13th International Workshop on Matrices and Statistics

Będlewo, Poland
August 18–21, 2004

Program and abstracts

Organizers

- Stefan Banach International Mathematical Center, Warsaw
- Committee of Mathematics of the Polish Academy of Sciences, Warsaw
- Faculty of Mathematics and Computer Science, Adam Mickiewicz University, Poznań
- Institute of Socio-Economic Geography and Spatial Management, Faculty of Geography and Geology, Adam Mickiewicz University, Poznań
- Department of Mathematical and Statistical Methods, Agricultural University, Poznań

This meeting has been endorsed by the International Linear Algebra Society.

Sponsors of the 13th International Workshop on Matrices and Statistics

Mathematical Research and Conference Center in Będlewo

edited by

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and

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Contents

Part I

Introduction

Poetical Licence The New Intellectual Aristocracy

Richard William Farebrother

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Edmund Taylor Whittaker (1887–1956) ↓
Alexander Craig Aitken (1895–1967) ↓
Nora Isobel Calderwood (1896–1985) ↘

Warren Milton Persons (1878–1937) ↘ Florian Cajori (1859–1930) ↙
Harold Thayer Davis (1892–1974) ↓
Theodore Wilbur Anderson (b.1918) ↙

George Peter Hansbenno Styan (b.1937)
↓
Simo Juhani Puntanen (b.1945)

On the first two pages of his autobiography *Goodbye to All That* (Jonathan Cape, 1929; Penguin Classics, 2000), the poet and mythographer Robert Graves (1895–1985) observed that

> Nor had I any illusions about Algernon Charles Swinburne, who often used to [...] pat me on the head and kiss me: [...] I did not know that Swinburne was a poet but knew that he was a public menace. Swinburne, by the way, when a very young man had gone to Walter Savage Lander, then a very old man, and been given the poet's blessing he asked for; and Landor when a child had been patted on the head by Dr Samuel Johnson; and Johnson when a child had been taken to London to be touched by Queen Anne for scrofula, the King's Evil; and Queen Anne when a child ...

During the Eighth International Workshop on Matrices and Statistics (held in Tampere, Finland, in August 1999), I presented the above intellectual genealogy. By contrast with the *Mathematics Genealogy Project* described in *Image* No. 23, this genealogy is not restricted to doctoral supervision but embodies other forms of intellectual contact provided that they go well beyond the laying on of hands described in the quotation.

On the left of the table we find that Edmund Taylor Whittaker supervised the D.Sc. thesis of Alexander Craig Aitken who, in turn, supervised the Ph.D. thesis or other research of Nora Isobel Calderwood. On the right of the table we find that Warren Milton Persons and Florian Cajori both acted as mentors to Harold Thayer Davis who, in turn, taught Theodore Wilbur Anderson. Meanwhile, at the bottom of the table we find that Nora Isobel Calderwood and Theodore Wilbur Anderson both taught George Peter Hansbenno Styan who, in turn, supervised the Ph.D. thesis of Simo Juhani Puntanen.

As it stands, this table will be of particular interest to the students, grand students, and great-grandstudents of Ted Anderson, George Styan, and Simo Puntanen. Further, when augmented by the descent

Edmund Whittaker → Alexander Craig Aitken → James Campbell →
→ Shayle Searle → Harold Henderson

see *Image* No. 22, it will also serve for the intellectual progeny of Shayle Searle and Harold Henderson. Hopefully, the organisers of this Thirteenth Workshop will be able to provide a similar intellectual genealogy for the descendants of Ingram Olkin.

Part II

Local Information

Purpose

The purpose of this Workshop is to stimulate research and, in an informal setting, to foster the interaction of researchers in the interface between statistics and matrix theory. This Workshop will include the presentation of both invited and contributed papers on matrices and statistics. It is expected that many of these papers will be published, after refereeing, in a Special Issue of *Linear Algebra and its Applications* associated with this Workshop.

The workshop will celebrate Ingram Olkin's 80th birthday. Professor Olkin is a world-wide leading expert in statistics. Although his prime research focus is multivariate statistics, his research contributions cover an unusually wide range from pure mathematics to educational statistics. Many of his papers and books are classics in their fields, like his book with Albert Marshall on majorization and related distributional and inequality results. His statistical work has applications in medicine, and his book with Larry V. Hedges on meta-analysis has become the basic methodology for combining the results of independent studies. His bibliography includes nearly 200 publications, five authorized books, seven edited books, and two translated books. He has served as a member of editorial boards of some major statistics journals as well as of Linear Algebra and Its Applications. Professor Ingram Olkin has promoted nearly forty PhD students in pure and applied statistics and in statistical methods in educational research.

Previous workshops

The previous twelve Workshops were held as follows:

- Tampere, Finland: August 1990.
- Auckland, New Zealand: December 1992.
- Tartu, Estonia: May 1994.
- Montréal, Québec, Canada: July 1995.
- Shrewsbury, England: July 1996.
- Istanbul, Turkey, August 1997.
- Fort Lauderdale, Florida, USA, December 1998, in Celebration of T. W. Anderson's 80th Birthday.
- Tampere, Finland, August 1999.
- Hyderabad, India, December, 2000 in Celebration of C.R. Rao's 80th Birthday.
- Voorburg, The Netherlands, August 2001.
- Lyngby, Denmark, August 2002, in Celebration of G.P.H. Styan's 65th Birthday.
- Dortmund, Germany, August 5-8, 2003.

Forthcoming workshop

14th International Workshop on Matrices and Statistics to be held at Massey University, Albany Campus, Auckland, New Zealand,
March 29 – April 1, 2005.
The Workshop will include invited and contributed talks. It is intended that refereed Conference Proceedings will be published.
Furher details will become available on the conference website
http://iwms2005.massey.ac.nz/.
The website will be updated on a regular basis.
IWMS-2005 is a Satellite Conference to the 55th Biennial Session of the International Statistical Institute to be held in Sydney April 5 – 12, 2005.

The Local Organising Committee is Chaired by Jeff Hunter
<j.hunter@massey.ac.nz>.

The International Organizing Committee consists of

- George Styan (Chair) <styan@math.mcgill.ca>,
- Hans Joachim Werner (Vice-Chair) <werner@united.econ.uni-bonn.de>,
- Simo Puntanen <Simo.Puntanen@uta.fi>.

Organizing Committees

The International Organizing Committee for this Workshop comprises

- R. William Farebrother (Shrewsbury, England),
- Simo Puntanen (Tampere, Finland; chair),
- George P. H. Styan (Montral, Canada; vice-chair),
- Hans Joachim Werner (Bonn, Germany).

The Local Organizing Committee comprises

- Jan Hauke, Institute of Socio-Economic Geography and Spatial Management, Faculty of Geography and Geology, Adam Mickiewicz University, Poznań,
- Augustyn Markiewicz (chair), Department of Mathematical and Statistical Methods, Agricultural University, Poznań,
- Tomasz Szulc, Faculty of Mathematics and Computer Science, Adam Mickiewicz University, Poznań,
- Waldemar Wołyński, Faculty of Mathematics and Computer Science, Adam Mickiewicz University, Poznań.

Location

The 13th International Workshop on Matrices and Statistics (IWMS-2004) will be held in Będlewo, about 30 km south of Poznań, Poland, from 18th by 21st August 2004. Będlewo is the Mathematical Research and Conference Center of the Polish Academy of Sciences; the setting is similar to Oberwolfach, with accommodation on site.

Poznań is one of the oldest cities and the greatest academic centers in Poland. It has over half million inhabitants and it is located about 300 km west of Warsaw. There is an airport which offers a number of international connections.

Call for Papers

We are pleased to announce a special issue of Linear Algebra and Its Applications devoted to this workshop. It will include selected papers strongly correlated to the talks of the conference. We encourage submissions on the theory of matrices and methods of linear algebra with statistical origin or possible applications in statistics.

All papers submitted must meet the publication standards of Linear Algebra and Its Applications and will be subject to normal refereeing procedure.

The deadline for submission of papers is the end of February, 2005, and the special issue is expected to be published in 2006.

Papers should be sent to any of its special editors, preferably by email in a PDF or PostScript format:

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Part III

Program

Program

Tuesday, August 17, 2004

13:00–15:00 Lunch

14:00–19:00 Registration

19:00– Reception

Wednesday, August 18, 2004

8:00–9:00 Breakfast

Opening:

9:00–9:20 *Z. Palka* - Dean of the Faculty of Mathematics and Computer Sciences of Adam Mickiewicz University

Session I – Chair with comments *G.P.H.Styan*

9:20–10:05 Opening lecture – *Gene H. Golub:* Numerical methods for solving least squares problems with constraints
10:05–10:15 Introduction to Nokia Lecturer – *G.P.H. Styan*
10:15–11:30 Nokia Lecture – *Ingram Olkin:* Inequalities: some probabilistic, some matric, and some both

11:00–11:30 Coffee break

Session II – Chair *R. Bru*

11:30–12:00 *L. Elsner:* Nonnegative matrices, max-algebra and applications
12:00–12:30 *V. Mehrmann:* Numerical solution of the eigenvalue problem for the Anderson Model
12:30–13:00 *C. Johnson:* Words in two positive definite letters

13:00–15:00 Lunch

Session IIIa – Chair *V. Mehrmann*

15:00–15:20 *U. Rothblum:* One-dimensional optimal bounded-shape partitions for Schur convex sum objective functions
15:20–15:40 *K. Ziętak:* Family of Gander's methods and approximation of matrices
15:40–16:00 *I. Wróbel:* On the numerical range of powers of matrices
16:00–16:20 *D. Wojtera-Tyrakowska:* Some properties of equiradial and equimodular sets
16:20–16:40 *V. Prokip:* On common divisors of matrices over principal ideal domain

Session IIIb – Chair *G. Trenkler*

15:00–15:20 *R. Kala:* Two local operators and the BLUE
15:20–15:40 *H. Drygas:* Linear minimax-estimation in the three parameter case
15:40–16:00 *Y. Takane:* Two interesting metric matrices in statistics
16:00–16:20 *J. Isotalo:* Linear prediction sufficiency for new observations in the general Gauss–Markov model
16:20–16:40 *P. Pordzik:* On linear sufficiency with respect to given parametric functions

16:40–17:10 Coffee break

Session IVa – Chair *E.P. Liski*

17:10–17:30 *T. Kollo:* Multivariate skewness and kurtosis measures
17:30–17:50 *H. Oja:* Some notes on scatter matrices and independent component analysis (ICA)
17:50–18:10 *S. Sirkia:* One-sample spatial sign and rank methods
18:10–18:30 *E. Ollila:* Unitary invariant random Hermitian matrices and complex elliptical distributions
18:30–18:50 *I. Malinowska:* Estimation of location and scale parameters using k-th record values
18:50–19:10 *T. Benesch:* Adaptive Designs for Clinical Trials: An Overview

Session IVb – Chair *R. Kala*

17:10–17:30 *H. Naseri:* Linear prediction for electricity consumption with Levy distribution
17:30–17:50 *R. Covas:* Exact distributions for certain linear combinations of Chi-Squares
17:50–18:10 *T. Ostrowski:* Population equilibrium and its fitness in evolutionary matrix games
18:10–18:30 *M. Wilkos:* Some properties of sample characteristics from nonnegative data
18:30–18:50 *K. Szuster:* Almost sure Central Limit Theorem for subsequences
18:50–19:10 *C.A. Coelho:* The singular value decomposition as a basic tool in generalized canonical analysis and related linear models

19:10– Dinner

Thursday, August 19, 2004

8:00–9:00 Breakfast

Session V – Chair *T. Caliński*

9:00– 9:30 *T.W. Anderson:* Asymptotic distribution of a set of linear restrictions on regression coefficients
9:30–10:00 *J.K. Baksalary:* Relationships between partial orders of Hermitian matrices and their powers
10:00–10:20 *S. Puntanen:* On decomposing the Watson efficiency of ordinary least squares in a partitioned weakly singular linear model
10:20–11:00 *G.P.H. Styan:* Inequalities and equalities for the generalized efficiency function in orthogonally partitioned linear models

11:00–11:30 Coffee break

Session VI – Chair *H.J. Werner*

11:30–12:00 *B.K. Sinha:* Some combinatorial aspects of a counterfeit coin problem
12:00–12:30 *C.M. Cuadras:* Parametric multiple correspondence analysis
12:30–13:00 *G. Tusnady:* Reconstruction of Kauffman networks applying trees
13:00–13:30 *B. Uhrin:* A problem in multivariate analysis

13:30–15:00 Lunch

15:00–18:30 Excursion – Kórnik Castle

19:00– Conference Dinner

Friday, August 20, 2004

8:00–9:00 Breakfast

Session VII – Chair *B.K. Sinha*

9:00– 9:30 *J. Kunert:* On optimal cross-over designs when carry-over effects are proportional to direct effects
9:30–10:00 *P. Druilhet:* Optimal designs for total effects
10:00–10:30 *R.J. Martin:* Optimal experimental designs when most treatments are unreplicated
10:30–11:00 *N.K. Mandal:* Optimum choice of covariates in BIBD setup

11:00–11:30 Coffee break

Session VIII – Chair *H. Oja*

11:30–12:00 *T. Ledwina:* Data driven score test of fit for semiparametric homoscedastic linear regression model
12:00–12:30 *E.P. Liski:* The MDL model choice for linear regression
12:30–13:00 *P. Major:* Sharp estimates on the tail behaviour of some random integrals and their application in statistics

13:00–15:00 Lunch

Session IXa – Chair *J. Kunert*

15:00–15:20 *C. Stępniak:* Canonical form of a linear model and its applications
15:20–15:40 *M. Grzdziel:* Quadratic subspaces and construction of admissible estimators of variance components
15:40–16:00 *K. Filipiak:* On optimality of binary designs under interference models
16:00–16:20 *T. Nahtman:* Permutation invariant covariance marices
16:20–16:40 *L. Koskela:* Analysis of growth curve data by using cubic smoothing splines

Session IXb – medical – Chair *J. Hauke*

15:00–15:45 *Ingram Olkin* lecture part I
15:50–16:00 break
16:00–16:40 *Ingram Olkin* lecture part II

16:40–17:10 Coffee break

Session Xa – Chair *J.J. Hunter*

17:10–17:30 *A. Klein:* An explicit expression for the Fisher information matrix of a multiple time series process
17:30–17:50 *J. Ledoux:* Criteria for the comparison of discrete-time Markov chains
17:50–18:10 *I. Traat:* Bias of regression estimator in survey sampling
18:10–18:30 *P. Comon:* Blind identification of linear mixtures
18:30–18:50 *T. Górecki:* Sequential method in discriminant analysis

Session Xb – Chair *L. Elsner*

17:10–17:30 *B. Schaffrin:* Reliability analysis in linear models
17:30–17:50 *R. Szwedek:* Interpolation of measure of non-compactness and applications to spectral theory
17:50–18:10 *A. Maćkiewicz:* A new rank revealing tri-orthogonalization algorithm and its applications
18:10–18:30 *M. Niezgoda:* On the structure of a class of normal decomposition systems
18:30–18:50 *Y. Tian:* On projectors with respect to seminorms

Session Xc – medical – Chair *J. Hauke*

17:10–17:50 *I. Roterman-Konieczna (SAS):* Survival analysis in SAS
17:50–18:00 break
18:00–18:30 *J. Hauke and W. Wołyński:* How to avoid an overinterpretation of the results of statistical analyzes in medical research

Presentation – plenary session

19:00–19:20 *SAS software presentation*

19:30– Barbecue

Saturday, August 21, 2004

8:00–9:00 Breakfast

Session XI – Chair *G.H. Golub*

9:00– 9:30 *R. Bru:* Schwarz iterations for singular systems of Markov chains
9:30–10:00 *J.J. Hunter:* Mixing times and their application to perturbed Markov chains
10:00–10:30 *D. von Rosen:* Some results on patterned matrices
10:30–11:00 *H. Yanai:* Non-negative determinant of a rectangular matrix: Its definition and applications to multivariate data analysis

11:00–11:30 Coffee break

Session XII – Chair *S. Puntanen*

11:30–12:00 *H.J. Werner:* BLUPs and BLIMBIPs in the general Gauss–Markov model
12:00–12:30 *J. Gross:* Restricted ridge estimation
12:30–13:00 *J.T. Mexia and M. Fonseca:* Statistical analysis of normal orthogonal models with emphasis on their algebraic structure in view of obtaining effcient statistics for inference
13:00–13:30 *W. Ratajczak:* Spectral matrix decomposition in geographical research

13:30–15:00 Lunch

Mitra session – Chair *J.K. Baksalary*

15:00–15:20 *G. Trenkler:* On generalized quadratic matrices
15:20–15:40 *P. Kik:* Characterizations of the commutativity of projectors referring to generalized inverses of their sum and difference
15:40–16:00 *K. Chylińska:* A specific form of the generalized inverse of a partitioned matrix useful in econometrics
16:00–16:20 *A. Kuba:* Invariance of matrix expressions with respect to specific classes of generalized inverses
16:20–16:40 *O.M. Baksalary:* Further generalizations of a property of orthogonal projectors
16:40–17:00 **Closing**

17:00–17:30 Coffee break

17:30–19:00 *Farewell - informal discussions*

19:00– Dinner

Part IV

Ingram Olkin

Ingram Olkin, Statistical Statesman *

Yadolah Dodge

University of Neuchâtel, Switzerland

Ingram Olkin was born on 23rd July 1924 in Waterbury, Connecticut, USA. He received a Bachelor's degree from the College of the City of New York in 1947, after serving in the army as a meteorologist. He then continued his studies, received a Master's degree in Mathematical Statistics from Columbia University in 1949, and a Ph.D. in Mathematical Statistics from the University of North Carolina in 1951. He taught at Michigan State University and the University of Minnesota before assuming from 1961 his present position at Stanford University as Professor of Statistics and Education.

Dr. Olkin has been an editor of *The Annals of Statistics* and an associate editor of *Psychometrika*, the *Journal of Educational Statistics*, the *Journal of the American Statistical Association*, *Linear Algebra and its Application* and other mathematics and statistical journals. His major interests are in multivariate analysis, inequalities, in the theory and application of metaanalysis and in models in the social, behavioral, and biological sciences. He has cowritten a number of books including: *Inequalities Theory of Majorization and its Applications* (1979), *Selecting and Ordering Populations* (1980), *Probability Models and Applications* (1980), and most recently, *Statistical Methods in Meta-Analysis* (1985). He has also served as chairman of the Committee of Applied and Theoretical Statistics, National Research Council, National Academy of Sciences; chairman of the Special Interest Group in Educational Statistics; President of the Institute of Mathematical Statistics, and Chair, Committee of Presidents of Statistical Societies. He is currently a member of the Technical Advisory Committee for the National Assessment of Educational Progress and a member of the Board of Trustees, National Institute of Statistical Sciences. He is a Fellow of the American Statistical Association and the Institute of Mathematical Statistics. He is a recipient of the Wilks Medal, and was awarded an honorary Doctor of Science degree by de Montfort University.

