# RAK-33060 Fracture mechanics and fatigue

## 1. Exercise

1. Consider the theoretical strength of a crystalline solid where the atoms are arranged into a regular cubic lattice with distance of $d_0$ between the neighbouring atom layers. The bonding force between two atoms can be obtained as $F = -\mathrm{d}\Psi/\mathrm{d}r$ where the following Lennart-Jones potential is adopted

$$\Psi = -A\left(\frac{d_0}{r}\right)^6 + B\left(\frac{d_0}{r}\right)^{12}.$$

The first terms represents attractive forces, while the second term describes repulsive ones. Determine the expression for the cohesive strength $\sigma_\mathrm{c}$. The stress could be defined as

$$\sigma = -\frac{F}{d_0^2}.$$

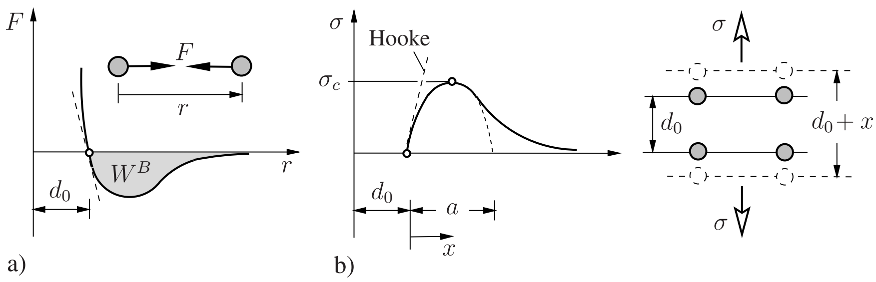
Strain $\varepsilon$ can be defined naturally as

$$\varepsilon = \frac{x}{d_0} = \frac{r - d_0}{d_0},$$

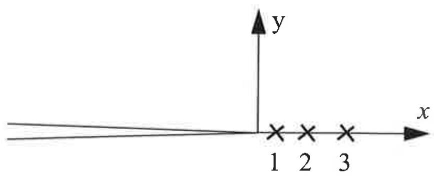
where $x$ is the distance from the equilibrium position. Determine also the expression for the surface energy $\gamma_0$

$$2\gamma_0 = \int_0^\infty \sigma \,\mathrm{d}x.$$

What are the values obtained for $\sigma_\mathrm{c}$ and $\gamma_0$ if the Young's modulus has the value $E = 210$ GPa and the distance between the atomic layers is $d_0 = 2.5 \cdot 10^{-10}$ m.




2. A two dimensional finite element analysis has been performed for a plate with a crack. The material is linearly isotropic elastic. In front of one of the crack tips the results shown in the table were obtained. Estimate the stress-intensity factors for this crack tip.




| point | $x$ [mm] | $y$ [mm] | $\sigma_x$ [MPa] | $\sigma_y$ [MPa] | $\tau_{xy}$ [MPa] |
|---|---|---|---|---|---|
| 1 | 0.10 | 0.0 | 1714.1 | 1712.1 | 1076.3 |
| 2 | 0.35 | 0.0 | 916.9 | 917.2 | 574.8 |
| 3 | 0.70 | 0.0 | 647.3 | 649.4 | 408.5 |

3. In which direction $\theta$ is the largest shear stress found ar the tip of a crack loaded in mode III? The material is linear, isotropically elastic. Stress field in the vicinity of the crack tip is

$$\tau_{zx} = -\frac{K_{\mathrm{III}}}{\sqrt{2\pi r}} \sin\tfrac{1}{2}\theta, \quad \tau_{zy} = -\frac{K_{\mathrm{III}}}{\sqrt{2\pi r}} \cos\tfrac{1}{2}\theta,$$

   and the other stress components vanish.

4. Calculate and sketch the distribution of the Tresca effective stress around a crack tip loaded in mode I. The material is isotropic and linearly elastic with $\nu = 0.3$. The stress components in the Cartesian coordinate system are

$$\sigma_x = \frac{K_{\mathrm{I}}}{\sqrt{2\pi r}} \left[\cos\tfrac{1}{2}\theta \left(1 - \sin\tfrac{1}{2}\theta \sin\tfrac{3}{2}\theta\right)\right],$$
$$\sigma_y = \frac{K_{\mathrm{I}}}{\sqrt{2\pi r}} \left[\cos\tfrac{1}{2}\theta \left(1 + \sin\tfrac{1}{2}\theta \sin\tfrac{3}{2}\theta\right)\right],$$
$$\tau_{xy} = \frac{K_{\mathrm{I}}}{\sqrt{2\pi r}} \left(\cos\tfrac{1}{2}\theta \sin\tfrac{1}{2}\theta \cos\tfrac{3}{2}\theta\right), \quad \tau_{zy} = \tau_{zx} = 0.$$

   Consider both plane stress $\sigma_z = 0$ and plane strain $\sigma_z = \nu(\sigma_x + \sigma_y)$.

   The Tresca effective stress is

$$\sigma_{\mathrm{e}} = \max(|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|),$$

   where $\sigma_1, \sigma_2$ and $\sigma_3$ are the principal stresses.