Tilastollinen tietojenkäsittely
Exercise 2
31.10.2006

1. Define $\boldsymbol{x} = (1,2,2)'$, $\boldsymbol{y} = (1,1,2)'$, $\boldsymbol{J} = \mathbf{1}_3(\mathbf{1}_3'\mathbf{1}_3)^{-1}\mathbf{1}_3'$, $\boldsymbol{X} = (\mathbf{1}_3, \boldsymbol{x})$ and $\boldsymbol{A} = (\boldsymbol{x}, \boldsymbol{y})$. Compute a) $\boldsymbol{Jx}$, b) $\boldsymbol{JA}$, c) $\hat{\boldsymbol{\beta}} = (\boldsymbol{X}'\boldsymbol{X})^{-1}\boldsymbol{X}'\boldsymbol{y}$, d) $\boldsymbol{R} = (diag\ \boldsymbol{T})^{-1/2}\boldsymbol{T}(diag\ \boldsymbol{T})^{-1/2}$, where $\boldsymbol{T} = \boldsymbol{A}'(\boldsymbol{I} - \boldsymbol{J})\boldsymbol{A}$.

2. Let
$$\boldsymbol{Y} = \begin{pmatrix} 7 & 26 & 78.5 \\ 1 & 29 & 74.3 \\ 11 & 56 & 104.3 \\ 11 & 31 & 87.6 \\ 7 & 52 & 95.9 \\ 11 & 55 & 109.2 \\ 3 & 71 & 102.7 \\ 1 & 31 & 72.5 \\ 2 & 54 & 93.1 \\ 21 & 47 & 115.9 \\ 1 & 40 & 83.8 \\ 11 & 66 & 113.3 \\ 10 & 68 & 109.4 \end{pmatrix}.$$
Calculate (using matrix operations) $\bar{\boldsymbol{x}} = n^{-1}\mathbf{1}'Y$, $\boldsymbol{S} = (n+1)^{-1}\boldsymbol{T}$ and $\boldsymbol{R}$.

3. Fit a third degree polynomial to the following data
$$\boldsymbol{y} = (1,3,3,7,5)' \text{ and } \boldsymbol{x} = (2,3,7,9,8)'.$$
Calculate also $\hat{y}$ and $e = y - \hat{y}$.

4. Solve
$$\begin{cases} x_1 + x_2 - x_3 = & 1 \\ -x_1 + x_2 + x_3 = & \text{-1} \\ x_1 - x_2 + x_3 = & 1. \end{cases}$$

5. Compute (avoid looping)
$$\text{a) } \sum_{i=1}^{10}\sum_{j=1}^{10}(\sin i\pi/6)(\cos j\pi/6), \text{ b) } \sum_{i=1}^{10}\sum_{j=1}^{10}\frac{i}{j+1}.$$

6. What is the number of ways 2 euros can be calculated as a sum of 1 euros, 50 , 20, 10 and 5 cents coins?