In "A Conversation with Ingram Olkin" (published in Gleser, L.J. et al. Eds. (1989), *Contributions to Probability and Statistics: Essays in Honor of Ingram Olkin*, Springer, pp. 7-33), Ingram Olkin answers to the question "What is your assessment of the current state of the health of the field of statistics, and where do you see the field heading?" as follows:

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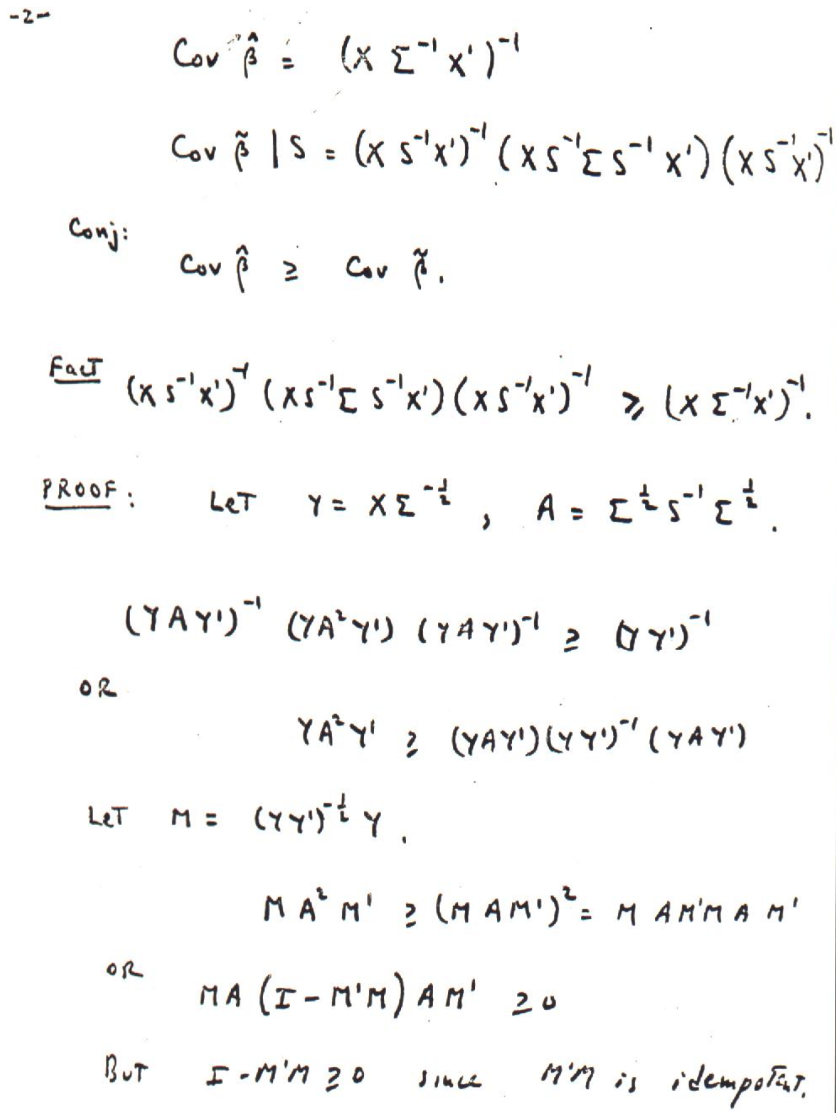
"I am a bit worried about statistics as a field. As you know, I come from mathematical community and I've always liked the mathematics of statistics. But I think that the connection with applications is an essential ingredient at this time. I say that because applications are crying out for statistical help. We currently produce approximately 300 Ph.D.'s in statistics and probability per year. This is a small number considering the number of fields of application that need statisticians. Fields such as geostatistics, psychometrics, education, social science statistics, newer fields such as chemometrics and legal statistics generate a tremendous need that we are not fulfilling. Inevitably this will mean that others will fulfill those needs. If that happens across fields of application, we will be left primarily with the mathematical part of statistics, and the applied parts will be carried out by others not well-versed in statistics. Indeed, I think that a large amount of statistics is now being carried out by non-statisticians who learn their statistics from computer packages and from short courses. So I worry about this separation between theory and practice and the fact that we are not producing the number of doctorates to fulfill needs in all of these other areas".

He then adds: "There is still an excitement in the field, but my impression is that, except for a few places, the growth in statistics departments has reached a plateau. I believe that this is true because we do not have a natural mechanism for statistics departments to create strong links to other departments of the academic community. Academic institutions have not been designed for cross-disciplinary research, and indeed may actually be antagonistic to cross-disciplinary research".

Memories from the International Statistical Institute Conference of 1961 in Paris.

From left to right, Sam Greenhouse, R.A. Fisher, an unknown participant, Carol Parzen, Ingram Olkin and Manny Parzen.

From bottom to top are on the left side of the table, an unknown participant, Elizabeth Scott, Jerzy Neyman, Anne Durbin, Jim Durbin, Miriam Chernoff and Herman Chernoff. From bottom to top are on the right side of the table, Jack Youden, Ingram Olkin, Dorothy Gilford, Manny Parzen, Carol Parzen, Ellen Chernoff and Judy Chernoff.



-2-

$$\text{Cov}\ \hat\beta = (X \Sigma^{-1} X')^{-1}$$

$$\text{Cov}\ \tilde\beta \mid S = (X S^{-1} X')^{-1} (X S^{-1} \Sigma S^{-1} X') (X S^{-1} X')^{-1}$$

Conj: $\text{Cov}\ \hat\beta \geq \text{Cov}\ \tilde\beta.$

Fact $(X S^{-1} X')^{-1} (X S^{-1} \Sigma S^{-1} X') (X S^{-1} X')^{-1} \geq (X \Sigma^{-1} X')^{-1}.$

PROOF: Let $Y = X \Sigma^{-\frac{1}{2}}$, $A = \Sigma^{\frac{1}{2}} S^{-1} \Sigma^{\frac{1}{2}}$.

$$(YAY')^{-1} (YA^2Y') (YAY')^{-1} \geq (YY')^{-1}$$

or

$$YA^2Y' \geq (YAY')(YY')^{-1}(YAY')$$

Let $M = (YY')^{-\frac{1}{2}} Y$.

$$MA^2M' \geq (MAM')^2 = MAM'MAM'$$

or $MA(I - M'M)AM' \geq 0$

But $I - M'M \geq 0$ since $M'M$ is idempotent.

Olkin's handwriting showing that one estimator is preferable to another by showing that the covariance matrices are ordered (from class notes about 1968).

Why is matrix analysis part of the statistics curriculum *

Ingram Olkin

Stanford University, USA

Abstract

We provide a discussion of several important areas of intersection between matrix theory and statistics.

Keywords

Matrix factorizations, extremal problems, multivariate distributions, inequalities.

1 Introduction

The Masters program at the Université de Neuchâtel lists only one mathematics course "Matrix Theory and Inequalities". One might argue for the inclusion of other mathematics courses, but it is hard to argue that this topic does not play a central role in many statistical contexts. Matrix analysis is part of a warehouse of handy mathematical tools.

In this note I review some areas in which matrix theory (including inequalities) have natural statistical origins:

1. Matrix factorizations,
2. Extremal problems,
3. Multivariate integrals,
4. Inequalities,
5. Independence properties.

Each of these topics is large, and we provide only a few examples to illustrate the intersection of statistics and matrix analysis.

2 Matrix factorizations

This is a topic that is currently in vogue, primarily because factorizations often help to reduce a problem to a simpler form. Only a few factorizations

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suffice for most statistical purposes. In the following all matrices are real; there are counterpart factorizations for complex matrices.

Theorem 1 *Every $p \times p$ symmetric matrix S can be factored as*

$$S = GDG',$$

where G is orthogonal, $D = diag(\lambda_1, \dots, \lambda_p)$, and $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_p$ are the ordered eigenvalues of S.

Theorem 2 *Every $p \times p$ positive semi-definite matrix S can be factored as*

$$S = LL',$$

where $L = (l_{ij})$ is lower triangular with $l_{ii} \geq 0$.

The matrix L is sometimes called the triangular square root of S, in contrast to the symmetric positive semi-definite square root V in $S = V^2$. In Theorem 1 if $S = GDG'$, then $V = GD^{1/2}G'$.

Theorem 3 *Every $p \times p$ matrix A can be factored as*

$$A = GDH,$$

where G and H are orthogonal, $D = diag(\alpha_1, \dots, \alpha_p)$, and $\alpha_1 \geq \cdots \geq \alpha_p \geq 0$ are the ordered singular values of A, that is

$$\alpha_i = \lambda_i^{1/2}(AA'), \qquad i = 1, \dots, p.$$

Other factorizations arise from numerical analytic problems. For example, every $p \times p$ matrix $A = QR$, where Q is lower triangular with $q_{ii} = 1$, and R is upper triangular.

3 Extremal problems

The method of maximum likelihood is a basic concept in the estimation of the parameters of a distribution. In univariate analysis the maximizations can often be carried out with standard calculus methods. However, in multivariate analysis we almost always are confronted with a matrix problem, as in the following:

$$\max_{\Omega} (\det \Sigma)^{-n/2} \exp\left[-\frac{1}{2} tr\,(\Sigma^{-1}S)\right], \tag{1}$$

where $\Omega = \{\, \Sigma : \Sigma$ is positive definite$\}$ and S is positive definite.

To simplify this problem note that $tr\,(\Sigma^{-1}S) = tr\,(V\Sigma^{-1}V)$, where $V = S^{1/2}$. With $\Psi = V\Sigma^{-1}V$ (1) becomes (except for a constant)

$$\max_{\Omega_1} (\det \Psi)^{-n/2} \exp\left[-\frac{1}{2} tr\,\Psi\right], \tag{2}$$

where now $\Omega_1 = \{\Psi : \Psi$ is positive definite$\}$.

Using the factorization of Theorem 1: $\Psi = GDG'$, where G is orthogonal and
$D = diag(\lambda_1, \ldots, \lambda_p)$ and the λ_i are the eigenvalues of Ψ we obtain from (2):

$$\max_{\lambda_i > 0} \left(\prod_1^p \lambda_i \right)^{-n/2} \exp\left[-\frac{1}{2} \sum_1^p \lambda_i \right] = \prod_1^p \max_{\lambda_i > 0} \left(\lambda_i^{-n/2} e^{-\frac{1}{2}\lambda_i} \right).$$

We have now reduced the multivariate problem to a product of univariate problems, for which the solution is $\widehat{\lambda}_i = n$. Consequently, the maximum likelihood estimator $\widehat{\Psi}$ of Ψ is

$$\widehat{\Psi} = G(nI)G' = nI,$$

from which

$$\widehat{\Sigma} = V\widehat{\Psi}^{-1}V = V^2/n = S/n.$$

This result is the multivariate version of the maximum likelihood estimator of the population variance.

4 Multivariate integrals

The normal distribution and the chi-square distribution are central distributions in univariate analysis. The multivariate counterparts are the multivariate normal distribution and the Wishart distribution. The latter generates an integral that we need to evaluate:

$$\int_{\Omega} (\det S)^{(n-p-1)/2} \exp\left[-\frac{1}{2} tr\, S \right] \prod_{i<j} ds_{ij}, \tag{3}$$

where $\Omega = \{ S : S$ is positive definite$\}$.

To simplify this problem we use the factorization of Theorem 2: $S = LL'$. Then

$$\det S = \prod l_{ii}^2, \qquad tr\, S = \sum_{i \leq j} l_{ij}^2.$$

Consequently, the integral (3) becomes

$$\int_{\omega} \prod l_{ii}^{n-p-1} \exp\left[-\frac{1}{2} \sum_{i \leq j} l_{ij}^2 \right] J \prod_{i<j} dl_{ij}, \tag{4}$$

where $\omega = \{ l_{ij} : l_{ii} > 0, -\infty < l_{ij} (i \neq j) < \infty \}$, and J is the Jacobian of the transformation involved in Theorem 2. We do not provide the details of the computation of

$$J = 2^p \prod_1^p l_{ii}^{p-i+1},$$

but urge the reader to verify this result for $p = 3$ or 4. Using this value of J the integral (4) reduces to a product of integrals

$$2^p \prod_1^p \left\{ \int_{l_{ii}>0} l_{ii}^{n-i} e^{-\frac{1}{2}l_{ii}^2}\, dl_{ii} \right\} \left(\int_{-\infty<l_{ij}<\infty} e^{-\frac{1}{2}l_{ij}^2}\, dl_{ij} \right)^{p(p-1)/2} , \tag{5}$$

where now each integral is a univariate integral. As

$$\int_0^\infty x^{n-i} e^{-\frac{1}{2}x^2}\, dx = 2^{\frac{n-i+1}{2}}\, \Gamma\left(\frac{n-i+1}{2}\right)$$

and

$$\int_{-\infty}^\infty e^{-\frac{1}{2}x^2}\, dx = \sqrt{2\pi},$$

the integral (5) is equal to

$$2^{pn/2} \pi^{p(p-1)/4} \prod_1^p \Gamma\left(\frac{n-i+1}{2}\right).$$

5 Inequalities

The Cauchy-Schwarz inequality is a cornerstone inequality that arises under a variety of guises. For vectors $x = (x_1, \ldots, x_n)$ and $y = (y_1, \ldots, y_n)$ the C-S inequality is

$$(xy')^2 \le (xx')(yy'). \tag{6}$$

If we let $x = uA^{1/2}$ and $y = vA^{-1/2}$, where A is positive definite, then (6) becomes

$$(uu')^2 \le (uAu')(vA^{-1}v').$$

Throughout Inequality (6) has an integral representation

$$\left(\int f(x)\, g(y)\, dx\, dy \right)^2 \le \int f^2(x)\, dx \int g^2(y)\, dy \tag{7}$$

from which we obtain that the squared correlation $\rho^2(x, y)$ between the random variables x and y is less than or equal to 1.

A more general inequality than (7) is the Hölder inequality

$$\int \prod_1^n f_i^{q_i}(x)\, dx \le \prod_1^n \left[\int f_i(x)\, dx \right]^{q_i}, \quad q_i \ge 0, \quad \sum q_i = 1. \tag{8}$$

In (7) and (8) we assume finite integrability.

A discussion of inequalities would be incomplete without a mention of the arithmetic mean-geometric mean inequality

$$\sum_1^n x_i g_i \geq \prod_1^n x_i^{g_i}, \quad x_i \geq 0, \quad g_i \geq 0, \quad \sum g_i = 1.$$

Even when the x_i may be negative, and $x_{(1)} \leq x_{(2)} \leq \cdots \leq x_{(n)}$, we have that

$$x_{(1)} \leq \sum_1^n x_i g_i \leq x_{(n)}. \tag{9}$$

In another context, if x is a vector and S is a positive definite matrix, then

$$\lambda_1 \leq \frac{xSx'}{xx'} \leq \lambda_n, \tag{10}$$

where $\lambda_1 \leq \cdots \leq \lambda_n$ are the eigenvalues of S. Inequality (10) can be obtained by using the factorization of Theorem 1.

The theory of least squares provides a variety of opportunities for the development of inequalities. Given the equation

$$xA = b,$$

if A is invertible, then the solution is $x = bA^{-1}$. However, if x is a $1 \times p$ vector and A is a $p \times n$ matrix, then A does not have an inverse. This leads to the development of a theory of generalized inverses, and permits us to find the vector x_0 for which $x_0 A$ is "closest" to b. That is, the Euclidean distance $\|x_0 A - b\|$ is minimized.

6 Independence properties

If $x_1, \ldots, x_n$ are independent random variables, with a common standard normal distribution, we ask when are two linear forms $L_1 = xa'$ and $L_2 = xb'$ independent. The answer is that a and b must be orthogonal, that is $ab' = 0$.

This suggests that we ask when is linear form $L = xa'$ independent of a quadratic form $Q = xAx'$, where A is symmetric. Now the answer is that $aA = 0$.

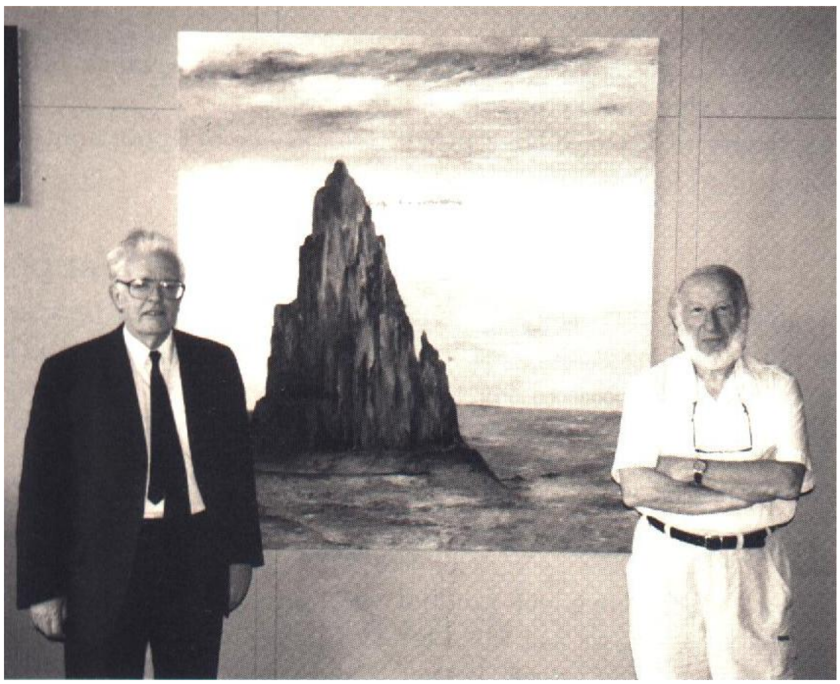
To move to the next step, when are two quadratic forms $Q_1 = xAx'$ and $Q_2 = xBx'$, where A and B are symmetric, independent. The natural conjecture is that $AB = BA = 0$. Indeed, this is the correct answer. But in order to prove this we have an intermediate step:

$$\det(I - \alpha A)\,\det(I - \beta B) = \det(I - \alpha A - \beta B) \tag{11}$$

must holds for all α and β. The proof that (11) implies $AB = BA = 0$ is not simple. A good exercise is to show this result for the special case $n = 2$.

7 Summary

The move from univariate statistical inference to multivariate statistical inference means that matrices will replace scalars. This is turn generates a wide class of problems. In this short summary we attempt to provide only a few prototype problems that arise in a statistical context. Matrix analysis is a useful tool that permits insights into the geometry of some of the models. It is difficult to imagine a curriculum in which a knowledge of linear algebra and matrix analysis is omitted.



Ingram Olkin with Sir David Cox and a mountain of yet unsolved problems.

A brief biography and appreciation of Ingram Olkin *

Ingram Olkin, known affectionately to his friends in his youth as "Red", was born July 23, 1924 in Waterbury, Connecticut. He was the only child of Julius and Karola (Bander) Olkin. His family moved from Waterbury to New York City in 1934. Ingram graduated from the Bronx's DeWitt Clinton High School in 1941, and began studying statistics in the Mathematics Department at the City College of New York. After serving as a meteorologist in the Air Force during World War II (1943-1946), achieving the lank of First Lieutenant, Ingram resumed his studies at City College. He received his B.S. in mathematics in 1947.

Ingram then began graduate study in statistics at Columbia University, finishing his M.A. in mathematical statistics in 1949. He completed his professional training at the University of North Carolina, Chapel Hill, by obtaining a Ph.D. in mathematical statistics in 1951.

During his tour of duty in the Air Force, Ingram met Anita Mankin. They were married on May 19, 1945. Their daughters Vivian, Rhoda and Julia were born, respectively, in 195O, 1953 and 1959. Ingram and Anita now are the proud grandparents of three grandchildren.

Ingram began his academic career in 1951 as an Assistant Professor in the Department of Mathematics at Michigan State University. He early on demonstrated his penchant for "visiting" by spending 1955-1956 at the University of Chicago and 1958-1959 at Stanford University. Ingram was promoted to Professor at Michigan State, but left in 1960 to become the Chairman of the Department of Statistics at the University of Minnesota. Shortly afterward in 1961 he moved to Stanford University to take a joint position, which he holds to this day, as Professor of Statistics and of Education. From 1973-1976, he was also Chairman of the Department of Statistics at Stanford.

Ingram's professional accomplishments span a broad spectrum, and have made and continue to make a significant impact upon the profession of statistics. He is an outstanding and prolific researcher and author, with nearly thirty PhD. students in both statistics and education. The professional societies in statistics and their journals have greatly benefited from his leadership and guidance. His contributions at the federal level include his work with the National Research Council, National Science Foundation, Center for Educational Statistics, and the National Bureau of Standards.

Over one hundred publications, five authored books, six edited books and two translated works are included in his bibliography. Although his prime

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research focus is multivariate statistics, his research contributions cover an unusually wide range from pure mathematics to educational statistics. Many of his papers and books are virtually classics in their fields - notably his work with Al Marshall on majorization and related distributional and inequality results. His statistical meta-analysis research and book with Larry Hedges are also extremely influential. His text books on probability and on ranking and selection have made novel pedagogical contributions, bringing statistics to a broader nontechnical audience. Also of substantial value to the profession has been his editing of the *Annals of Statistics Index* and the three volume set *Incomplete Data in Sample Surveys* which derived from the Panel on Incomplete Data, which he chaired (1977-1982) for the National Research Council.

