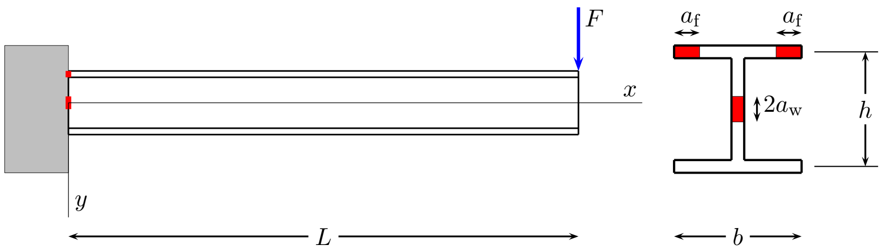
RAK-33060 Fracture mechanics and fatigue

2. Exercise

1. A tip loaded cantilever I-beam has cracks at the clamped end. The cross-section can be considered as an ideal I-profile. In the web there is a crack of length $2a_\mathrm{w} = 2t$ positioned symmetrically about the neutral axis. At the flange tips there are two symmetrically positioned cracks. How long (a_f =?) these flange tip cracks should be in order to be more dangerous than the crack in the web? Thickness of the web is t and the flanges $3t/2$, respectively. The other dimensions are related as $L/h = 10$, $h/t = 50$ and $b = h/2$. The fracture toughness in the mode II is $K_\mathrm{IIc} = (\sqrt{3}/2)K_\mathrm{Ic}$, where K_Ic is the mode I fracture toughness.

   You can assume that the shear stresses are distributed uniformly in the web. As the cross-section is assumed to be an ideal I-section, the moment of inertia for the web can be neglected. The bending stresses can also be assumed to have a constant value in the flanges.

   Tables of stress intensity factors are at the end of this paper.



2. Determine the stress intensity factor K_I for a penny-shaped crack of radius a in an infinite domain under uniaxial stress σ. Use the Griffith energy approach assuming that stresses are relaxed in a ball of radius a around the crack. Compare to the values you have found in the literature.

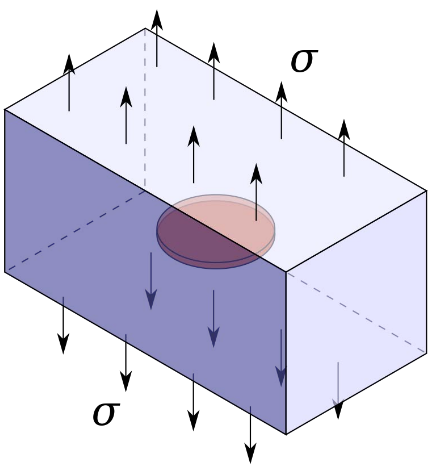


Figure: Bbanerje - Own work, CC BY-SA 3.0,
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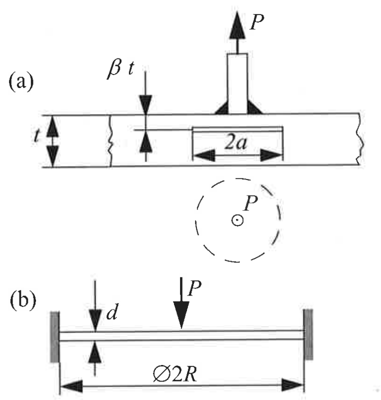
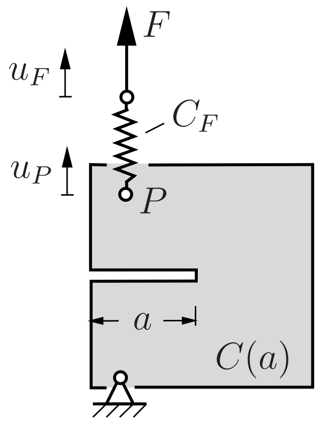
3. A thick plate containg a circular delamination crack is loaded by a point force according to figure (a). Determine the stress intensity factor K_{I} and decide the critical load for fracture if $K_{\mathrm{Ic}} = 200$ MPam$^{1/2}$, $\beta = 0.1, t = 10\,\mathrm{cm}, a = 20\,\mathrm{cm}$.

   Hint. For a circular rigidly fixed plate according to figure (b) the displacement of the loading point due to a force is

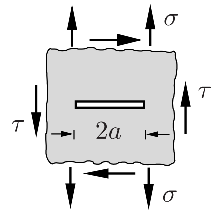
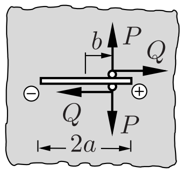
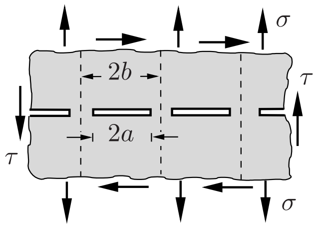
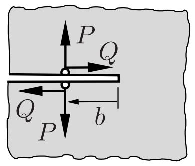
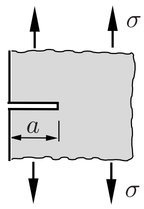
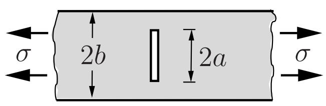
$$\Delta = \frac{3(1-\nu^2)}{4\pi}\frac{PR^2}{Ed^3}.$$



4. Consider a DCB-test (Double Cantilever Beam) in a flexible testing apparatus. Draw the dimensionless crack driving force $\mathcal{G}/\mathcal{G}_{\mathrm{c}}$ as a function of the dimensionless crack length a/a_0 using different values of the flexibility ratio $C_F/C(a_0)$. How flexible has the testing machine to be to produce unstable crack growth for a materal which has a shallow (nearly constant) R-curve?



Gross, Seelig: Fracture Mechanics, figure 4.44.

1		$\begin{Bmatrix} K_I \\ K_{II} \end{Bmatrix} = \begin{Bmatrix} \sigma \\ \tau \end{Bmatrix} \sqrt{\pi a}$
2		$\begin{Bmatrix} K_I^{\pm} \\ K_{II}^{\pm} \end{Bmatrix} = \begin{Bmatrix} P \\ Q \end{Bmatrix} \frac{1}{\sqrt{\pi a}} \sqrt{\frac{a \pm b}{a \mp b}}$
3		$\begin{Bmatrix} K_I \\ K_{II} \end{Bmatrix} = \begin{Bmatrix} \sigma \\ \tau \end{Bmatrix} \sqrt{2b \tan \frac{\pi a}{2b}}$
4		$\begin{Bmatrix} K_I \\ K_{II} \end{Bmatrix} = \begin{Bmatrix} P \\ Q \end{Bmatrix} \frac{2}{\sqrt{2\pi b}}$
5		$K_I = 1.1215\, \sigma \sqrt{\pi a}$
6		$K_I = \sigma \sqrt{\pi a}\, F_I(a/b)$ $F_I = \frac{1 - 0.025(a/b)^2 + 0.06(a/b)^4}{\sqrt{\cos(\pi a/2b)}}$