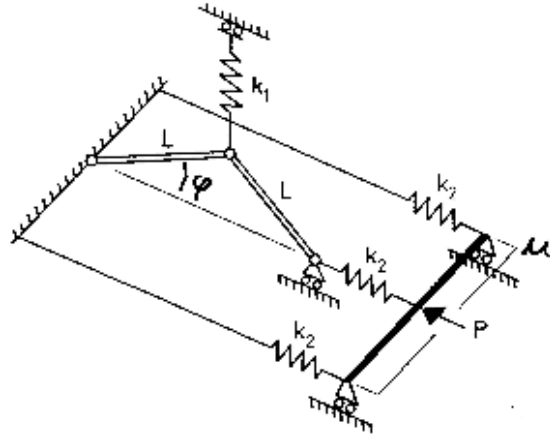


Stability of structures

3. exercise – imperfection sensitivity

Problem 1. Investigate the effect of imperfections for the structure in Exercise 2, Problem 3, consisting of rigid bars and elastic springs and shown below. Investigate also the stability of the equilibrium paths. Investigate the case $k_1 = 5k_2$ and draw the imperfection sensitivity diagram.



Solution. Let us determine the critical load (the limit load of the imperfect structure) as a function of the imperfection amplitude φ_0 . Now the strain energy is

$$U = \frac{1}{2}k_1L^2(\sin \varphi - \sin \varphi_0)^2 + k_2u^2 + \frac{1}{2}[u - 2L(\cos \varphi_0 - \cos \varphi)]^2$$

and the total potential energy has the expression

$$\begin{aligned} \frac{\partial \Pi}{\partial \varphi} &= k_1L^2(\sin \varphi - \sin \varphi_0) \cos \varphi + k_2[u - 2L(\cos \varphi_0 - \cos \varphi)](-2L \sin \varphi) = 0 \\ \frac{\partial \Pi}{\partial u} &= 2k_2u + k_2[u - 2L(\cos \varphi_0 - \cos \varphi)] - P = 0 \end{aligned}$$

Solving u from the equation above and substitute it into the equation below, gives

$$\begin{aligned} u &= \frac{k_1}{2k_2} \frac{\sin \varphi - \sin \varphi_0}{\tan \varphi} L - 2L(\cos \varphi - \cos \varphi_0) \\ P &= 3k_2u - 2k_2L((\cos \varphi - \cos \varphi_0)) = \frac{3k_1}{2} \frac{\sin \varphi - \sin \varphi_0}{\tan \varphi} L - 8k_2L((\cos \varphi - \cos \varphi_0)) \end{aligned}$$

Programming the equations into matlab the figure (1) can be obtained.

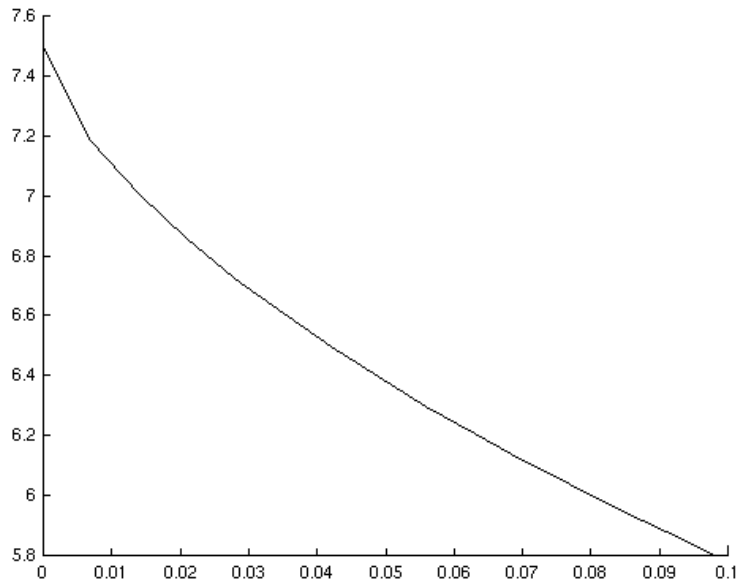


Figure 1: The maximum load $\lambda_{max} = P_{max}/kL$ as a function of the imperfection amplitude φ_0

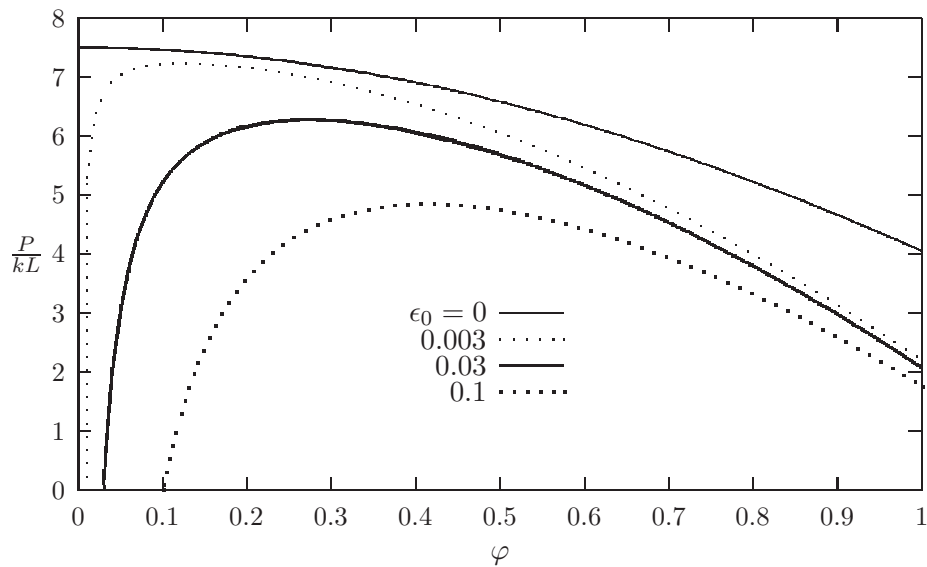


Figure 2: Equilibrium paths with different imperfection amplitude.