Among Ingram's significant contributions to the statistical profession has been his fostering of the growth of quality journals of statistics. He was a strong proponent of splitting the *Annals of Mathematical Statistics* into the *Annals of Statistics* and the *Annals of Probability*. He oversaw this transition as the last editor (1971-1972) of the *Annals of Mathematical Statistics* and the first editor (1972-1974) of the *Annals of Statistics*. As President of the Institute of Mathematical Statistics (1984-1985), he was instrumental in initiating the journal *Statistical Science* and has served in the capacity of co-editor since its inception. He was also influential in introducing the IMS Lecture Notes - Monograph Series. Furthermore, he was heavily involved in the establishment of the *Journal of Educational Statistics*, for which he served as Associate Editor (1977-1985) and as Chair of the ASA/AERA Joint Managing Committee. In all these and numerous other editorial activities, he strongly supports and encourages the major statistics journals to publish applications of statistics to other fields and to build ties with other scientific societies' publications.

Ingram's activities also extend to his work on governmental committees. He was the first Chair of the Committee on Applied and Theoretical Statistics (1978-1981) of the National Research Council, and also was a member for six years of the Committee on National Statistics (1977-1983). He currently is involved with a major project to construct a national data base for educational statistics.

As Ingram will happily admit, he is a prolific traveler. He has given seminars at more than sixty American and Canadian universities, and at numerous universities in twenty five other countries. He also has attended statistical meetings throughout the world, and has been a visiting faculty member or research scientist at Churchill College (Cambridge University), Educational Testing Service (Princeton, NJ), Imperial College, The University of British Columbia, the University of Copenhagen (as a Fulbright Fellow), Eidgenüssische Technische Hochschule (Switzerland), the National Bureau of Standards, Hebrew University, and the Center for Educational Statistics.

Anyone wishing to call Ingram has to be prepared to be forwarded from one phone number to another.

In his travels, Ingram has tirelessly promoted and advanced the discipline of statistics. On an outside review committee at a university, he will convince the dean to take steps to form a new department of statistics. On a governmental panel, he will persuade an agency to seek input from statisticians. He has been an effective advocate for increased interdisciplinary ties both in universities and in government, and has been equally successful in convincing deans and statistics department heads of the need to reward statistical consulting. At most statistics meetings, you will find Ingram in constant conversation - perhaps promoting a new journal, encouraging progress of a key committee, or giving advice about seeking grants or allocating funds. His public accomplishments are many and impressive, but equally important are his behind-the-scenes contributions.

Ingram flourishes when working with others. Many of his published papers are collaborations, and his collaborative relationships tend to be long lasting. Ingram is always bursting with new ideas and projects, and delighted when a common interest develops. His enthusiasm is contagious, and his energy and positive outlook (which are legendary in the field of statistics) are tremendously motivating to all around him.

In describing Ingram, one cannot simply list his personal accomplishments. He is above all a remarkably charming and unpretentious person, who gives much of himself to his family, friends and colleagues. For his former students and the many young statisticians he has mentored, he is a continual source of wisdom, guidance and inspiration. All of us whose lives have been touched by Ingram view him with deep personal affection and great professional admiration.

A conversation with Ingram Olkin ⋆

Early in 1986, a new journal *Statistical Science* of the Institute of Mathematical Statistics appeared. This is a journal Ingram Olkin was intimately involved in founding. One of the most popular features of *Statistical Science* is its interviews with distinguished statisticians and probabilists. In the spirit of those interviews, the Editors of this volume wanted to include an interview with Ingram. However, one does not "interview" Ingram; one simply starts him talking, and sits back to listen and enjoy.

The following conversation took place at the home of S. James Press in Riverside, California in November of 1988.

Press: I am pleased to have this opportunity to interview you. How did you initially get interested in the subject of statistics?

Olkin: To tell the truth, I'm not quite sure. What I do know is that in my high school year book dated 1941 each student listed the profession that he wanted to follow; mine was listed as a statistician. I am quite sure that at that time I did not know what a statistician did, nor what kind of profession it was.

I was a mathematics major in DeWitt Clinton High School, which was an all male school, and then went to CCNY - The College of the City of New York, now called City University of New York. At City College I was a mathematics major and took a course in mathematical statistics. This was taught by Professor Selby Robinson, who became quite well known for having indoctrinated many of the statisticians who are currently at various universities, in government, or in industry.

It was through this course that I became interested in the subject. Selby was not a great teacher, but he was a lovely person who somehow managed to communicate an interest in the field. It may have been that I was challenged to find out more about the subject.

Press: I would like to hear more about Selby Robinson, and your courses with him.

Olkin: I believe that he got his degree at Iowa. He did publish a paper in 1937 on the chi-square distribution. The book we used in class was Kenney and Keeping, which was one of the few mathematically oriented texts.

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In the applications course we used Croxton and Cowden, which was a classic applied statistics text.

Anyone who was at CCNY and took a course in mathematical statistics probably studied with Selby; Kenneth Arrow, Herman Chernoff, Milton Sobel, Herbert Solomon, and many others were students in his class. I don't know how he managed to instill such an interest in statistics, but I'm grateful that he did.

Some years ago I learned that Selby had retired to California. Several of us invited Selby and his wife for a weekend to Stanford at a time that the Berkeley-Stanford Colloquium was scheduled. He and his wife had a marvelous time with us.

College Days

Press: Tell me more about City College, and how statistics was taught there.

Olkin: Statistics was not taught in a single department at City College. It was taught in part by the Mathematics Department. As a matter of fact, the name of one statistics course taught by the Economics Department was "Unattached, 15.1." The terminology "unattached" indicated its status at City College, that is, it was not basically part of a structured departmental discipline. It was the first in a sequence of three discrete courses, all of an applied nature. I left CCNY in 1943 in my junior year, during the war, and became a meteorologist in what was then the United States Army Air Force. (Shortly thereafter the Air Force became a separate branch of the military.) I returned from the service in 1946 and finished my bachelor's degree at City College. In 1947 I went to Columbia University to continue my studies, because by then I knew I was interested in statistics, and Columbia was a major center.

Press: Was there a Statistics Department at Columbia at that time?

Olkin: The Department of Mathematical Statistics was formed formally about 1946. The faculty at Columbia consisted of Ted Anderson, Howard Levene, Abraham Wald, and Jack Wolfowitz. I had most of my courses from Wald and Wolfowitz and a number of visitors; Anderson was on leave during my stay. That was a heyday for visitors. Henry Scheffé, Michel Loève, R.C. Bose, and E.J.G. Pitman were visitors about that time.

Press: How long were you at Columbia?

Olkin: I stayed at Columbia for my master's degree, and then went to Chapel Hill to continue my studies for the doctorate. Harold Hotelling started his career at Stanford University from 1924-1931, at which time he moved to Columbia. In 1946 he moved to Chapel Hill to form a new department. I left Columbia for Chapel Hill in 1948.

Press: Why did you go to Chapel Hill?

Olkin: It was partially for personal reasons. I was married to Anita in 1945 while I was in the service. When we returned to New York after my discharge from the army, the country was faced with a severe housing shortage. In fact, it was almost impossible to find an apartment at that time. Even telephones were rationed after the war. If you were a doctor you could get a telephone, but there was a very long waiting list for the general public.

My parents had a small apartment, but Anita's parents had an extra bedroom, so we lived with her parents in Manhattan for about two years. After living in California for our first year of marriage, we were not as enamored with New York as before. This prompted me to look for an alternative to Columbia, and I learned that Chapel Hill was another major center. I was offered a Rockefeller Fellowship at Chapel Hill which made such a move very attractive. But despite our desire to leave New York, I was not at all disenchanted with Columbia. Quite to the contrary. We had started a graduate student group that generated a sense of community among the students. There were virtually no books on statistics at this time, certainly not on advanced topics, and one of our accomplishments was the publication of class lecture notes. So I have fond memories of Columbia.

Press: Tell me about Chapel Hill.

Olkin: In 1948 there were very few places where you could get a Ph.D. in statistics. Berkeley didn't have a department, though you could get a doctorate in statistics. Iowa State had a department; Chicago had a program, but not a department. Princeton, though small, generated an amazing number of doctorates within the mathematics department. Chapel Hill had an Institute of Statistics with two departments, one at Chapel Hill and one at Raleigh. It had a galaxy of stars on the faculty. On the East Coast, Columbia and Chapel Hill were really the large centers and there was a lot of interaction between the two.

Press: So you ended up following Hotelling?

Olkin: In a certain sense, that's right. The faculty at Chapel Hill in 1948 when I arrived, consisted of Hotelling as chair, R.C. Bose, Wassily Hoeffding, P.L. Hsu, William Madow, George Nicholson, and Herbert Robbins. Gertrude Cox was Director of the Institute.

Hsu was on the faculty, but was on leave in China for a year. He never did return, and S.N. Roy joined the department the following year. The faculty together with visitors formed a phenomenally large group. At Raleigh, there was a Department of Experimental Statistics, with Bill Cochran and many others. The Chapel Hill-Raleigh group was really one of the great faculties.

Press: So you spent about three years there?

Olkin: Yes, from 1948 until 1951 when I graduated.

The Doctoral Dissertation at Chapel Hill

Press: What was the subject of your dissertation?

Olkin: Well, there is a story to my dissertation. I had planned to take a class in multivariate analysis from P.L. Hsu, but he was in China. That year

Hoeffding gave a beautiful set of lectures in multivariate analysis, after which I wanted to continue working in this area. A fellow colleague, Walter Deemer, and I asked Hotelling about continuing our studies as a reading course. He suggested that we use student notes from previous courses given by Hsu. My memory is vague on this, but I recall that we had notes from Al Bowker and Ralph Bradley who had previously taken such a course. Walter and I formalized the material on Jacobians of matrix transformations, and extended many of the results. This was the basis of my joint paper with Walter Deemer on Jacobians of matrix transformations, and really set the stage for my later work. The next year when S.N. Roy arrived, I continued my work with him and with Hotelling on multivariate distribution theory. The object was to develop a methodology for deriving a variety of multivariate distributions. I was able to obtain new derivations for the distribution of the rectangular coordinates, for various beta-type distributions related to the Wishart distribution; for the joint distribution of singular values of a matrix and for the characteristic roots of a random symmetric matrix.

The singular value decomposition was not used much at that time, but this has now become a common decomposition used by numerical analysts. I believe that this was one of the earliest statistical uses of singular values.

Press: The dissertation was formally under Roy and Hotelling?

Olkin: They were both readers, but Roy served as principal advisor.

Press: That else can you tell me about Columbia and Chapel Hill?

Olkin: Both Columbia and Chapel Hill had great students. You have to remember that these were the first post-war classes. So there was a tremendous backlog of individuals who had been away during the war and were returning immediately thereafter. If you catalog the statisticians who received doctorates at both Columbia and Chapel Hill during those early years, you will find a large number who are leaders in the field today. It was a very exciting period at Chapel Hill, both in terms of faculty and in terms of what the students were doing.

Press: Who were some of your fellow students?

Olkin: The list of students at Columbia and Chapel Hill was very long, and my memory is not good enough to remember everyone. But I do recall many with whom I interacted.

At Columbia the list includes Raj Bahadur, Robert Bechhofer, Allan Birnbaum, Thelma Clark, Herbert T. David, Cyrus Derman, Charles Dunnett, Harry Eisenpress, Lillian Elveback, Peter Frank, Mina Haskind, Leon Herbach, Stanley Isaacson, Seymour Jablon, William Kruskal, Roy Kuebler, Gottfried Noether, Monroe Norden, Ed Paulson, G.R. Seth, Rosedith Sitgreaves, Milton Sobel, Henry Teicher, and Lionel Weiss.

At Chapel Hill-Raleigh there were Raj Bahadur, Isadore Blumen, Colin Blyth, Ralph Bradley, Uttam Chand, Willard Clatworthy, William Connor, Meyer Dwass, Sudhish Ghurye, Bernard Greenberg, Max Halpern, Jim Hannan, Gopinath Kallianpur, Marvin Kastenbaum, Paul Minton, Sutton Munro, D.N. Nanda, Joan (Raup) Rosenblatt, Shared Shrikhande, Morris Skibinsky, Paul Somerville, Robert Tate, Milton Terry, Geoffrey Watson, and Marvin Zelen.

Press: Did you do any statistics during the war, before you returned?

Olkin: No, I did not. I was trained at MIT and Chanute Air Force Base to be a meteorologist, and subsequently was a weather forecaster at several airports. At one point I thought of combining the two fields, since a variety of statistical procedures were being used to forecast weather. But somehow this merger did not materialize. Actually quite a number of statisticians and mathematicians were in the meteorology program - for example, those I remember are Kenneth Arrow, Jim Hannan, Gil Hunt, Selmer Johnson, Jack Kiefer, Sam Richmond, and Charles Stein, but I am sure there were many others.

Press: Did subjectivity enter weather forecasting at that time?

Olkin: Not in a formal way. Some of the good forecasters were old timers, who happened to remember similar weather patterns from previous years. They were able to retrieve information from old maps and use that as a basis for forecasting. As you may know, it is rather difficult to beat a forecast of continuity, that is, forecast for tomorrow what the weather is today. How to evaluate weather forecasts in terms of accuracy is also an interesting area.

Early Years

Press: Can we shift gears a bit and have you tell me about your childhood and your family?

Olkin: I was born in Waterbury, Connecticut. My father came to the United States from Vilna in Lithuania - probably to escape being inducted in the Tsarist Russian Army. This was a common sequence at that time. My mother was born and lived in Warsaw, and met my father there.

The move to Waterbury was primarily because some colleagues in my father's occupation - he was a jeweler - were in Waterbury and they had arranged a job for him. When the depression period in the early 1930's came, jewelry was one of the first professions to feel the financial pinch, because it was a luxury item. My family then moved to New York City. I suspect that the move to New York was also prompted by a concern about my future education. Connecticut did not have any tuition-free state universities. Of course, it had Yale University, but for immigrants Yale was totally out of the question, whereas City College was free. We moved to New York in 1934 and my formative years of high school and college were really there.

New York City was quite an exciting place. I went to DeWitt Clinton High School, which at that time had a mathematics team. There was also a football team, but I don't remember it. The math team was a good one.

We used to have meets on Saturdays at one of the high schools, and two different high school mathematics teams would compete. It was very much like the Olympiad and Putnam competitions.

Press: What kind of high school was this?

Olkin: DeWitt Clinton was a very large school with an enrollment of about 4000. It was located in the Bronx bordering on a park area. My graduating class of 1941 boasts of James Baldwin, who wanted to be a writer and became a distinguished one, Julius Irving, who became Managing Director of the Vivian Beaumont Theater at Lincoln Center, and Charles Silberman, who wrote several books, including "Crisis in the Classroom." I am sure that many others have become success stories. With such a large enrollment there were opportunities to pursue many different avenues.

Papers That I Like

Press: I'd like to discuss your publications. You've published more than 100 papers that I know about. Which ones would you regard as your particular favorite ones?

Olkin: That's a hard question, Jim. Certainly the first one with Walter Deemer was a favorite. Walter and I spent a lot of time together, and it was an invigorating, productive, and enjoyable collaboration. It also was my first paper, and that often is special. In retrospect, the papers I tend to like most are the ones that brought me into a new area, ones that I had not worked on before. There is a tendency to continue working in the same research area, and it is not easy to move into different fields.

Chronologically, probably the next paper that I like was the one with John Pratt on Chebychev-type inequalities. That started me in a research area that I continued with Albert Marshall for approximately ten years. My association with Al came about by accident. He had completed his dissertation at the University of Washington. His thesis was also on Chebychev inequalities, and was related to my work with Pratt. In 1958 Al was a post-doctoral fellow at Stanford and I was on sabbatical leave from Michigan State University. We had corresponded before we met, and we were both immersed in the ideas related to Chebychev inequalities. We had adjacent offices, which made it easy to work together. We wrote several papers that year and generated ideas for later work. That started a long history of collaboration. The paper on this subject that I like most is the one in which we were able to obtain multivariate Chebychev inequalities in a rather general framework.

Earlier on I had given some lectures at Michigan State University on independence properties and characterizations of distributions. This led me to think about multivariate versions, and it started a collaborative effort with Sudhish Ghurye and with Herman Rubin. The key point here is that multivariate characterizations often introduce an ingredient that is quite different from the univariate case. With Ghurye the multivariate characterizations dealt with the normal distribution, and with Rubin the Wishart distribution. Each of these papers had novel aspects in their multivariate versions.

The paper with Al Marshall on the multivariate exponential distribution seemed to fill a niche in terms of being a non-normal distribution that had some very nice properties. That paper has probably been referenced more than any of my other papers.

Press: Why is that?

Olkin: It may be because the problem of constructing bivariate distributions with given marginals is rather tantalizing. We generated this particular bivariate exponential distribution from several disparate points of view, and they all converged to the same result. The bivariate exponential distribution has now been applied in different contexts - in reliability theory, in hydrology, and in medicine. Recently Neils Keiding in Denmark has used our bivariate distribution as a model in which cancer can occur individually or simultaneously at several sites. I think that this will become an important application.

A long-time interest of mine has been matrix theory. I think this started when I took a course with Alfred Brauer at Chapel Hill. He was the kind of teacher who was able to command an interest and excitement about the field. At that time he had obtained some nice new results on estimating the eigenvalues of a matrix. I studied matrix theory rather extensively, used it in my dissertation, and subsequently in my work in multivariate analysis.

I've enjoyed trying to mesh some probabilistic results with matrix theory results. For example, a quadratic form can be considered as the first moment of a distribution on the eigenvalues of the matrix of the quadratic form. Consequently, Chebychev inequalities can provide estimates for the location of eigenvalues of a symmetric matrix. There have been several papers of that type; one in particular, with Al Marshall, dealt with scaling of matrices.

An area that I've probably spent the most time on is majorization. I am not sure how Al and I started, but I believe that it was a natural follow up of the work on Chebychev inequalities. From probabilistic inequalities we moved into a variety of real variable inequalities, such as the Hölder, Minkowski and matrix eigenvalue inequalities. At one time we thought of trying to update the Hardy, Littlewood and Pólya book on inequalities. It didn't take long to realize such a plan was rather presumptuous. But we did discover that majorization was a fundamental notion with a rich theory that could be applied to a wide range of topics. On and off we spent approximately 15 years in writing our book on majorization. The reception of this work and its continued use in different areas is most gratifying. The reviews of our book were very laudatory.

Another general area that I enjoy is to try statistically or probabilistically to model a practical problem and to develop procedures for handling the statistical analysis. That has been an ongoing process throughout my career. The applications have been in the behavioral and social sciences, including education. Many of my papers have a genesis in an application. Most recently I've been working on meta-analysis, which, again, deals with a different area. I became intrigued in this through my connections in education. One of my colleagues, Nate Gage, who is an expert in teacher education, pointed out that in education you rarely see profound or strong effects. What you see are small or modest effects that are consistent over repeated studies. The question he posed was whether there was a way in which one could strengthen the conclusion of the composite of studies, even though each particular study was non-significant. At that time the procedure that might be called "vote counting" (that is, counting the number of significant results) was in vogue. My first results in 1972 dealt with the development of an alternative method of analysis to vote counting. Later in 1976-1978 Gene Glass coined the term meta-analysis and proposed a quantification in combining results from independent studies. This served as a catalyst to work more seriously in this area. Larry Hedges was a doctoral student at Stanford and in 1980 wrote his dissertation with me on meta-analysis. Subsequently he continued to work in this area and contributed a lot to the field. Meta-analysis had begun to be somewhat of a fad, and the statistical procedures available or used were not always rigorous. So we decided to write a book that focused on the statistical methods for meta-analysis. This book, titled *Statistical Methods for Meta-Analysis*, was completed in 1985, and has had extensive use, in education, psychology and medicine.

