, P. D. (1990). Pairwise-balanced, variance-balanced and resistant incomplete block designs revisited. *Ann. Inst. Statist. Math.* **42**, 163-171.
Baksalary, J. K. and Shah, K. R. (1992). Some properties of two-way elimination of heterogeneity designs. In: R. R. Bahadur, Ed., *Probability, Statistics and Design of Experiments.* Wiley Eastern, Delhi.
Baksalary, J. K. and Siatkowski, I. (1992). Decomposability of the C-matrix of a two-way elimination of heterogeneity designs. *J. Statist. Plann. Inference* **36**, 301-310.
Baksalary, J. K. and Tabis, Z. (1985). Existence and construction of connected block designs with given vectors of treatment replications and block sizes. *J. Statist. Plann. Inference* **12**, 285-293.
Baksalary, J. K. and Tabis, Z. (1987a). Connectedness of PBIB designs. *Canad. J. Statist.* **15**, 147-150.
Baksalary, J. K. and Tabis, Z. (1987b). Conditions for the robustness of block designs against the unavailability of data. *J. Statist. Plann. Inference* **16**, 49-54.
Caliński, T. and Kageyama, S. (2000). *Block Designs: A Randomization Approach, Volume I: Analysis.* Lecture Notes in Statistics, Volume 150, Springer, New York.
Caliński, T. and Kageyama, S. (2003). *Block Designs: A Randomization Approach, Volume II: Design.* Lecture Notes in Statistics, Volume 170, Springer, New York.
Eccleston, J. A. and Hedayat, A. (1974). On the theory of connected designs: characterization and optimality. *Ann. Statist.* **2**, 1238-1255.
Fisher, R. A. (1940). An examination of the different possible solutions of a problem in incomplete blocks. *Ann. Eugen.* **10**, 52-75.
Ghosh, S. (1982). Robustness of BIBD against the unavailability of data. *J. Statist. Plann. Inference* **6**, 29-32.
Graf-Jaccottet, M. (1977). Comparative classification of block designs. In: J. R. Barra, F. Brodeau, G. Romier and B. van Cutsem (eds.), *Recent Developments in Statistics.* North-Holland, Amsterdam, 471-474.
Jones, R. M. (1959). On a property of incomplete blocks. *J. Roy. Statist. Soc. B* **21**, 172-179.
Kageyama, S. (1982). Connectedness of two-associate PBIB designs. *J. Statist. Plann. Inference* **7**, 77-82.
Kageyama, S. (1987). Some characterization of locally resistant BIB designs of degree one. *Ann. Inst. Statist. Math. Ser. A* **39**, 661-669.
Kageyama, S. and Saha, G. M. (1987). On resistant t-designs. *Ars Combin.* **23**, 81-92.
Kageyama, S. and Tsuji, T. (1980). Some bounds on balanced block designs. *J. Statist. Plann. Inference* **4**, 155-167.
Kageyama, S. and Tsuji, T. (1984). A condition for the validity of Fisher's inequality. *J. Japan. Statist. Soc.* **14**, 85-88.
Nair, K. R. and Rao, C. R. (1948). Confounding in asymmetrical factorial experiments. *J. Roy. Statist. Soc. Ser. B* **10**, 109-131.

Ogawa, J., Ikeda, S., and Kageyama, S. (1984). Connectedness of PBIB designs with applications. *Proc. Sem. Combinatorics and Applications in Honour of Professor S. S. Shrikhande on His 65th Birthday* (Indian Statistical Institute, 14-17 December 1982), pp. 248-255.

Pearce, S. C. (1976). Concurrences and quasi-replication: An alternative approach to precision in designed experiments. *Biom. J.* **18**, 105-116.

Puri, P. D. and Nigam, A. K. (1975a). On patterns of efficiency balanced designs. *J. Roy. Statist. Soc. B* **37**, 457-458.

Puri, P. D. and Nigam, A. K. (1975b). A note on efficiency balanced designs. *Sankhyā B* **37**, 457-460.

Raghavarao, D. (1971). *Constructions and Combinatorial Problems in Design of Experiments.* John Wiley, New York.

Roy, P. M. (1953-1954). Hierarchical group divisible incomplete block designs with m associate classes. *Sci. and Culture* **19**, 210-211.

Vartak, M. N. (1963). Disconnected balanced designs. *J. Indian Statist. Assoc.* **1**, 104-107.

Williams, E. R. (1975). Efficiency-balanced designs. *Biometrika* **62**, 686-689.