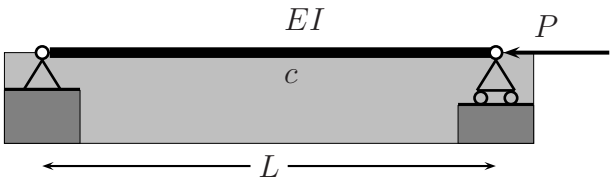
Stability of structures

Home exercises 7 and 8

Home exercise 7. Consider a beam on an elastic foundation. Assume that the foundation can be modelled as the Winkler foundation and thus the the eigenvalue problem can be written as

$$EI\frac{\mathrm{d}^4 v}{\mathrm{d}x^4} + P\frac{\mathrm{d}^2 v}{\mathrm{d}x^2} + kv = 0.$$

The foundation coefficient is $k = cb = \beta\pi^2 EI/L^4$, where β is a dimensionless constant and b is the width of the beam. Express the compressive force P as $P = \lambda\pi^2 EI/L^2$.



Make a small program using the finite difference method (or the finite element method) to compute the critical load of the beam.

1. Investigate the effect of the foundation stiffness c on the buckling load and mode when the foundation coefficient c varies in the range from soft soil 5 kN/mm^2 to hard rock 1 MN/mm^2.

2. Investigate the effect of mesh size h.

3. Based on the knowledge you have, how do you think the post-buckling behaviour and imperfection sensitivity changes with varying foundation stiffness.

You can use the values of C30 concrete for the beam and $L = 6$ m, and $b = 0.3$ m and the height of the beam $H = 2b$.

Home exercise 8. Consider a simply supprted elastic plate on an elastic foundation. Now the eigenvalue problem has the form

$$D\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) + N_x \frac{\partial^2 w}{\partial x^2} + cw = 0.$$

Express the foundation coefficient as $c = \beta\pi^2 D/b^4$ and the compressive force (per length) $N_x = \lambda\pi^2 D/b^2$.

Make a small program using the finite difference method to compute the critical load of the plate.

1. Investigate the effect of the foundation stiffness c on the buckling load and mode when the foundation coefficient c varies in the range from soft soil 5 kN/mm^2 to hard rock 1 MN/mm^2.

2. Investigate the effect of mesh size h.

3. Based on the knowledge you have, how do you think the post-buckling behaviour and imperfection sensitivity changes with varying foundation stiffness.

You can use the values of C30 concrete for the plate and $b = 6$ m, and the thickness of the plate $t = 0.3$ m.