People I've Known

Press: Let's talk about individuals and your relationships with them. Which ones were the closest? Prom what you've said so far, it's clear that you've spent a great many years working with Al Marshall. But what about your early years? For example, who was your mentor when you got started, and what was your relationship with people such as S.N. Roy and Harold Hotelling?

Olkin: Hotelling certainly had a great influence. He valued research, and did not emphasize personalities. He was a very strong advocate for the profession at large, and I think that this characteristic rubbed off on me. I don't think I ever heard Hotelling diminish anyone's work. He always built up individuals if they were productive. Also, he fought very strongly for the teaching of statistics by statisticians. We see this most clearly from his articles which have been reproduced in *Statistical Science*. But it was hard for a student to become close to him on a personal level. In part this was because he was such an esteemed figure, and his manner was somewhat on the distant side. I would say that one could feel respectful, fond, loyal, and appreciative.

Roy was much more approachable. He had just come to the United States, and I was rather close to him. He and I wrote a joint paper immediately after my thesis. But then he died at quite an early age. This was quite a shock to me.

There were others who influenced me, in particular, Wolfowitz at Columbia, Bose, Hoeffding, and Robbins at Chapel Hill. This was in terms of their scholarship and as role models.

Shortly thereafter I became a faculty member at Michigan State University. Prom the beginning I have written a lot of joint papers and my collaborators became close associates. At one time I counted over thirty collaborators. Milton Sobel has been a continual collaborator. He and I, together with Jean Gibbons, did write a book titled *Selecting and Ordering Populations*.

I've really enjoyed the collaboration and the closeness with almost all of my students. With some I've written a number of papers after they completed their doctoral degrees. One of my first students was Leon Gleser. I've written a book with him titled *Probability Models and Applications* and a number of papers. Today we keep up socially and professionally and are still involved on several papers. You were the second student, and we have collaborated on several papers. This was lso true with Joe Eaton, Mike Perlman, Allan Sampson, Tom Stroud and others. So I've continued both the collaboration and friendship with students throughout my career. It is pleasing to me that I have had a total of 28 doctoral students in statistics and education, many of whom have had very successful careers.

Probably in terms of the individuals with whom I've collaborated most, I don't think there's any question but that Al and I have the longest history. We've worked together for 30 years, which is quite a long time. Joe Eaton, Leon Gleser, Mike Perlman, and Milton Sobel are the others that I have worked with the longest.

Press: I don't hear you mention any one person whom I would call a mentor, people who drove you on, or from whom you sought advice.

Olkin: They were mentors in a different sense, more as friends who were supportive. For example, when I went to Michigan State University, Leo Katz was the senior faculty member. Leo visited Chapel Hill where we first met, and he recruited me. Statistics at Michigan State was part of the mathematics department in 1951; a separate statistics department was formed about five years later. Leo and I were quite close, though we only wrote one or two papers together. There was no question that in many ways Leo played the role of a mentor or a senior advisor with whom I could talk about a number of problems.

There were a number of people of that type, but I didn't necessarily work with them. This was true to some degree when I visited Stanford in 1958 and when I joined the faculty in 1961. The faculty were young with strong collegial relationships. There were lots of discussions and you could get advice from colleagues. I was quite close to Bob Bechhofer during my sabbatical and later established a special closeness with Jack Kiefer until his death and with Jerry Lieberman, which has continued to this day.

There are other individuals who influenced me in different ways. For example, I took more courses from Jack Wolfowitz at Columbia than from any other single person. Again, I can't say that he was a mentor in the sense that I would ask him for advice. Jack was not that approachable. But he was a mentor in the sense of being a role model in his emphasis on publishing, on being active, and on having students. While I am reminiscing about Wolfowitz, I can think back to an incident that is now humorous, but wasn't at the time. It was not very easy to meet with either Wald or Wolfowitz. First of all, they were well-known and busy, and the secretary considered it her duty to keep them sheltered. Wald was a very kind person, but somewhat formidable for a young student.

At Columbia you had to write a dissertation for the master's degree. Wolfowitz was my adviser, so I occasionally needed to see him. He had office hours from 12:45 to 1:00, and so one would queue up for a long time in order to see him. I remember once waiting to discuss my dissertation with him. He invited me into his office and then there was something like a quartet in a Verdi opera, except that no one sang. Jack asked me to state my problem at the blackboard, which I did. As I was speaking, the phone rang. Jack started a conversation with the other person on the telephone and would periodically tell me to continue speaking. While this was going on, he was reading his mail. So Jack and the other person were speaking, I was talking to the blackboard, and Jack was reading his mail. This kind of interaction had a salutary effect. It kept students from coming back to see him, and certainly was successful in my case.

Chapel Hill was quite different from Columbia. It was much more intimate because it was not a commuting community. There was a little snack bar where the faculty and students could buy ice cream and sit around. And so the faculty were very approachable. There's no question that there was a lot of interaction between the faculty and the students.

Both Robbins and Hoeffding were very active researchers. Robbins frequently would call students to his office or to a classroom to discuss his current research. Often, colloquia speakers were invited to faculty houses and occasionally a few students were also invited. I felt a part of a community, and the atmosphere at Chapel Hill fostered this feeling of community.

Press: When you had questions about your career, to whom did you speak at North Carolina?

Olkin: Mostly my own peer group. The students were a closely-knit group, but I don't recall talking to the faculty about non-technical matters. There were not that many people in the field. Statistics was a new field, so there was not much previous experience or previous track record concerning career opportunities.

For example, when I graduated in 1951 there were very few job openings. This was before the many statistics departments were formed. So most positions were in mathematics departments. This meant that there were at most a few statisticians in each mathematics department, and it was not unusual to be the only statistician in the department. I felt that it was important to get a position in a university where there were plans for building a nucleus of statisticians.

Press: Is that why you went to Michigan State?

Olkin: As I mentioned, Leo Katz was there, and he wanted to build a group. Chuck Kraft was just getting his doctorate from Berkeley, and was an instructor at the time. A year later Ken Arnold came from Wisconsin, and then Jim Hannan came from Chapel Hill. Leo managed to attract a large number of visitors. For example, Alfred Rènyi visited, as did R.A. Fisher. And so, within a period of three or four years we had a critical mass on the faculty.

Press: After Michigan State you were at the University of Minnesota.

Olkin: I spent a year and a half at the University of Minnesota. That was also a very nice period, though it wasn't for long. While at Michigan State I was invited to join the faculty at Minnesota, which I did. At that time Leo Hurwicz, Palmer Johnson, and Richard Savage were on the faculty. A statistics department was being formed and I was asked to serve as Chair. Within a short period thereafter Meyer Dwass, Sudhish Ghurye, Gopinath Kallianpur, and Milton Sobel joined the faculty.

Press: Then there was Stanford?

Olkin: As you know, my years at Stanford comprised a major part of my life and I will soon mark my thirtieth anniversary at Stanford. I visited Stanford in 1958 while on sabbatical, and then joined permanently in 1961. The Bay Area with Berkeley and Stanford was phenomenally active. You have to remember that this was almost 30 years ago. Many of us were in our 30's or early 40's and a tremendous amount of energy and electricity flowed in the two places.

Press: How about the Berkeley Symposium?

Olkin: The Berkeley Symposium, held every five years, was very strong. In addition, we had frequent joint Berkeley-Stanford colloquia. These were very exciting years. Berkeley and Stanford each was trying to build its department, and there was a lot of research activity. The students were first rate. I think it's an interesting commentary that so many of our students have become the statistical leaders at this time.

This was also a period of a lot of visitors, both in summer and during the academic year. If you waited on the steps of Sequoia Hall or the department at Berkeley, you probably would meet almost every statistician at some point. Over the years Anita and I have entertained a very large number of visitors. It is not surprising for someone to tell Anita that he and his family had dinner in our house twenty years ago.

The faculty at Stanford in 1961 was great: Bowker, Chernoff, Chung, Johns, Karlin, Lieberman, Miller, Moses, Parzen, Solomon, Stein. During the first 15 years at Stanford my main energy was devoted to my own research in both departments and to helping build the statistics department and my program to train educational statisticians in the School of Education.

Current Interests

Press: What are your current research interests? You've been involved with many different research directions.

Olkin: Two topics seem to follow me. I often receive letters on inequalities and majorization, and I think I'm ready to say that I don't want to stare another inequality in the face. But I must also confess that I cannot keep away from a new inequality. I also receive letters concerning meta-analysis, in which I still have a strong interest, especially the meta-analyses being conducted in medicine.

Al Marshall and I are becoming more involved in an area that we had worked on earlier, namely the bivariate exponential distribution. That was a specialized result, and we are now concerned with more general questions. For some time now we've been intrigued by the question of how to build dependencies into bivariate distributions. We have developed several unifying themes, some of which have been published, and we are contemplating writing a book that brings the subject into better focus. Since the last book took 15 years to write, we each are cautious about beginning a venture that could take a long time to complete. However, I suspect that subconsciously we each have a book in mind, but are reluctant to state so too openly.

Let me add a bit about this project about dependencies. A natural question that needs to be resolved is how to simulate or generate distributions with these given dependencies. Computer simulation is an area that has recently interested me. In addition to my work with Al, I have written joint papers with Ted Anderson and George Marsaglia on generating correlation matrices and generating random orthogonal matrices. In order to apply these multivariate distributions, we need to develop methods for computer simulations. But it is not always clear how to generate observations with a particular type of dependency. This is quite different from, say, fitting data using the Pearson families. The families that we have in mind arise more from models than from data.

That's our main current work. Al and I are trying to get together more often, which should make it easier to keep working on a single project.

Press: How have you and Al managed to work together so much?

Olkin: I must say that in retrospect, I don't quite know how he and I have managed to collaborate as much as we have, inasmuch as we are not at the same university. In our early collaborations we would visit each other for periods ranging from three to seven days. This would give us a chance to get started on a project.

These visits occurred approximately once every six weeks, so we really had a continuing connection with one another. This was particularly the case when Al was at the Boeing Scientific Research Laboratories, at the University of Rochester and the University of British Columbia, and I was at Stanford. We managed to meet this way over the years, but we also had longer periods of time together.

We spent one year together at Cambridge University and one year at Imperial College. I visited the University of British Columbia and Al visited Stanford for longer periods. We also spent three months in Zurich. During these periods we had an opportunity to work intensively. That has been the modus operandi - namely, working for short stretches, and then meeting for a longer period of time when we could put things into perspective and write things up.

Press: Do you have any other major books or projects under way at this time?

Olkin: The project with Al has a high priority. But there are several other projects that I have in mind.

As you know, I have a fine collection of photographs of well-known statisticians and probabilists that adorn Sequoia Hall. Over the years visitors have suggested that I publish these so that others might enjoy them. I plan to do this as a joint venture with H.O. Lancaster, who is more of a historian than I.

The medical profession has taken to meta-analysis. There often are many studies dealing with the same illness, but with varying conditions and patients, and meta-analysis offers a method for combining results. In my book with Larry Hedges on meta-analysis we did not include a discussion of medical applications, and we now plan a sequel to do that. Dr. Thomas Chalmers of the Harvard School of Public Health and Mt. Sinai Hospital will join us in this project. He has engaged in a number of medical meta-analyses, and will bring a first-hand knowledge base of medicine to the project.

Another project that I have had in the back of my mind goes back to my Chapel Hill days, namely to write a book on matrix theory applications in statistics. But now I would like to add applications in operations research and numerical analysis. This project was to be a collaborative effort with Richard Cottle in the Department of Operations Research, and Gene Golub in the Department of Computer Science. We started to meet on Saturdays to discuss this project, but you have to recognize that these three participants include some of the world's heaviest travelers. So the absences became more and more frequent, and we did not make much headway. But I like to imagine that the future will bring some free time to all of us and that this project will come to fruition.

Press: I notice that you have not mentioned your work with the Department of Education in Washington. Tell me about that connection.

Olkin: Thanks for bringing that up. I have concentrated on the statistics part of my activities. But I would like to tell you about some of the education activities.

As you know, I have a joint appointment between Statistics and the School of Education at Stanford. This has been the case from the time I came to Stanford. My role in the School of Education has been primarily in the doctoral program, to train educational statisticians, in a comparable manner to the training of biostatisticians, psychometricians, econometricians, or geo-statisticians. I have also been involved in research arising from an educational context. The meta-analysis work is an example of research that started from my education connections. But there have been many other instances where a problem has arisen and led to a research study. Several of these have dealt with correlational models; others have involved statistical inference arising from achievement test models.

Recently the American Statistical Association started a wonderful Fellows program designed to bring academia and government closer. This program is supported by the National Science Foundation. The first Fellows program was with the Census Bureau, and later was extended to the National Center for Education Statistics (NCES) and to the Bureau of Labor Statistics. There is a similar program in the Department of Agriculture, and the potential for one at the National Bureau of Standards.

I was invited to be a Fellow at the Center, and before accepting this opportunity, it was important to me that I be engaged in a project that could have an impact in education. It would have been easy to become involved in particular studies at the Center, but over the years I've been on so many panels and studies that I did not see going to Washington for yet another study, even though it might be an important one. The NCES collects data in the form of studies or surveys. For example, the NCES sponsors a School and Staffing Survey, A National Educational Longitudinal Study, and Common Core of Data. There is also a major longitudinal study in mathematics and reading called the National Assessment of Educational Progress (NAEP). ETS is the contractor for NAEP, and periodically issues a report to the nation on the state of mathematics and reading learning. I am currently a member of the NAEP Technical Advisory Committee.

In addition to these national data bases, considerable data is collected by the states. Much data is required to fulfill federal legislation requirements. Thus, the states and the federal government collect data, but there is little integration among states or with the federal government. Ultimately, if not immediately, we need to have enough information to permit us to answer broad issues about education, and to make policy intelligently. This all pointed to the need for a national education data base. The idea of designing such a system was intriguing. It was an area in which I could serve as a link in bringing together the academic and governmental constituencies.

There are 50 states, approximately 16,000 school districts, 100,000 schools and 4 million teachers. It seemed reasonable as a beginning to focus on states and school districts. Each of these constituencies is of a manageable size. This was the thread of my thinking.

It seemed to me that this general area of a statistical education data base would be an interesting challenge that could have a great impact in terms of the statistics for the future. It combined my interests in both statistics and education. It was clear from the outset that such a project was a very ambitious kind of program, and that it would be addressed better by a small group of individuals. I proposed to two colleagues, Ed Haertel, who is a test and measurement specialist at Stanford University, and Larry Hedges, an educational statistician at the University of Chicago, to join me in this project. They both accepted the challenge. We were fortunate in being able to obtain help and advice from a number of colleagues more knowledgeable than we are about data bases. In particular, Nancy Flournoy, who had been instrumental in setting up a multi-disciplinary medical data base information system at the Fred Hutchinson Cancer Research Center, helped a lot. John McCarthy of Lawrence Laboratory, Berkeley, had developed a meta data base for the military, and his experiences were very informative. We Fellows have been holding a series of conferences to help us more fully understand the information and policy needs in a data base system and some of the caveats to worry about. The first conference was with the educational constituency - teachers, principals, administrators, educators, etc. The second conference was with data base specialists, and third was with state data base representatives. We are now in the process of amalgamating this information. But already we have had an effect in bringing some of these constituencies together.

Press: How did you get into the field of education 27 years ago, or earlier?

Olkin: The Department of Statistics at Stanford was modeled on some structural principles. You may have read about this in the interview with Al Bowker that appeared in *Statistical Science*. Some of the structure came from the Statistical Research Group at Columbia University during World War II, and some came from local needs. In effect, there was a strong outreach program. The word "outreach" is in vogue today, but in 1961 it wasn't. But the basic idea was that statistics should be intimately connected to substantive fields. During the 1960's approximately nine out of the sixteen faculty members in the Department of Statistics at Stanford had joint appointments with other fields: three with the medical school, one with operations research, one with electrical engineering, one with economics, one with education, one with geology, and one with mathematics. Later we had a connection with the linear accelerator center. These joint appointments were a guiding principle. Each joint appointee was supposed to develop a program in the other department.

An opportunity arose about 1958-1960 for a joint appointment with the School of Education. Since my work had dealt with multivariate analysis and with models in the social sciences, I was one of the candidates who could fit into both the Statistics Department and the School of Education. The key was to try to find individuals who would be acceptable to two departments as varying as the ones I've mentioned. That's been a very difficult task. We have been successful until now, and I hope that we continue in this vein. In a certain sense this guarantees the development of cross-disciplinary research.

Press: You have had many special satisfactions during your career. Which ones do you value or appreciate the most?

Olkin: There are several aspects of my professional career that I really have enjoyed. I think that what I've enjoyed most of all is the connection with the students I've had and the collaborations I've had. I've enjoyed keeping up with individuals, and I've enjoyed being able to work with them. It's been fruitful in many ways.

In general, I have enjoyed the opportunity to create some programs and to effect changes. Being President of the Institute of Mathematical Statistics (IMS) afforded me an opportunity to tackle a number of needs for the profession. For example, the creation of *Statistical Science* came out of that period, and I hope that this journal remains as a successful legacy. I think it is fair to say that *Statistical Science* came about from the concerted efforts of Morrie DeGroot, Bruce Trumbo, who was treasurer of the IMS at that time, and myself. Morrie DeGroot deserves a tremendous amount of credit for making the journal a success during its formative years.

Those are activities that have given me a lot of satisfaction over the years.

Press: How about your editorships?

Olkin: The editorship of *The Annals* is the kind of job where it is best if you're not asked to be Editor, but it is hard to say no if you are asked. It was a phenomenal amount of work. I was Editor at the time when the two journals - *The Annals of Statistics* and *The Annals of Probability* were still combined into the one, *The Annals of Mathematical Statistics*. I used to receive two new submissions every day. This meant that there were approximately 700 new papers a year to handle, not to mention the ones that were revised several times.

It was a monumental task, and it became quite clear that it was too much for a single Editor to deal with in a responsible way, and that a split was in order. I was pleased to be able to have an effect in starting the two offspring journals. At that time there was a lot of controversy as to whether a split was reasonable, and there were valid arguments on each side. But I doubt that many would want to combine the two *Annals* at this time. Each journal has become a leading publication and is a success. In fact, each journal is now sufficiently large that it is becoming a burden on each Editor. My impression is that professional societies have been too conservative in their publication policies. Because of the tremendous rise in both population and research we need more journal outlets.

Comments About Statistics

Press: I'd like to move on to a very broad question for the field of statistics as you see it. What is your assessment of the current state of the health of the field of statistics, and where do you see the field heading?

Olkin: I'm a bit worried about statistics as a field. As you know, I come from the mathematical community and I've always liked the mathematics of statistics. But I think that the connection with applications is an essential ingredient at this time. I say that because applications are crying out for statistical help. We currently produce approximately 300 Ph.D.'s in statistics and probability per year. This is a small number considering the number of fields of application that need statisticians. Fields such as geostatistics, psychometrics, education, social science statistics, newer fields such as chemometrics and legal statistics generate a tremendous need that we are not fulfilling. Inevitably this will mean that others will fulfill those needs. If that happens across fields of application, we will be left primarily with the mathematical part of statistics, and the applied parts will be carried out by others not well-versed in statistics. Indeed, I think that a large amount of statistics is now being carried out by non-statisticians who learn their statistics from computer packages and from short courses.

So I worry about this separation between theory and practice and the fact that we are not producing the number of doctorates to fulfill needs in all of these other areas.

Press: Has the number of doctorates been going up or down?

Olkin: The number is going up. There were approximately 150 Ph.D.'s in statistics and probability in 1970, 240 in 1975, and there are now about 300. Thus we have doubled in about 15 years. But this growth is not commensurate with the needs and growth of other fields.

Press: Do you mean that we are not producing enough doctorates to meet the demand?

Olkin: We are definitely not producing enough doctorates to meet the demand. An example of an area of high demand is the drug companies, which could use almost all the doctoral students produced each year. Education is another area in which the number of statisticians involved is relatively small, but for which there is a large potential. Statistical needs are growing at a rapid rate in the legal profession, and I don't think there are many law schools that have either statisticians on the faculty or connections with statistics departments. That's an area I would like to see statisticians become involved in early on, to avoid the inevitable turf battle as to who teaches statistics for law.

Press: Is the field of statistics heading now toward more applications?

Olkin: I think a number of individuals in the profession are heading more towards applications, but the field as a whole does not have enough faculty and students working in applied areas. Biostatistics is probably our only big success story in the sense that there are a lot of statisticians in medical schools, though perhaps not enough. This came about in part because of the prevalence of training grants, and in part because of federal regulations mandating clinical trials or other statistical procedures. But there are few, if any, statisticians in law schools, in social science departments, pharmacy, dentistry, education, business, and so on.

Industry used to be a big user of statistics; this diminished considerably about twenty years ago, and now has become a high demand area. Sample surveys are used a lot but this specialization is totally undernourished. The number of universities that teach sampling is small, and we have trained few experts. I am sure that we could expand the research effort and doctorate production in sampling theory.

There is still an excitement in the field, but my impression is that, except for a few places, the growth in statistics departments has reached a plateau. I believe that this is true because we do not have a natural mechanism for statistics departments to create strong links to other departments of the academic community. Academic institutions have not been designed for cross-disciplinary research, and indeed may actually be antagonistic to cross-disciplinary research.

Press: Whereas, by nature, statistics tends to be used and needed in other fields?

Olkin: Yes, indeed. It is particularly important because problems are now becoming much larger. For example, the study of a large scale problem such as pollution or acid rain with a small group of researchers is really not very realistic. We will need a lot of connections with other disciplines. Except for a few places, we are not fostering that connection. The National Science Foundation has recognized this need by creating centers that have a strong cross-disciplinary component, but these do not have strong statistical components. I think that the time has come for the profession to have a Statistical Sciences Institute that would focus on cross-disciplinary research.

It's interesting to note that when I was first at Chapel Hill, Gertrude Cox was a strong advocate for learning a substantive field. We were all encouraged, almost pushed, to become not only statisticians, but to gain a knowledge base in biology, sociology, political science, etc. - any area in which we could apply statistics. Except for the medical field, not many took this route. Remember that starting in the late 1940's the decision theory orientation was strong so that many of us studied mathematics, rather than a substantive field, and we became mathematical statisticians rather than socio-statisticians or geo-statisticians. That was fine up to a point, but the needs in 1990 center much more on our connections and usefulness in these other fields. We need to expand our vision.

Press: How about the direction of growth of the field with computers and data analysis? Are we moving in that direction, are we moving enough or too much?

Olkin: I am more comfortable with the previous general question than I am with the question about statistics and computers. I am not well versed about the field as a whole, but I do have some general impressions. I believe that statistical packages have had serious positive and negative components. The positive, of course, is that people now can carry out more sophisticated analyses than they would be able to if they had to learn programming on their own. There's no doubt that it's been a great service. On the other hand, there is a tendency for people not to learn the statistical underpinnings, but only to learn how to use a statistical computer package. Indeed, my experience in reading doctoral dissertations is that the availability of packages is what drives the choice of analysis. The availability of statistical packages also drives the curriculum and may emphasize how to generate numbers, rather than interpretation and understanding.

More recently there's been a strong development in statistical graphics and resampling schemes. Again I think that in principle these are positive developments. What worries me is that they will be overly used and subsequently abused, as is the case with almost any new area for which there is a lot of use. It doesn't take long before there's a certain amount of abuse, and it becomes a serious problem.

The result of the availability of computers and packages is that statistical analyses are being carried out by non-statisticians much more than ever before. This is fine when done well, but this is not always the case - probably not the case most of the time - so that the public may be faced with erroneous conclusions. The statistical community is not intimately involved in this growth of the use of computers. We provide some of the packages and some of the theory, but, in effect, its big use is elsewhere.

To illustrate the high use by non-statisticians, not long ago Stu Geman gave a talk on image processing, an area in which he and others have been working. Stu mentioned that he had published a paper in an I.E.E.E. (Institute of Electrical and Electronics Engineers) journal, for which he received well over a thousand reprint requests. It's hard to imagine any statistician of my acquaintance who publishes in any of the standard statistical journals receiving that many reprint requests. I usually get six reprint requests - especially if I ask my relatives to write for them. Of course, photocopying confounds these numbers, but the fact remains that little theory is translated to usable methods except through packages.

Press: That certainly makes the point.

Olkin: That's a good example of an area in which the statistical community had a large input, but it is being developed, extended, and used by other fields.

In Spain I Am a Bayesian

Press: Here's a difficult question. Are you Bayesian or not?

Olkin: I think I'm a part-time Bayesian. My inclination in dealing with a problem is to use classical procedures, but when I get more deeply involved and need to obtain information about the parameters I do not hesitate to incorporate some of the Bayesian ideas. I have found that some problems can be formulated in a manner that calls for a Bayesian approach. In other instances, this is not the case and a Bayesian approach would seem forced. I am not a Bayesian in the sense that I feel compelled to use a Bayesian approach, nor am I a classicist in not using a Bayesian procedure.

I suspect that I am begging the question a bit. I don't have too many papers in which the word "Bayesian" appears in the title, but since I want to be invited to the Bayesian conferences, especially when they are in Spain, I will say I'm 75 percent non-Bayesian and 25 percent Bayesian.

Press: You've certainly written papers with avowed Bayesians so you cannot be anti-Bayesian. Have you become increasingly sympathetic over the years?

Olkin: I am not sure how to answer that. Recently I have been working with Irwin Guttman on a model for interlaboratory differences. I had previously written a paper with Milton Sobel in which we used a ranking and selection procedure. At that time this seemed to be a reasonable formulation, but ranking and selection procedures have not been accepted very much in applied work. With Irwin we looked at an alternative formulation that led naturally to a Bayesian point of view, and I was interested to know how different the answers would be. I have no antipathy in using either approach, and try to understand what one gains from each method.

Visits to Other Universities

Press: How about some of your travels. What are some of the universities that you visited, and some of the people there?

Olkin: In 1955 I spent a year at the University of Chicago. At that time Allen Wallis had a program in which they invited two visitors every year, and Don Darling and I spent the year at Chicago. This was a very productive year for me. I taught a course in multivariate analysis and in sampling theory, as I recall, and this gave me an opportunity to renew old acquaintances and to begin new ones.

Chicago had a small but good student body; for example, Herb T. David, Morrie DeGroot, Al Madansky, and Jack Nadler were students at that time. The faculty consisted of Raj Bahadur, Pat Billingsley, Alec Brownlee, Leo Goodman, John Pratt and Dave Wallace. Bill Kruskal was on leave that year, and we lived in his house. Allen Wallis was there as Dean of the School of Business. Also, Meyer Dwass and Esther Seiden were at Northwestern, and we used to get together quite often.

Press: Tell me about your visits to other Universities. I know that you travel a lot.

Olkin: In 1967 I was on sabbatical leave from Stanford and was an Overseas Fellow at Churchill College, Cambridge. That was a great year. Al Marshall and Mike Perlman were also at Cambridge, and Alfred Rényi was a visitor for one quarter. Cambridge had a vigorous group led by David Kendall and Peter Whittle. We had a seminar on inequalities that got us much more deeply into the field. In fact, that year was a very active one in England with lots of visitors in London. We used to visit London quite often for seminars.

In 1971 I spent a year at the Educational Testing Service. Fred Lord, one of the leading researchers in tests and measurement, was head of a very active group. He invited visitors to participate in the program, and that year in addition to myself, Murray Aitkin, Leon Gleser, Karl Jöreskog, and Walter Kristof were in the group. There were some lively discussions. Also, since Princeton was nearby we were able to interact with some of their faculty. This was a period when Leon and I were able to work closely, and we wrote several papers and completed our book.

I again visited England in 1976-77, this time at Imperial College. For several years thereafter Anita and I tended to spend a month every year in London. These visits gave us an opportunity to maintain and to renew European contacts. I was a Fulbright Fellow during the fall of 1979 at the University of Copenhagen. It was a thriving place with Steen Andersson, Hans Brons, Anders Hald, Martin Jacobsen, Soren Johansen, Neils Keiding, Stefan Lauritzen and others. I gave some lectures there and was able to start some new projects.

In the spring of 1981 I was a visitor at the Eidgenüssische Technische Hochschule (ETH) in Zurich. Frank Hampel and his group, Elvezio Ronchetti, Peter Rousseeuw, Werner Stahel, were working on robust estimation. Others on the faculty were Hans Buhlmann and Hans Foellmer. Chris Field from Dalhousie, Bob Staudte from Melbourne, and Al Marshall were visitors. We each gave individual lectures, and I gave a series of lectures on multivariate analysis.

In the early 1950's the National Bureau of Standards was a center for applied mathematics and statistics. They had many postdoctoral and student visitors during those early days. The Bureau was trying to revitalize this program of visitors, and in the fall of 1983 I spent a quarter there. As a consequence of interactions with John Mandel, I again became involved in finding the expected value and covariances of the ordered characteristic roots of a random Wishart matrix. This moved me in the direction of some numerical work that was new to me. Recently, together with Vesna Luzar, who was a Fulbright visitor from Yugoslavia, we have compared several alternative modes of computation.

It was at the Bureau that I met Cliff Spiegelman, and he and I later started a collaboration on semi-parametric density estimation. This is work that we are both continuing.

I visited Hebrew University as a Lady Davis Fellow in the spring of 1984. Our family had visited Israel in 1967, and this gave me an opportunity to renew acquaintances. The statistics group was very lively with lots of activity. The faculty consisted of Louis Guttman, Yoel Haitovsky, Gad Nathan, Samuel Oman, Danny Pfeffermann, Moshe Pollak, Adi Raveh, Yosef Rinott, Ester Samuel-Cahn, Gideon Schwarz, and Josef Yahav. Larry Brown was a visitor for the year. I gave some lectures on inequalities, which generated a collaboration with Shlomo Yitzhaki, who is on the faculty of the Economics Department. This work had a basis in economics and the primary focus was on Lorenz curves and subsequently, on concentration curves. This also introduced me more intimately with some of the results of Gini, which we were able to use to greater advantage. My collaboration with Shlomo has continued, especially when he visits the U.S.

My Family

Press: You haven't had a chance to speak about your immediate family. Can you tell me more about them and bring me up to date?

Olkin: Anita and I were married in 1945 while I was in service. I was being transferred from LaGuardia Airport to San Francisco at the time, so we spent our honeymoon on the train from New York to San Francisco. We very much enjoyed our stay in California, and I am not sure why we returned to New York after my discharge. It never occurred to me to continue my studies at Berkeley. In any case, we did return and I attended CCNY, Columbia, and UNC. Anita and I now look back to our three years at Chapel Hill as a very happy period. We were one of few married couples, and Anita made our house available to many of the graduate students. Also, at Michigan State University we were very much involved in a University community, and our house was often a meeting place for visitors.

We have three daughters. The oldest, Vivian, was born in Chapel Hill in 1950. She and her husband, Sim, live in Austin, Texas and have two children. Vivian was a career counselor at the University of Texas, and is now getting her master's degree in Human Resources Development. Sim received a doctorate from Stanford and is now on the faculty of the Graduate School of Business, University of Texas. Our second daughter, Rhoda, was born in 1953 when I was at Michigan State University. She and her husband, Michael, live in Walnut Creek, California and have one child. Rhoda received a doctorate in counseling psychology from the University of California, Santa Barbara. She is now on the faculty of California State University, Sacramento. Michael is a bio-medical engineer with a company housed in Berkeley. Our youngest, Julia, was born in California in 1959. She and her husband, Juan, live in Castro Valley, California. Julia and Juan both received doctorates in mathematical sciences from Rice University, and are each working as numerical analysts - Julia at SRI and Juan at Sandia in Livermore. My family of females has taught me and trained me in the women's movement, and I have been sensitized to difficulties that women have in the workplace, and the prejudices that exist.

Other Activities

Press: Let me move away from statistics. What do you like to do when you're not doing statistics?

Olkin: I do enjoy traveling, which I think is well-known to a number of people. Whenever I do travel, I generally try to find museums, symphonies, operas, and theaters. I almost always do that, wherever I go. You also know that I'm generally a people person, which is one of the reasons why I've enjoyed students and collaborators. Over the years, the professional contacts have merged with the personal contacts. I enjoy hiking. We used to go to Yosemite regularly when the children were young. More recently I have visited state parks near meeting places.

Press: Do you hike alone?

Olkin: I have tried to entice colleagues to join me, and years ago when we went to Yosemite our youngest daughter Julia would always go with me. But more recently I have gone alone.

Press: How about sports?

Olkin: I enjoy tennis and swimming. The tennis is a social event, but I am a non-social swimmer.

Press: I have a vague recollection of having been told that you were the ping-pong champion on a ship. Tell me about that.

Olkin: That was so long ago that I had totally forgotten it. I mentioned that I was born in Waterbury, Connecticut. There's a resort near New Haven called Woodmont, and even though we were relatively poor, we used to go to Woodmont and rent a room during the summer time. There was a ping-pong room in one of the hotels, for which they used to charge an hourly fee. But if you would help clean up, you had access to the ping-pong tables when they were not being rented, so I used to play a lot when I was young. In 1967 we went to England on the Dutch ship, Rotterdam, which had table tennis contests in both tourist and first class. I played ping-pong in tournaments for both classes and won both. At Stanford there were ping-pong tables in the student union, but the building was altered and there are no tables there at this time, so I haven't played for years.

From 1989 On

Press: What does the future hold for you?

Olkin: The word retirement is a curious word in that it implies that you will stop working at a certain date. We need an alternative descriptor. As I understand the current California and federal laws, if you were born after August 31, 1923 you do not have to retire. The law may change in 1994, but as of now I will not have to retire. Of course, it may not make sense financially or intellectually not to retire. However, I don't see retirement as a problem. I have several projects that I'd like to work on, and I don't have enough time for these without giving up other activities. So retirement is one way of reallocating one's time to the activities that one likes. I am also deeply involved with the Center for Education Statistics project and even though the fellowship will be over within the near future, I would like to continue my involvement. If one has a high metabolic rate, it's difficult not to continue working. All my retired colleagues tell me that I will probably be doing more, rather than less.

Press: Thanks very much for the opportunity to review this part of your history. It was of interest to me and I am sure that it will be of great interest to many of our colleagues.

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Part V

Abstracts

Asymptotic distribution of a set of linear restrictions on regression coefficients

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Abstract

Reduced rank regression analysis provides maximum likelihood estimators of a matrix of regression coefficients of a specified rank and of corresponding linear restrictions on such a matrix. These estimators depend on the eigenvectors of an "effect" matrix in the metric of an error covariance matrix. It is shown that the maximum likelihood estimator of the restrictions can be approximated by a function of the effect matrix alone. The procedures are applied to a block of simultaneous equations. The block may be over-identified in the entire model and the individual equations just-identified within the block.

On the optimality of a class of designs with three concurences

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Abstract

In the present paper we consider a class of unequally replicated designs having concurrence range 2 and spectrum of the form $\mu_1(\mu_2)^{v-3}\mu_3$. Now, Jacroux's (1985) Proposition 2.4 says that a design with spectrum of the above form, if satisfies some further conditions, is type 1 optimal. Unfortunately, this proposition does not apply to our designs since they have a poor status regarding E-optimality. Yet we are able to prove the A-optimality (in the general class) of these designs using majorisation technique. A method of construction of an infinite series of our A-optimal designs has also been given.

The first and only known infinite series of examples of designs satisfying Jacroux's conditions appears to be the first one in section 4.1 of Morgan and Srivastava (2000) -hitherto referred to as [MS]. In this paper we use majorisation technique to prove stronger optimality properties of the above mentioned designs of [MS] as well as to present simpler proof of another optimality result in [MS].

Keywords

Majorisation, A-optimality, E-optimality, Type 1 optimality.

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Relationships between partial orders of Hermitian matrices and their powers

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Abstract

Baksalary and Pukelsheim (1991) considered the problem of how an order between two Hermitian nonnegative definite matrices $\mathbf{A}$ and $\mathbf{B}$ is related to the corresponding order between the squares $\mathbf{A}^2$ and $\mathbf{B}^2$, in the sense of the star partial ordering, the minus partial ordering, and the Löwner partial ordering. Baksalary and Mitra (1991) and Groß (2001) developed characterizations of these orderings referring to the concept of the space preordering. Professor Ingram Olkin, who was the editor of the Baksalary and Pukelsheim's paper, asked several interesting questions concerning the results contained in it. Answers to some of them are presented here, showing possible generalizations of the above-mentioned results from two points of view: by relaxing the nonnegative definiteness assumption and by replacing the squares by arbitrary powers. The results obtained, accompanied by a set of comments, form a complete solution to the problem.

Keywords

Star partial ordering, Minus partial ordering, Löwner partial ordering, Space preordering, Nonnegative definite matrix, Hermitian matrix, Power of a matrix.

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Further generalizations of a property of orthogonal projectors

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Abstract

Generalizing the result in Lemma of Baksalary and Baksalary (2002), Baksalary, Baksalary, and Szulc (2002) have shown that if $\mathbf{P}_1$ and $\mathbf{P}_2$ are orthogonal projectors, then in all nontrivial cases a product of any length having $\mathbf{P}_1$ and $\mathbf{P}_2$ as its factors occurring alternately is equal to another such product if and only if $\mathbf{P}_1$ and $\mathbf{P}_2$ commute, in which case all products involving $\mathbf{P}_1$ and $\mathbf{P}_2$ reduce to the orthogonal projector $\mathbf{P}_1\mathbf{P}_2$ $(= \mathbf{P}_2\mathbf{P}_1)$. In the present paper, we propose two further generalizations of this property. The first of them consists in replacing a product of the type described above, appearing on one of the sides of the equality under considerations, by an affine combination of two or three such products, while the second generalization is obtained by replacing the products appearing on the two sides of the original equation by linear combinations of two chain-products of $\mathbf{P}_1$ and $\mathbf{P}_2$, where the scalars specifying these combinations have equal sums.

Keywords

Hermitian idempotent matrix, Commutativity, Product of projectors.

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Adaptive designs for clinical trials: An overview

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Abstract

Statistical inference based on adaptive designs allows to implement design adaptions without inflating the type I error rate. Adaptations may be based on the unblinded data collected so far as well as external information and the adaptation rules need not be specified in advance. We will explain some unexpected result (e.g. safety problems, large treatment effect, effect only in a subgroup) and possible adaptation strategies (e.g. reducing the dose, adapting the population).

Keywords

Adaptive designs.

References

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Schwarz iterations for singular systems of Markov chains

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Abstract

A convergence analysis is presented for additive Schwarz iterations when applied to consistent singular systems of equations $Ax = b$. The theory applies to singular M-matrices with one-dimensional null space, and is applicable in particular to systems representing ergodic Markov chains. The results are based on an algebraic formulation of Schwarz methods, in particular the convergence theorem for additive Schwarz iterations and the existence of a splitting of the matrix A with the same iteration matrix as the additive Schwarz scheme. This work complements the results of [Marek and Szyld, LAA, in press], where multiplicative Schwarz iterations are shown to converge for singular systems.

Keywords

Schwarz method, Markov chains, Iterative methods.

References

Marek, I. and B.D. Szyld (2004). Algebraic Schwarz Methods for the Numerical Solution of Markov Chains, *Linear Algebra Appl.. 386*, 67-81.

A specific form of the generalized inverse of a partitioned matrix useful in econometrics

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Abstract

Faliva and Zoia (2002) provided an explicit formula for the inverse of the matrix

$$\mathbf{M} = \begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C}' & \mathbf{0} \end{pmatrix},$$

where $\mathbf{A} \in \mathbb{R}_{m\times m}$ and nonzero $\mathbf{B}, \mathbf{C} \in \mathbb{R}_{m\times n}$ are both of full column rank. They also showed the applicability of their result in econometrics. The additional assumption adopted by Faliva and Zoia is that the matrix $\mathbf{B}'_{\perp}\mathbf{A}\mathbf{C}_{\perp}$, with $\mathbf{B}_{\perp}$ and $\mathbf{C}_{\perp}$ spanning the orthogonal complements of the column spaces of $\mathbf{B}$ and $\mathbf{C}$, is of full rank. The purpose of the present paper is to generalize this formula from two points of view: firstly, by pointing out that the nonsingularity of $\mathbf{B}'_{\perp}\mathbf{A}\mathbf{C}_{\perp}$ is not necessary for the nonsingularity of $\mathbf{M}$ and, secondly, by removing the assumptions that $\mathbf{B}$ and $\mathbf{C}$ are of full column rank and considering the Moore-Penrose inverse of $\mathbf{M}$ instead of its usual inverse.

Keywords

Generalized inverse, Moore-Penrose inverse, Partitioned matrix.

References

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The singular value decomposition as a basic tool in generalized canonical analysis and related linear models

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Abstract

The Generalized Canonical analysis (GCA) may be seen as an umbrella model for linear models. The usual or simple Canonical Analysis or Multivariate Regression, MANOVA, ANOVA, Multiple Linear Regression and Covariance Analysis models, as well as Principal Components, Discriminant and Correspondence Analysis models may easily be seen as particular cases of GCA both under the geometric-algebraic and inferential approaches.
When a joint multivariate Normal distribution is assumed for the variables involved, general inference for all these models both for the general fit of the model and for testing between a model and a submodel, as well as for testing the significance of particular directions are either particular cases of the generalized Wilks Lambda statistic or closely related with it.
The use of the Singular Value Decomposition to implement GCA (or any of the above models) is highly advantageous since it will enable us to overcome problems related with quasi-multicollinearity in an easy, elegant and efficient way, enabling both the computation of the above mentioned statistics through the computation of the singular values and the estimation of the parameters in the models, through the computation of the corresponding singular vectors or eigenvectors.
A few examples are shown and worked out.

Keywords

Linear models, Generalized canonical analysis, Generalized Wilks Lambda statistic, Singular value decomposition.

Blind identification of linear mixtures

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Abstract

The problem consists of identifying a $N \times P$ matrix, $\mathbf{A}$, from the sole (possibly noisy) observation of realizations of the N-dimensional random variable $\mathbf{x} = \mathbf{As}$, where $\mathbf{s}$ is an unknown non Gaussian P-dimensional random variable with independent components. Here, all variables take their values in the field of real or complex numbers. This paper addresses the case where $N < P$, which is referred to as the *under-determined* case, in certain communities. This problem is not new, but perhaps surprisingly, it has still not received a general answer of practical value, even if some identifiability results are available for a long time; e.g. see Kagan et al. (1973) and references therein. A summary of identifiability results is given. Then two practical approaches are proposed. The first makes use of the Pth derivatives of the joint second characteristic function of variables x_i. The main diffculty lies in the solution of a system of homogeneous polynomial equations of degree N in PN unknowns. A constructive numerical algorithm is proposed, and has been implemented for $2 \leq N \leq 4$ and $3 \leq P \leq 6$. The second approach makes use of the tensor of cumulants of order r of variable $\mathbf{x}$. Because of the observation model assumed, this tensor is structured, and it can be shown that matrix $\mathbf{A}$ can be recovered by exploiting the redundancies hidden in it, provided $r > 3$ and $2N \leq r(P-1)+2$. The algorithm again terminates within a finite number of steps, and has been implemented for both $r = 4$ and $r = 6$. This second family of algorithms can be seen to be related to the canonical decomposition of tensors, already addressed suboptimally in Kruskal (1977).

Keywords

Cumulants, Linear mixture, Characteristic function, Polynomial system, Deviation from Normality.

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Exact distributions for certain linear combinations of Chi-Squares

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Abstract

Exact distributions for linear combinations of Chi-Squares with positive coefficients and even number of degrees of freedom are obtained. Both central and non-central Chi-Squares are considered. Applications to orthogonal mixed models are given.

Keywords

Linear combinations, Chi-Squares distributions.

Parametric multiple correspondence analysis

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Abstract

We perform an unweighted multidimensional scaling on the Burt matrix in multiple correspondence analysis, with the diagonal frequencies parametrized to reduce their influence. With this approach, the off-diagonal contribution is smaller than the standard method, based on correspondence analysis on the Burt matrix. Two real data examples are studied to illustrate this approach.

Keywords

Correspondence Analysis, Burt matrix, Categorical data, Multidimensional Scaling.

Optimal designs for total effects

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Abstract

We study optimality of circular neighbour-balanced block designs when neighbour effects are present in the model. In the literature, many optimality results are established for direct effects and neighbour effects separately, but few for total effects, that is, the sum of direct effect of treatment and relevant neighbour effects. We show that circular neighbour-balanced designs are universally optimal for total effects among designs with no self neighbour. Then, we use some adaptations of the methods developed by Kunert and Martin (2002) to derive efficiency factors of these designs, and show some situations where a design with self neighbours is preferable to a neighbour-balanced design.

Keywords

Neighbour designs, Cross-over design, Universal optimality, Total effects.

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Kunert, J. and Martin, R. J. (2000). On the determination of optimal designs for an interference model. *Ann. Stat.* pp. 1728–1742.

Linear minimax-estimation in the three parameter case

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Abstract

Consider the linear regression model with regression-matrix $\mathbf{X}$ and parameter-vector $\boldsymbol{\beta}$. Assume that there are circular restrictions on the parameter-vector. Let, moreover, $\mathbf{B}^T\mathbf{B}$ be the loss-matrix corresponding to the square loss-function. The linear minimax-problem is considered under the assumption that $\mathbf{X}^T\mathbf{X}$ and $\mathbf{B}^T\mathbf{B}$ possess a joint eigenvector. The problem not yet solved was the case that the maximal eigenspace of the bias-matrix possesses the dimension 2. If in the spectral decomposition of the bias-matrix the unit-vector not belonging to the maximal eigenspace is assumed to be of the form $(\mathbf{u}_1, \mathbf{u}_2, \mathbf{0})$, then it will shown that the solution is found by solving a non-linear equation for $\mathbf{u}_1$.

Nonnegative matrices, max-algebra and applications

Ludwig Elsner

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Abstract

We consider nonnegative matrices as matrices in the max-algebra, i.e. we replace in all operations the usual addition by the max-operation. After an overview we consider some newer applications, in particular in the context of the analytic hierarchy process.

On optimality of binary designs under interference models

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Abstract

We present properties of information matrices of binary designs, especially neighbor balanced designs, in several interference models. Since we are interested in optimality of designs, we analyse such properties of information matrices as complete symmetry and maximality of the trace.

We study one- and two-dimensional interference models, where neighbor effects are fixed or random, and where observations can be correlated.

Keywords

Circular neighbor balanced design at distances 1 and 2; Interference model; Information matrix.

References

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Numerical methods for solving least squares problems with constraints

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Abstract

In this talk, we discuss the problem of solving linear least squares problems and Total Least Squares problems with linear constraints and/or a quadratic constraint. We are particularly interested in developing stable numerical methods when the data matrix is singular or near singular. Of particular interest are matrices which are large and sparse and for which iterative methods must be employed. The quadratically constrained problems arise in problems where regularization is required. For such problems, a Lagrange multiplier is required and that calculation may be quite intensive. The method we propose will quickly yield an estimate of the parameter and allow for finding the least squares solution.

Sequential method in discriminant analysis

Tomasz Górecki

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Abstract

This work describe a some type of combining method in discriminant analysis. It is sequential method which good work for many datasets. We used stacked regression and a nearest neighbors method. These methods are combining to obtain better results. We used regression with restriction to obtain positive values of discriminant index in stacked regression to make use of this method in sequential process.

Keywords

Discriminant analysis, k-nn method, Combining methods.

References

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Restricted ridge estimation

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Abstract

Under the linear regression model with additional linear restrictions on the parameter vector, a ridge estimator for the vector of parameters is introduced. This estimator is a generalization of the well known restricted least squares estimator and is confined to the (affine) subspace which is generated by the restrictions. Necessary and sufficient conditions for the superiority of the new estimator over the restricted least squares estimator are derived. The new estimator is not to be confounded with the restricted ridge regression estimator introduced by Sarkar (1992). Eventually, an estimator for the ridge parameter is proposed.

Keywords

Least Squares, Linear Restrictions, Matrix Risk, Ridge Estimator, Shrinkage.

References

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Quadratic subspaces and construction of admissible estimators of variance components

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Abstract

Let $A_1, \ldots, A_m$ be given, symmetric $n \times n$ matrices. We are interested in finding a basis of Q, the smallest subspace of the space of all $n \times n$ symmetric matrices containing $A_1, \ldots, A_m$ and satisfying the "quadratic subspace condition":

$$A \in Q \Longrightarrow A^2 \in Q.$$

Solutions for some cases corresponding to certain classes of linear mixed models are given e.g. in Zmyślony (1976) or Wojtasik and Zontek (2000). Malley (1994) proposed several algorithms for solving this problem in general case. However, implementation of these algorithms may pose some numerical challenges. In this paper we discuss these issues. We show that by using appropriate numerical techniques it is possible to obtain satisfactory results. We also present numerical results illustrating the efficiency of this approach in constructing admissible quadratic estimators of variance components in linear mixed models.

Keywords

Quadratic admissible estimator, Quadratic subspace.

References

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How to avoid an overinterpretation of the results of statistical analyzes in medical research

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Abstract

A significant inference-facilitating tool often applied in medical research is a statistical analysis of the experimental data obtained. Owing to the high cost of research or an insufficient number of experimental cases analyzed, the obtained body of data may happen to be rather modest. Another problem is the great variability of patient parameters. The above makes us realize that there are limitations to the various statistical methods employed. In the presentation, attention is paid to a correct choice of statistical methods and a way of their application that will limit their overinterpretation. The comments will largely refer to uni- and multivariate statistical methods. Practical examples leading to invalid inferences will be shown as well as ways of correcting them.

Keywords

Statistical methods in medical research.

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Mixing times and their application to perturbed Markov chains

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Abstract

A measure of the "mixing time" or "time to stationarity" in a finite irreducible discrete time Markov chain is considered. The statistic, $\eta_i = \sum_{j=1}^{m} m_{ij}\pi_j$, where $\{\pi_i\}$ is the stationary distribution and m_{ij} is the mean first passage time from state i to state j of the Markov chain, is shown to be independent of the state i that the chain starts in (so that $\eta_i = \eta$ for all i), is minimal in the case of a periodic chain, yet can be arbitrarily large in a variety of situations. An application concerning the affect perturbations of transition probabilities have on the stationary distributions of Markov chains leads to a new bound, involving η, for the 1-norm of the difference between the stationary probability vectors of the original and the perturbed chain. When η is large the stationary distribution of the Markov chain is very sensitive to perturbations of the transition probabilities.

Keywords

Markov chains, Stationary distribution, Mean first passage times, Mixing times, Perturbation theory, Time to stationarity.

Linear prediction sufficiency for new observations in the general Gauss–Markov model

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Abstract

We consider the prediction of new observations in a general Gauss–Markov model. We state the fundamental equation of the best linear unbiased prediction, BLUP, and consider some properties of the BLUP. Particularly, we focus on such linear statistics, which preserve enough information for obtaining the BLUP of new observations as a linear function of them. We call such statistics linearly prediction sufficient for new observations, and introduce some equivalent characterizations for this new concept.

Keywords

Gauss–Markov model, Linear prediction, Linear sufficiency, Linear prediction sufficiency, BLUE, BLUP, Fundamental equations of the best linear unbiased prediction.

References

Baksalary, J.K. and R. Kala (1986). Linear sufficiency with respect to a given vector of parametric functions. *Journal of Statistical Planning and Inference 14*, 331–338.

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Words in two positive definite letters

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Abstract

A word in two letters **A** and **B** is symmetric if it reads the same right to left as left to right. We interpret juxtaposition as matrix multiplication and the two letters **A** and **B** are independent positve definite n-by-n matrices. A symmetric word $S(\mathbf{A}, \mathbf{B})$ is itself positive definite for any substitution of positive definite letters **A** and **B**. We call the equation $S(\mathbf{A}, \mathbf{B}) = \mathbf{P}$ a symmetric word equation and we take one of the letters, say **A** and the right hand side **P** to be given positive definite matrices. The question then arises whether and how many positive definite solutions **B** such an equation has. We also discuss methods for finding solutions to symmetric word equations and systems of symmetric word equations in more variables.

Two local operators and the BLUE

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Abstract

For a given matrix $\mathbf{A}$, a matrix $\mathbf{P}$ such that $\mathbf{PA} = \mathbf{A}$ is said to be a local identity, and such that $\mathbf{P}^2\mathbf{A} = \mathbf{PA}$ is said to be a local idempotent. In the paper some simple properties of such operators are presented. Their relation to the Best Linear Unbiased Estimation in the general Gauss-Markov model is demonstrated.

Characterizations of the commutativity of projectors referring to generalized inverses of their sum and difference

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Abstract

It is obvious that the commutativity of projectors (idempotent matrices) $\mathbf{P}_1$ and $\mathbf{P}_2$ is a sufficient condition for the products $\mathbf{P}_1\mathbf{P}_2$ and $\mathbf{P}_2\mathbf{P}_1$ to be projectors as well. The commutativity property becomes also a necessary condition when $\mathbf{P}_1$ and $\mathbf{P}_2$ are orthogonal projectors (Hermitian idempotent matrices). An extensive collection of results concerning both algebraic and statistical aspects of this property has been given by Baksalary (1987). Two of his algebraic results characterize the equality $\mathbf{P}_1\mathbf{P}_2 = \mathbf{P}_2\mathbf{P}_1$ of orthogonal projectors by referring to generalized inverses of the sum $\mathbf{P}_1 + \mathbf{P}_2$. The purpose of the present paper is to establish several generalizations of these results from two points of view: firstly, by relaxing the assumption of the orthogonality of projectors $\mathbf{P}_1$ and $\mathbf{P}_2$, and secondly, by considering also generalized inverses of the difference $\mathbf{P}_1 - \mathbf{P}_2$.

Keywords

Idempotent matrix, Hermitian idempotent matrix, Sum of projectors, Difference of projectors, Commutativity of projectors.

References

Baksalary, J.K. (1987). Algebraic characterizations and statistical implications of the commutativity of orthogonal projectors, in: T. Pukkila, S. Puntanen (Eds.), *Proceedings of the Second International Tampere Conference in Statistics*, University of Tampere, Tampere, Finland, pp. 113-142.

An explicit expression for the Fisher information matrix of a multiple time series process

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Abstract

The principal result in this paper is concerned with the derivative of a vector with respect to a block vector or matrix. This is applied to the asymptotic Fisher information matrix (FIM) of a stationary vector autoregressive and moving average time series process (VARMA). Representations which can be used for computing the components of the FIM are then obtained. In a related paper of Klein (2000), the derivative is taken with respect to a vector. This is obtained by vectorizing the appropriate matrix products whereas in this paper the corresponding matrix products are left unchanged.

Keywords

Matrix differential rules, Matrix polynomial, Fisher information matrix, VARMA process.

References

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Multivariate skewness and kurtosis measures

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Abstract

Classical measures of skewness and kurtosis were introduced by Mardia (1970). Unfortunately these scalar characteristics may have the same numerical values for multivariate distributions with different shape. There have been suggestions to solve this problem by defining multivariate characteristics of skewness and kurtosis (see Móri, Rohatgi & Székely, 1993, and Koziol, 1989, for example). But these characteristics do not take into account all mixed cumulants of the third and fourth order. In recent years multivariate kurtosis has become a topic of special interest in independent component analysis. In this paper we suggest a multivariate skewness measure as a p-vector and a kurtosis characteristic in the form of an $p \times p$-matrix which are defined with help of the star-product of matrices. An application to the independent component analysis will also be discussed.

Keywords

Multivariate skewness, Multivariate kurtosis, Independent component analysis.

References

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Analysis of growth curve data by using cubic smoothing splines

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Abstract

Longitudinal data arises frequently in various fields of applied sciences where individuals are measured according to some ordered variable, e.g. time. A common approach used to model such data is based on the mixed models for repeated measures. This model provides an eminently flexible approach to modeling of a wide range of mean and covariance structures. However, such models are forced to rigidly defined class of mathematical formulas which may not be well supported by the data within the whole sequence of observations. A possible non-parametric alternative is a cubic smoothing spline which is very flexible and has useful smoothing properties. It is shown that the solution of the penalized log-likelihood equations is the cubic smoothing spline and this solution can be further written under the mixed model framework [Verbyla et al. (1999)]. In the present paper we show that under some special class of covariance structures these solutions can be written in closed forms. It is further investigated how these formulas can be utilized under balanced complete (growth curve) data.

Keywords

Covariance structures, Cubic smoothing splines, Growth curves, Longitudinal data, Mixed models.

References

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Trip matrix estimation for suburban quarters

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Abstract

In the paper an estimation is made of a trip matrix for relations between a Poznań suburb (Różany Potok), where the university campus is located, and the city center. Estimating a trip matrix is important from the point of view of city transport management, but it is not always possible in a direct way (i.e., on the basis of actual traffic measurements). To this end many various mathematical models are used, for example linear ones or of maximum entropy, which also allow an indirect estimation of traffic intensity on parts of the street network.

Keywords

Travel demand, Suburbs, Traffic, Trip table, Estimation.

References

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Invariance of matrix expressions with respect to specific classes of generalized inverses

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Abstract

Several results are known in the literature concerning the invariance of the product $\mathbf{AB}^{(1)}\mathbf{C}$ itself and some expressions involving it with respect to the choice of a generalized inverse $\mathbf{B}^{(1)}$ of $\mathbf{B}$, i.e., with respect to all matrices satisfying $\mathbf{BB}^{(1)}\mathbf{B} = \mathbf{B}$. The collection of other expressions mentioned above contains in particular the range, rank, spectrum, trace, and Frobenius norm of $\mathbf{AB}^{(1)}\mathbf{C}$. The purpose of the present paper is to revisit these results from the view-point of weakening the invariance requirement to the subsets of $\{\mathbf{B}^{(1)}\}$ comprising the matrices, which in addition to $\mathbf{BB}^{(1)}\mathbf{B} = \mathbf{B}$ satisfy also further conditions from a definition of the Moore-Penrose inverse of $\mathbf{B}$, i.e., $\mathbf{B}^{(1)}\mathbf{BB}^{(1)} = \mathbf{B}^{(1)}$ and/or $\mathbf{BB}^{(1)} = (\mathbf{BB}^{(1)})^*$ and/or $\mathbf{B}^{(1)}\mathbf{B} = (\mathbf{B}^{(1)}\mathbf{B})^*$.

Keywords

Generalized inverse, Range, Rank, Spectrum, Trace, Frobenius norm.

On optimal cross-over designs when carry-over effects are proportional to direct effects

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Abstract

There is a number of different models for crossover designs which take account of carryover effects. Since it seems plausible that a treatment with a large direct effect should generally have a larger carryover effect, Kempton, Ferris and David (2001) considered a model where the carryover effects are proportional to the direct effects. The advantage of this model lies in the fact that there are fewer parameters to be estimated. Its problem lies in the non-linearity of the estimates. Kempton, Ferris and David (2001) considered the least squares estimate. They point out that this estimate is asymptotically equivalent to the estimate in a linear model, which assumes the true parameters to be known.

For this estimate they numerically determine optimal designs for some cases. The present paper generalizes some of their results. Our results are derived with the help of a generalization of the methods used in Kunert and Martin (2000).

Keywords

Cross-over designs, Universal optimality, Carry-over effects.

References

Kempton, R. A., Ferris, S. J., and David, O. (2001): Optimal change-over designs when carry-over effects are proportional to direct effects of treatments. *Biometrika 88*, 391 – 399.

Kunert, J. and Martin, R. J. (2000): On the determination of optimal designs for an interference model. *Ann. Statist. 28*, 1728 – 1742.

Criteria for the comparison of discrete-time Markov chains

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Abstract

In this paper, we develop an approach to compare two discrete-time Markov chains which are not assumed to have the same state space. To do this, we introduce a binary relation between probability vectors or marginal laws which is not a partial order in general. This binary relation generalizes the notion of stochastic ordering for discrete random variables (Muller and Stoyan 2002). We obtain geometric criteria for the comparisons of discrete-time Markov chains. These criteria have the form of inclusion of polyhedral sets. Then, an algebraic form of the previous criteria may be derived from Haar's lemma (Haar 1918). In this context, the so-called concept of positive invariance is in force.

Our result allow us to reexamine some previous works on stochastic comparison of Markov chains. Moreover, we also discuss the comparison of Hidden Markov chains.

Keywords

Markov chain, Polyhedral set, Set invariance, Hidden Markov chain.

References

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Data driven score test of fit for semiparametric homoscedastic linear regression model

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Abstract

We shall present new test for asserting validity of the following semiparametric linear regression model M(0)

$$Y = \beta[v(X)]^T + \epsilon,$$

where X and ϵ are independent with unknown densities g and f, supported on [0,1] and R, respectively. We assume $E_f\epsilon = 0$ and $E_f\epsilon^2 < \infty$. $\beta \in R^q$ is a vector of unknown parameters while $v(x) = (v_1(x), ..., v_q(x))$ is a vector of known functions. The symbol ^{T} denotes transposition. Throughout we consider row vectors.

The test construction combines classical ideas with some modern smoothing methods.

The classical, mostly analytical, part relies, first of all, on following the idea of overfitting and replacing the basic problem by a series of auxiliary subproblems. More precisely, we embed M(0) into extended model $\mathtt{M_k}(\theta)$

$$Y = \theta[u(X)]^T + \beta[v(X)]^T + \epsilon,$$

where, for each given k, $\theta \in R^k$ is a vector of unknown parameters while $u(x) = (u_1(x), ..., u_k(x))$ is a vector of known functions. Note that the joint density of (X, Y), under $\mathtt{M_k}(\theta)$, has the form

$$p(z;\kappa) = g(x)f(y - (u,v)(\theta,\beta)^T) \quad \text{with} \quad \kappa = (\theta, \beta, f, g) \quad \text{and} \quad z = (x, y).$$

Next classical, in principle, idea is to construct efficient score test for testing $\theta = 0$ against $\theta \neq 0$ in $\mathtt{M_k}(\theta)$. This requires a derivation of suitable derivative of $p(z;\kappa)$ over κ from appropriate Banach space. The derivative is determined by a vector, which is called the score vector. Additionally, one has to calculate efficient score vector which results as residual from projections [derived under the null hypothesis] of scores for the parameters of interest on scores for nuisance parameters. Finally, an auxiliary statistic is defined as quadratic form of the efficient score vector and the inverse if its covariance matrix. This is analytical part of the work, which is discussed in details in Inglot and Ledwina (2004). In that paper the above programme is carried out for heteroscedastic model as well.

Probabilistic part relies on exploiting some ideas of adaptive semiparametric estimation to propose suitable estimators of the involved parameters of the auxiliary statistic, defined above. The basic focus is on ensuring that the limiting null distribution of resulting object shall be independent of unknown nuisance parameters β, f, g. We call such statistic efficient score statistic.

Last step of our construction is to propose data driven selection rule to choose the right subproblem. So, the final result is the efficient score statistic with the dimension k fitted by the selection rule.

The construction shall be presented in some details. Also some simulations shall be shown, to demonstrate good performance of our test.

Keywords

Data driven test, Efficient score, Linear regression, Selection rule, Semiparametric model.

References

Inglot, T., and T. Ledwina (2004). Semiparametric regression: Hadamard differentiability and efficient score functions for some testing problems. Shall be submitted to *Linear Algebra Appl.*, the workshop volume.

Inglot, T., and T. Ledwina (2003). Data driven score test of fit for semiparametric homoscedastic linear regression model. Submitted for publication.

The MDL model choice for linear regression

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Abstract

In this talk, we discuss the principle of *Minimum Description Length (MDL)* for problems of statistical modeling. By viewing models as a means of providing statistical descriptions of observed data, the comparison between competing models is based on *the stochastic complexity (SC)* of each description. *The Normalized Maximum Likelihood (NML)* form of the SC (Rissanen 1996) contains a component that may be interpreted as the parametric complexity of the model class. Once the SC for the data, relative to a class of suggested models, is calculated, it serves as a criterion for selecting the optimal model with the smallest SC. This is the MDL principle (Rissanen 1978, 1983) for model choice.

If the parametric complexity of a model family is unbounded, then one must deviate from the clean definition of the SC. The most important example of this phenomenon is the Gaussian family. One approach to bound the parametric complexity is by constraining the sample space. We calculate the SC for the Gaussian linear regression by using the NML density and consider it as a criterion for model selection. The final form of the selection criterion depends on the method for bounding the parametric complexity. As opposed to traditional fixed penalty criteria, this technique yields adaptive criteria that have demonstrated success in certain applications.

Keywords

Minimum description length, Stochastic complexity, Normalized maximum likelihood, Parametric complexity, Adaptive selection criteria.

References

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A new rank revealing tri-orthogonalization algorithm and its applications

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Abstract

In this paper we discuss some new numerical methods that are suited for regularization of Linear Least Squares (LS) problems with a numerically *rank–deficient* coefficient matrix A. Such problems arises frequently, when we consider inverse problems that need to be solved numerically. The main feature of these problems is that the matrix A in the corresponding overdetermined system of linear equations $Ax = b$ is having a cluster of small singular values, and there is a well determined gap between its large and small singular values.

Usually singular value decomposition of A (SVD) is used for that regularization. When A is large and sparse its SVD is not very useful, so we propose to apply a new triorthogonalization algorithm followed by the recent versions of Lanczos iterative bidiagonalization with selective reorthogonalization to economize the process of searching for solution of the problem considered. Some numerical results are presented.

Keywords

SVD decomposition, One-sided Householder, Linear Least Squares, Regularization, Divide and Conquer.

References

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Sharp estimates on the tail behaviour of some random integrals and their application in statistics

Péter Major

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Abstract

I met the problems discussed here in an investigation when I tried to adapt the methods of maximum likelihood estimates to some non-parametric problems. One can give a good and simple approximation for the error of the maximum likelihood estimate by means of an appropriate linearization in the maximum-likelihood equation. It can be shown that a really good approximation is obtained in such a way with the help of the observation that the coefficient of the second term in the Taylor expansion we apply in this linearization is bounded. This linearization argument can be adapted to the study of several interesting non-parametric problems, but in the non-parametric case some multiple random integrals have to be bounded instead of Taylor coefficients. This led to the following problem described below.

Let us have a sequence of iid. random variables $\xi_1, \dots, \xi_n$ on a space $(X, \mathcal{X})$ with distribution μ, and let μ_n denote their empirical distribution. Given a real valued function $f(x_1, \dots, x_k)$ of k variables on the space $(X, \mathcal{X})$ consider the k-fold random integral

$$J_{n,k}(f) = \frac{n^{k/2}}{k!} \int' f(u_1, \dots, u_k)(\mu_n(\,du_1) - \mu(du_1)) \dots (\mu_n(\,du_k) - \mu(\,du_k)),$$

where prime means that the diagonals are omitted from the domain of integration. We want to give a good bound on the probability $P(|J_{n,k}(f)| > x)$ for all $x > 0$. More generally, given a nice class of functions $f \in \mathcal{F}$ of k variables we are interested in the probability $P\left(\sup_{f \in \mathcal{F}} |J_{n,k}(f)| > x\right)$ for $x > 0$.

The tail-behaviour of $J_{n,k}(f)$ is similar to that of the k-th power of a Gaussian random variable with expectation zero and variance of the same order as the k-th root of the variance of the random variable $J_{n,k}(f)$ provided that this variance is not too small. More explicitly, $P(|J_{n,k}(f)| > x) < Ce^{-B(x/\sigma)^{2/k}}$ with some universal constants $C > 0$ and $B > 0$ and $\sigma^2 = \int f^2(u_1, \dots, u_k)\mu(\,du_1) \dots \mu(\,du_k)$ if the absolute value of the function f is bounded by 1, and $0 < x < n^{k/2}\sigma^{k+1}$. Beside this, the variance of $J_{n,k}(f)$ has the same order as σ^2. The same estimate holds with possibly different uni-

versal constants $C>0$ and $B>0$ for the probability $P\left(\sup\limits_{f\in\mathcal{F}}|J_{n,k}(f)|>x\right)$ if $\mathcal{F}$ is a nice class of functions of k variables whose elements are such functions which are bounded in supremum norm by 1 and in L_2 norm by σ. Such a result holds for instance if $\mathcal{F}$ is a so-called Vapnik–Červonenkis class of functions. The only additional restriction we have to impose for the validity of such an estimate is the condition $\left(\frac{x}{\sigma}\right)^{2/k}>D\log n$ with some $D>0$ to exclude the possibility that the supremum of relatively small random variables be large.

Estimation of location and scale parameters using *k*-th record values

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Abstract

We present the minimum variance linear unbiased estimators of the location parameter and scale parameter of Burr and Gumbel distribution using k-th record values. The estimators obtained by the least-squares method contain as particular cases the estimators given by Ahsanullah (1995). Applications of generalized order statistics in this subject is also discussed.

Keywords

Burr distribution, Estimation, k-th record values, Generalized order statistics, Gumbel model.

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Optimum choice of covariates in BIBD setup

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Abstract

The problem considered is that of finding optimum designs for the estimation of covariate parameters and the treatment and block-contrasts in a block-treatment design set-up in the presence of non-stochastic covariates. This is an extension of the work considered by Das et al. (2003). Here we deal with the situation wherein the block design setup admits existence of a balanced incomplete block design (BIBD) with parameters v, b, r, k and λ. It is very difficult to investigate the underlying combinatorial problems in the context of an arbitrary BIBD in order to accommodate maximum number of covariates in an optimal manner. Here we try to investigate the problem for some given series of BIBDs obtained through Bose's method of differences. Also we give some results for arbitrary BIBD and some particular BIBDs viz. resolvable BIBD, irreducible BIBD etc.

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Das, K., N.K. Mandal and Bikas K. Sinha (2003). Optimum experimental designs for models with covariates. *J. Statist. Plann. Infer. 115*, 273-285.

Optimal experimental designs when most treatments are unreplicated

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Abstract

n early generation variety trials, large numbers of new varieties may be compared, and little seed is usually available for each variety. A so-called unreplicated trial has each new variety on just one plot at a site, but includes several (often around 5) replicated check or control (or standard) varieties. The total proportion of check plots is usually between 10% and 20%. The aim of the trial is to choose some (around 1/3) good performing varieties to go on for further testing, rather than precise estimation of their mean yield.

Now that spatial analyses of data from field experiments are becoming more common, there is interest in an efficient layout of an experiment given a proposed spatial analysis. The usual C-matrix is very large, and hence it is time-consuming to calculate its inverse. However, since most varieties are unreplicated, the variety incidence matrix has a simple form, and some matrix manipulations can dramatically reduce the computation needed. Some possible design criteria are discussed, and efficient layouts under spatial dependence are considered.

Numerical solution of the eigenvalue problem for the Anderson Model

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Abstract

We discuss the application of modern eigenvalue algorithms to an eigenvalue problem arising in quantum physics, namely, the computation of a few interior eigenvalues and their associated eigenvectors for the large, sparse, real, symmetric, and indefinite matrices of the Anderson model of localization. This seemingly innocuous problem presents a major challenge for all modern eigenvalue algorithms. We show why this is the case and also discuss some remedies although so far none of these has turned out to be really successful.

Statistical analysis of normal orthogonal models with emphasis on their algebraic structure in view of obtaining effcient statistics for inference

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Abstract

The algebraic structure of normal orthogonal models is presented using Jordan algebras. Such presentation gives both sufficient complete statistics and pivot variables. From the first minimum variance unbiased estimates may be obtained while the second, through the use of the Caratheodory theorem, induces probability measures in the parameter spaces. These will be *a-posteriori* measures since they will depend on the values of sufficient complete statistics, despite no *a-priori* distribution having been assumed. The heavy computations required when dealing with the induced measures may be replaced by the use of Monte Carlo methods validated by the Glivenko-Cantelli theorem and related results.

Moreover, the use of Jordan algebras to describe the structure of the models leads to broader classes of models than those obtained starting with the factors, either fixed effect or random-effect factors, that we consider. Thus, this model formulation leads to a more robust inference. Besides this, a cleaner insight into certain results, such as negative variance components estimators, may be achieved. We point out that factor formulation of the models may be obtained from the algebraic formulation imposing restrictions to the parameters. These restrictions can be tested, thus leading to an integrated inference of both model formulations that may lead to the trimming of non significant factors.

The general results will be applied to models such as those having balanced cross-nesting.

Keywords

Jordan algebras, Mixed models, Algebraic formulation, Factor based formulation, Inductive pivot variables, Duality.

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Permutation invariant covariance marices

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Abstract

Certain types of patterned (structured) matrices which are generated via statements about invariance are considered. It follows that invariance with respect to the group of permutations implies a specific structure on the covariance matrix. These structures of patterned matrices which arise have been studied by a number of authors (Wilks, 1946; Votaw, 1948; Olkin, 1973; Searle and Henderson, 1979; Jennrich and Schluchter, 1986; etc.).
We are going to study permutation invariant covariance matrices which arise from k-way analysis of variance tables. We also study the spectrum and eigenvectors of the permutation invariant covariance matrix, i.e. we shall provide the insight into the structure of the eigenvalues and eigenvectors of such patterned matrices.

Keywords

Covariance matrix, Permutation invariance, Spectrum.

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Linear prediction for electricity consumption with Levy distribution

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Abstract

Symmetric alpha stable distributions with $\alpha < 2$ could not be applied by classical statistics methods for model fitting and prediction (Autocorrelation matrix analysis, MSE analysis, ...). Levy distribution is one element of this class of distributions with $\alpha = 1/2$. Today many methods have been introduced for analysing these distributions but infinite variance and unknown density function are the most important characteristic for this family which can provide many problems for statistical analysis. In this paper it have been shown a singular method for prediction in a time domain by real example for electiricity consumption in Iran. Electricity consumption in a specific level in Iran have Levy distribution (it will be shown simply) and by analysing on large data in this section and also applying the new concept of statistical parameter which is named *dispersion* we try to fit the model for prediction.

Keywords

Stable distribution, Dispersion, Regular variation, ARMA models, Domains of attraction.

On the structure of a class of normal decomposition systems

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Abstract

A normal decomposition system (NDS) is connected with a decomposition statement for vectors of a linear space and with an inequality related to the decomposition. A typical example is the Singular Value Decomposition for matrices provided with the trace inequality of von Neumann. In the paper, we study the problem of generating such systems. The structure of a certain class of NDS is given. As a corollary, we show that the SVD and von Neumann inequality are implied by analogous results of Miranda ad Thompson related to the special orthogonal group.

Keywords

Normal decomposition system, Eaton triple, G-majorization, Group induced cone ordering, Finite reflection group, Singular value, Eigenvalue.

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Some notes on scatter matrices and independent component analysis (ICA)

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Abstract

A matrix finctional $C(F)$ (for a p-variate distribution F) is called a scatter matrix if it is a positive definite symmetric $p \times p$-matrix with the affine equivariance property. Some interesting M-functionals are discussed in more details. Finally, the use of different scatter matrices $C(F)$ in the independent component analysis (ICA) is discussed.

Inequalities: some probabilistic, some matric, and some both

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Abstract

It is interesting that a theory of equations has been developed, for example, differential equations, functional equations, and so on, but not for inequations (inequalities). This partially borne out by the large number of books on the theory of equations versus the few books on inequalities. Indeed, there are only a few general methods for obtaining inequalities. In this survey we touch on the connection between probabilistic or statistical inequalities and matrix inequalities, and also discuss some new matrix inequalities.

Meta-analysis: combining information from independent studies

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Abstract

Meta-analysis enables researchers to synthesize the results of independent studies so that the combined weight of evidence can be considered and applied. Increasingly meta-analysis is being used in medicine and other health sciences to augment traditionals methods of narrative research by systematically aggregating and quantifying research literature.

Meta-analysis requires several steps prior to statistical analysis: formulation of the problem, literature search, coding and evaluation of the literature, After these steps one can address the statistical issues.

In this workshop we will review some of the history of meta-analysis and discuss some of the problematic issues such as various forms of bias that may exist. A summary of statistical techniques will be reviewed, in particular, nonparametric methods, combining proportions and combining effect sizes from continuous data. The discussion of proportions will include comments about alternative metrics, such as odds ratios, risk ratios, risk difference.

References

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Unitary invariant random Hermitian matrices and complex elliptical distributions

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Abstract

Complex random vectors and complex random hermitian matrices which are invariant in distribution under certain transformations by the group of unitary matrices are studied. It is shown that unitary invariance implies a certain structure on their covariance matrix and the pseudo-covariance matrix. (For a zero mean complex random vector $\mathbf{Z}$, the covariance matrix and the pseudo-covariance matrix are defined as $E(\mathbf{Z}\mathbf{Z}^H)$ and $E(\mathbf{Z}\mathbf{Z}^T)$, respectively, where the superscipts T and H denote transpose and conjugate transpose.) This result is then used in the derivation of the finite sample and asymptotic covariance matrix and the pseudo-covariance matrix of *any* affine equivariant estimates of location vector and scatter matrix when sampling from Complex Elliptically Symmetric (CES) distributions of Krishnaiah and Lin (1986). As an example we consider the sample mean and the sample covariance matrix (SCM) in detail. It is well known that when sampling from real elliptically symmetric distributions, the asymptotic covariances between the elements of the SCM can be expressed as a function of the kurtosis and the underlying true covariance matrix, or, as a function of multivariate cumulants (Muirhead, 1982). We show that this is the case also for the covariances and pseudo-covariances of the SCM when sampling from CES distributions. To accomplish this we need to define complex kurtosis and complex multivariate cumulants for complex random variables, and in particular, calculate these for CES distributions.

Keywords

Complex random vectors and matrices, Covariance matrix, Pseudo-covariance matrix, Complex multivariate cumulants, Kurtosis.

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Population equilibrium and its fitness in evolutionary matrix games

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Abstract

We consider an evolutionary game $[A, A^T]$ with no dominant behaviour, where $A \in R^{n \times n}$ (a population is divided into n number of phenotypes). We give formulae for the proportion of the i-th phenotype to the j-th phenotype in the population equilibrium and for its fitness.

We illustrate these results by the well known Hawk-Dove Game, which is similar to many games in economics, politics, etc. At first we consider 2×2 game, including the cases when the game converts in the famous Prisoner's Dilemma or in the Chicken Game.

After that we consider the three-by-three Skyrms Modest-Fair-Greedy Game.

On linear sufficiency with respect to given parametric functions

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Abstract

Linear sufficiency under the general linear model is considered in the context of a given set of parametric functions. Early research on this subject was carried out by Baksalary and Kala (1986). Presenting new characteristics of linearly sufficient statistic with respect to a given vector of parametric functions we provide further contribution to the theory of linear transformations preserving best linear unbiased estimators in the general linear model. Relation between linear sufficiency and misspecification of dispersion matrix in the model is also investigated.

References

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On common divisors of matrices over principal ideal domain

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Abstract

Let R be a principal ideal ring with the identity $e \neq 0$ (Newman, 1972) and R_n the ring of $n \times n$ matrices over R. It is said that the matrices $B, C \in R_n$ have a common left divisor if $B = DB_1,\ C = DC_1$, where $D \in R_n$, $\mathbf{det} D = d \neq 0$ and $D \notin GL(n, R)$.

The problem of common left divisors of matrices over a principal ideal domain is investigated. Necessary and with certain restrictions sufficient conditions are established for existence of a common divisor $D \in R_n$ with a prescribed $\mathbf{det} D = d$ of matrices $B \in R_n$ and $C \in R_n$. In the case, when the desired divisor exists, the method of it constructing is specified. The results are true for elementary divisors rings.

Notation: $B, C \in R_n$; d_A^k – the greatest common divisor of the minors of order k, $1 \leq k \leq n$, of matrix $A = \|B\ C\|$.

Proposition. Let $\mathbf{rank} A \geq n-1$ and d_A^n admits representation in the form $d_A^n = dg$, where $R \ni d \neq 0$ and d is not unit. If $(d, g, d_A^{n-1}) = e$, then for the matrices B and C there exists a left common divisor $D \in R_n$ with given $\mathbf{det} D = d$. If the matrices B and C admit another representations $B = D_1 B_2$, $C = D_1 C_2$ such that $D_1 \in R_n$ and $\mathbf{det} D_1 = d$, then $D = D_1 W$, where $W \in GL(n, R)$. $1 \leq k \leq n$.

Keywords

Matrix, Common divisor.

References

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On decomposing the Watson efficiency of ordinary least squares in a partitioned weakly singular linear model

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Abstract

We consider the estimation of regression coefficients in a partitioned weakly singular linear model and focus on questions concerning the Watson efficiency of the ordinary least squares estimator of a subset of the parameters with respect to the best linear unbiased estimator. Certain submodels are also considered. The conditions under which the Watson efficiency in the full model splits into a function of some other Watson efficiencies is given special attention.

Keywords

BLUE, Efficiency multiplier, Frisch–Waugh–Lovell theorem, Linear sufficiency, OLSE, Reduced linear model, Splitting the efficiency.

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Spectral matrix decomposition in geographical research

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Abstract

In geographical research, especially in the field of socio-economic geography, a significant role is played by the spectral decomposition of special types of matrices, e.g., a covariance or a transition matrix, because it allows specific properties of spatial processes to be captured. A case in point is the process of population migration. That is why the issue addressed in more detail here is the effects of the spectral decomposition of the above-mentioned matrices.

Some results on patterned matrices

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Abstract

We are going to consider patterned matrices as subsets of matrix elements without tieing the notion of patterned matrix to any specific relation among the elements of the matrix. A patterned matrix $\mathbf{A}(K)$ is a matrix where any element or a certain part of the original matrix, defined by an index-set K, has been excluded from $\mathbf{A}$, i.e. a certain pattern has been "cut out" from the original matrix. The major part of the applications of the approach concerns symmetric, skew-symmetric, diagonal, Toeplitz, triangular, etc. matrices.

Let $\mathbf{A}$ be an $p \times q-$matrix and K a set of pairs of indices:

$$K = \{(i,j) : i \in I_K, j \in J_K; I_K \subset \{1,\ldots,p\}; J_K \subset \{1,\ldots,q\}\}.$$

We call $\mathbf{A}(K)$ a patterned matrix and the set K a pattern of the $p \times q-$matrix, if $\mathbf{A}(K)$ consists of elements a_{ij} of $\mathbf{A}$ where $(i,j) \in K$.

The notation $\mathbf{A}(K)$ does not represent a matrix in a strict sense since it is not a rectangle of elements. One should just regard $\mathbf{A}(K)$ as a convenient notion for a specific collection of elements. When the elements of $\mathbf{A}(K)$ are collected into one column by columns of $\mathbf{A}$ in a natural order, we get an r-vector, where r is the number of pairs in K. Let us denote this vector by vec$\mathbf{A}(K)$. Clearly, there exists always a matrix which transforms vec$\mathbf{A}$ into vec$\mathbf{A}(K)$.

We are going to apply a vector space approach and also define several useful matrices in order to present a systematic treatment of patterned matrices. The results turn out to be useful when we are interested in finding Jacobians or want to derive moments of higher order.

Survival analysis in SAS

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Abstract

There is a great need for methods allowing prediction in medicine. Survival analysis is a method increasingly applied to medical data analysis. The conclusions derived from survival analysis differ depending on the medical discipline. Cases revealing these differences will be presented, and the usefulness and applicability of survival analysis in medical practice, particularly in diagnostics and therapy will be discussed.

Keywords

Survival analysis, Diagnostics, Therapy.

One-dimensional optimal bounded-shape partitions for Schur convex sum objective functions

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Abstract

Recently, Hwang and Rothblum solved the problem in the title for the special case that the lower bounds and upper bounds over the parts have the same ordering. In this paper we consider the general case. While a unique optimal solution may not exist, we give a set of at most $(\frac{p^2}{8}+1)\binom{p}{\lfloor p/2 \rfloor}$ candidates, where p is the number of parts the n elements are partitioned into. We conjecture that the candidate set actually has at most $\binom{p-1}{\lfloor (p-1)/2 \rfloor}$ members.

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Reliability analysis in linear models

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Abstract

In geodetic science, reliability measures are used to determine the potential of detecting any outliers in the respective observations ("internal reliability"), and to limit the impact of any undetected outliers on the estimated parameters ("external reliability"). Here we study the change of these reliability measures, due to modifications in the experimental design, within various linear models that include the rank-deficient Gauss-Markov Model with and without effective constraints, and the so-called Gauss-Helmert Model. We shall conclude this study with some examples from photogrammetry, resp. "computer vision", where sometimes up to 15% of the automatic measurements may be affected by outliers or blunders.

Some combinatorial aspects of a counterfeit coin problem

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Abstract

Considered is the set-up of availability of n coins and the possibility of at the most one of these coins being counterfiet in the sense of possessing slightly more or slightly less weight than that of the rest, though all the coins have remarkably identical appearance ! There has been considerable interest in the underlying combinatorial problem of checking the existence of such a coin and identifying the same [in case it exists] in minimum number of efforts with an ordinary 2 - pan balance.

After quickly reviewing the literature, we propose to discuss some combinatorial aspects of this problem in situations wherein two or more 2-pan balances are made available to us.

One-sample spatial sign and rank methods

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Abstract

Consider a sample $x_1, \ldots, x_n$ from a p-variate elliptically symmetric distribution with density $f(x; \mu, \Omega) = |\Omega|^{-\frac{1}{2}} g(x^T \Omega^{-1} x)$. We wish to estimate the location vector μ and the scatter matrix Ω or the so called shape matrix $V = \frac{p}{Tr(\Omega)} \Omega$. In this talk several affine equivariant estimates for location and shape are considered. These are based on the concepts of multivariate, or spatial, signs, $S(x_i) = ||x_i||^{-1} x_i$, and ranks, $R(x_i) = ave_{j \neq i} \{S(x_i - x_j)\}$. Hettmansperger and Randles (2002) proposed a special case of M-estimates for location and shape, namely, a combination of the so called transformation-retransformation median (Chakraborty et al., 1998) and Tyler's M-functional (Tyler, 1987). Unfortunately, the simultaneous existence and uniqueness of these estimators have not been proven, although the algorithm seems to work very well in practice. Another possibility for shape estimation is to use pairwise differences of observations and spatial signs of those (Dümbgen, 1998). This Kendall's Tau -type estimate can be shown to exist uniquely, there is a simple algorithm for it and it is highly efficient. Yet another estimate can be based on spatial ranks, resulting in a Spearman's Rho -type estimate, but proofs for its existence and uniqueness seem to be much more difficult and have not yet been completely found.

Keywords

Multivariate location and shape, Multivariate signs and ranks, Affine equivariance.

References

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Canonical form of a linear model and its applications

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Abstract

Arbitrary linear model of type $L(A\beta, V)$, $L(A\beta, \sigma V)$ or $N(A\beta, \sigma V)$ for a random sample $X_1, \ldots, X_n$ is considered. It is well known that if the model is *regular* in the sense $R(A) \subseteq R(V)$ then it admits a representation by a sample $Y_1, \ldots, Y_m$ $(m \leq n)$, where $EY_i = \eta_i$ for $i = 1, \ldots, k$ and zero for $i = k+1, \ldots, m$, while $Cov(Y_i, Y_j) = \delta_{ij}$ or $\sigma\delta_{ij}$, respectively. This canonical form, introduced by Kołodziejczyk (1935), was used, among others, by Schéffe (1959) and Lehmann (1959, 1986) for linear estimation and testing linear hypotheses. It appears that this technique may be extended for *arbitrary*, not necessarily regular model and, what is more, it may be applied for quadratic estimation. This enables us to derive many results in a simple way.

References

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Inequalities and equalities for the generalized efficiency function in orthogonally partitioned linear models

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Abstract

We consider the estimation of regression coefficients in orthogonally partitioned linear models and focus on the Watson efficiency of the ordinary least squares estimator of the full set of the parameters with respect to the best linear unbiased estimator and how this full Watson efficiency relates to the product of the Watson efficiencies of two subsets of the parameters. Building upon our recent paper Chu et al. (2004), we introduce a new and apparently very useful generalized efficiency function and show how it is related to the Watson efficiency.

References

Chu, K.L., J. Isotalo, S. Puntanen, and G.P.H. Styan (2004). On decomposing the Watson efficiency of ordinary least squares in a partitioned weakly singular linear model. *Sankhyā 66*, in press.

Almost sure Central Limit Theorem for subsequences

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Abstract

The aim of the speech is to present the almost sure central limit theorem (ASCLT) for sequences of independent, nonidentically distributed random variables. Let $S_n, n \geq 1$ be a partial sum of the sequence of independent random variables with zero mean and finite variances and let $a(x)$ be the real function, satisfying certain conditions. Starting from the functional version of ASCLT we will arrive at the ASCLT presenting sufficient conditions, under which

$$\lim_{N\to\infty} \frac{1}{N} \sum_{k=1}^{N} a\left(S_{n_k}/(ES_{n_k}^2)^{1/2}\right) = \int_{-\infty}^{\infty} a(x)d\Phi(x), \quad P\text{−p.p..}$$

The method of constructing subsequences $\{n_k,\ k \geq 1\}$ will be shown.

Keywords

Almost sure central limit theorem, Functional central limit theorem.

References

Rychlik, Z., and K.S. Szuster (2003). Some remarks on the almost sure central limit theorem for independent random variables. *Probab. Math. Statist. 23*, 241–249.

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Interpolation of measure of non-compactness and applications to spectral theory

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Abstract

We study interpolation of the measure of non-compactness of operators between couples of compatible Banach spaces. We show as a consequence of the obtained results that an interpolation estimate for the essential spectral radius of interpolated operators can be derived.

Two interesting metric matrices in statistics

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Abstract

Metric (weight) matrices often play an important role in the estimation of linear and bilinear models as a way of differentially weighting, and/or of decorrelating correlated observations. In this paper, we discuss two examples of metric matrices useful in statistics, one arising from a ridge type of regularization technique (Takane & Hwang, 2004; Takane & Yanai, 2003), and the other arising from an instrumental variable estimation (Takane & Yanai, 1999) in linear models. We show some interesting properties of these matrices. In both cases, they are closely related to the transformations necessary to go from primal bases (of a data matrix) to dual bases (of its generalized inverse), and vice versa (Yanai & Takane, 2003).

Keywords

Rank additivity, Oblique projectors, Dual bases, Ortho-normalizing metrics.

References

Takane, Y. and H. Hwang (2004). Regularized multiple-set canonical correlation analysis. In: J. Blasius, and M.J. Greenacre (Eds.), *Multiple Correspondence Analysis and Related Methods*. London: Academic Press, (in press).

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Yanai, H. and Y. Takane (2003, July). On generalized canonical correlation analysis among M sets of variables. A paper presented at IMPS 2003, Sardinia, Italy.

On projectors with respect to seminorms

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Abstract

Let A be an $m \times n$ matrix and V be an $m \times m$ nonnegative definite matrix. A square matrix $P_{A:V}$ is called a projector into the column space $\mathcal{R}(A)$ of A with respect to the seminorm $\|\mathbf{z}\|_V$ defined by $\|\mathbf{z}\|_V = (\mathbf{z}'V\mathbf{z})^{1/2}$ if

$P_{A:V}\mathbf{y} \in \mathcal{R}(A)$ and $\|\mathbf{y} - P_{A:V}\mathbf{y}\|_V \leq \|\mathbf{y} - A\mathbf{x}\|_V$ for any $\mathbf{x} \in \mathbb{R}^{n\times 1}$, $\mathbf{y} \in \mathbb{R}^{m\times 1}$.

The projector $P_{A:V}$ is not necessarily idempotent for a given pair of matrices A and V. In this paper, we investigate various properties of $P_{A:V}$ under the three conditions: (i) V is positive definite, (ii) $\text{rank}(VA) = \text{rank}(A)$, (iii) $\text{rank}(VA) < \text{rank}(A)$.

Keywords

Seminorm, Moore-Penrose inverse, Weighted Moore-Penrose inverse, Projector, Matrix equality, Rank equality, Range equality, Commutativity.

References

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Bias of regression estimator in survey sampling

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Abstract

The regression estimator (see Särndal, Swensson, Wretman 1992) is now widely used in official statistics. In estimating finite population totals it borrows strength from the auxiliary variables. The estimator is more effective than the classical Horvitz-Thompson estimator is. Nevertheless, it is a biased estimator, though for big samples the bias is negligible.

We develop a general matrix expression for the bias of regression estimator. The Taylor expansion with matrix derivatives is used. The bias depends on the covariances of involved random quantities. Special cases for different sampling designs and different relationships between variables are drawn from the general formula. A ratio estimator as a particular special case is considered. Numerical illustration is given.

Keywords

Regression estimator, Ratio estimator, Bias.

References

Särndal, C.-E., B. Swensson, and J. Wretman (1992). *Model Assisted Survey Sampling* . New York: Springer-Verlag.

On generalized quadratic matrices

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Abstract

Extending an approach considered by Radjawi and Rosenthal (2002), we investigate the set of square matrices whose square equals a linear combination of the matrix itself and an idempotent matrix. Special attention is paid to the Moore-Penrose and group inverse of matrices belonging to this set.

References

Radjavi, H. and P. Rosenthal (2002). On commutators of idempotents. *Linear and Multilinear Algebra 50*, 121-124.

Reconstruction of Kauffman networks applying trees

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Abstract

According to Kauffman's theory the enzymes have two possible behavior in living organisms: they may be active and passive. Behaviors of enzymes at a given moment together form the actual state of the organism. The states change step by step following prescribed rules dictated by a system called Kauffman network. After a possible initial phase one previous state returns and then the whole process is systematically repeated. The repeated states together form an attractor of the organism. In microarray measurements the activity of clones is measured. In the present investigation we reduce the experimental results to two possible outputs: one clone may be active or passive likewise the enzymes themselves. The problem what we are discussing here is the reconstruction of the structure of enzymatic interactions of the living organism from microarray data. The task resembles recapitulating the whole story of a film from unordered and perhaps not complete collections of its pieces. We shall use two basic ingredients in tackling the problem. In our earlier works we used an evolutionary strategy called Tierra which was proposed by Tom Ray for investigating complex systems. Here we apply the method together with the tree-structure of clones found in our earlier statistical analysis of microarray measurements.

Keywords

Regulatory networks, Kauffman's theory, Microarrays, Tierra.

References

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A problem in multivariate analysis

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Abstract

In the computer science the following "hidden subgroup problem" is nowadays intensively studied (see, e.g., [1], [2]): Let G be a finite group, $f : G \to R$ be a function and assume that there exists a nontrivial subgroup $H \subset G$ such that the f is periodic w.r.t. the H. Determine the H. This suggests the following question: What about a similar problem with R^n and L instead of G and H, resp., where L is a discrete subgroup in R^n ? The talk is about first steps in studying the latter question, based on new notions and results proved (in [3], [4], [5]) for periodic properties of functions $f : R^n \to R$ w.r.t. $L \subset R^n$.

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BLUPs and BLIMBIPs in the general Gauss–Markov model

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Abstract

In this talk, we discuss two powerful prediction concepts in the general (possibly singular) Gauss–Markov model. We study their basic properties and connections. Most observations are obtained by employing rather elementary, yet powerful, matrix algebra. The relationships to the estimation concepts BLUE and BLIMBE, being discussed in detail in Schönfeld and Werner (1986), are also mentioned.

Keywords

BLUEs, BLIMBEs, BLUPs, BLIMBIPs, η-inverses, Gauss-Markov model, Singular model.

References

Schönfeld, P. and H. J. Werner (1986). Beiträge zur Theorie und Anwendung linearer Modelle. In: W. Krelle (Ed.), *Ökonomische Prognose-, Entscheidungs- und Gleichgewichtsmodelle* (pp. 251–262). Weinheim: VCH Verlagsgesellschaft.

Some properties of sample characteristics from nonnegative data

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Abstract

By sample we mean a collection of data. There are two objects in our view. At first we tend to reach the classical characteristics of arbitrary sample in a formal way. Next we focus on a sample of nonnegative data and derive some properties of the characteristics in this case.

Some properties of equiradial and equimodular sets

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Abstract

Our goal is to describe some properties of matrices belonging either to the set of matrices equiradial with a matrix A, or to the set of matrices equimodular with A. The first set is specified by the standard Gerschgorin-type information about A, i.e. by the vector

$$I^r_{tG}(A) = (|a_{11}|, R_1(A), |a_{22}|, R_2(A), \ldots, |a_{nn}|, R_n(A)),$$

where $R_i(A)$ is the sum of moduli of the off-diagonal entries in the i'th row, and the second set is distinguished by the spreaded Gerschgorin-type information about A, i.e. by the vector

$$I^m_{tG}(A) = (|a_{11}|, |a_{12}|, \ldots, |a_{1n}|, |a_{21}|, |a_{22}|, \ldots, |a_{2n}|, \ldots, |a_{n1}|, |a_{n2}|, \ldots, |a_{nn}|).$$

References

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On the numerical range of powers of matrices

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Abstract

The numerical range of a matrix $A \in \mathbb{C}^{n\times n}$ is the set defined by

$$W(A) = \{\langle Ax, x\rangle : x \in \mathbb{C}^n, \|x\| = 1\}$$

where $\langle x, y\rangle = y^*x$ is the inner product of $x, y \in \mathbb{C}^n$ and $\|x\| = \sqrt{\langle x, x\rangle}$ is the Euclidean norm. One of the basic properties of $W(A)$ is that it contains all eigenvalues of A.

We consider the following problem. Suppose the numerical range $W(A^k)$ of all powers $k = 1, 2, \ldots$ of a given matrix $A \in \mathbb{C}^{n\times n}$ is contained in some strip that is not parallel to the real axis. We show that in this case A is power bounded. The extension of this result to linear operators on a Hilbert space is also given.

Keywords

Numerical range, Operator norm, Power bounded operators.

References

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Non-negative determinant of a rectangular matrix: Its definition and applications to multivariate data analysis

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Abstract

It is well known that the determinant of a matrix can only be defined for a square matrix. In this paper, we propose a new definition of the determinant of a rectangular matrix, and examine its properties. We first apply the properties to squared canonical correlation coefficients and also squared partial canonical correlation coefficient and so on. Furthermore, the proposed definition of the determinant of a rectangular matrix allows us to decompose the likelihood ratio quite easily when the given set of variables X and Y are partitioned into $X = (X_1, X_2, ..., X_p)$ and $Y = (Y_1, Y_2, ..., Y_q)$. The last section discusses an intuitive method which allows determining necessary and sufficient conditions of redundancy of sets of variables measured in terms of the likelihood ratio of a partitioned matrix.

Keywords

Determinant, Rectangular matrix, Canonical correlation, Partial canonical correlation, Likelihood ratio, Redundancy of variables.

Family of Gander's methods and approximation of matrices

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Abstract

The polar decomposition of a complex matrix $A \in \mathcal{C}^{m \times n}$ is defined as follows (see for example Ben-Israel and Greville (2003), p. 220)

$$A = EH,$$

where H is Hermitian nonnegative definite matrix of order n, $E \in \mathcal{C}^{m \times n}$ is a subunitary matrix (partial isometry). Here we assume that $m \geq n$.

The polar factors E and H have interesting applications (see for example Higham (1990)). We consider the following approximation problems for matrices:

P1: minimal rank approximation,
P2: approximation by subunitary matrices.

There are known several iterative methods for computing the unitary polar factor E of full rank matrix A (see for example Gander (1995) and the review paper Zieliński and Ziętak (1995)). In the talk we show how Gander's methods can be adapted for solving the approximation problems P1 and P2.

Keywords

Polar decomposition, Approximation by subunitary matrices, Minimal rank approximation, Numerical algorithms.

References

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Part VI

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