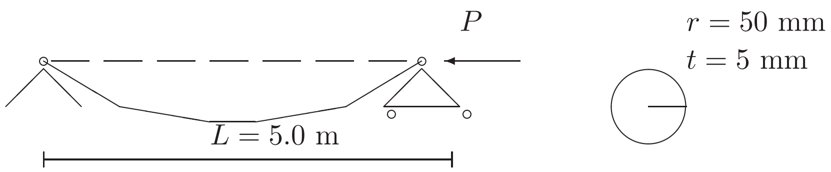
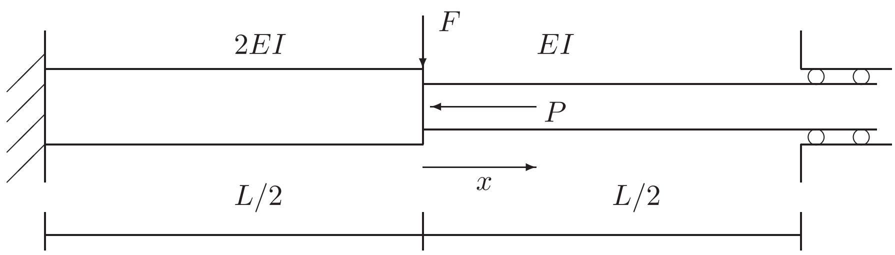
# Stability of structures

## 5. exercise – beam-columns, inelastic buckling

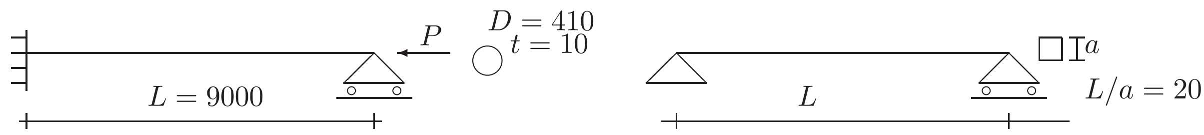
1. A beam with circular cross-section has an initial deflection $v_0(x) = v_0 \sin(\pi x/L)$. What is the safety factor with respect to the yield limit if the compressive load has the value $P = 50$ kN? The yield stress is $\sigma_{\mathrm{y}} = 220$ MPa and the Young's modulus $E = 210$ GPa. The amplitude of the initial deflection is $v_0 = L/1000$.




2. Determine the bending moment distribution at the load levels $P/P_E = 0.25$, $0.50$ and $0.75$, where $P_E$ is the critical load of the buckling problem. Determine also the expressions of the support moments at both ends and the bending moment in the midspan as a function of the compressive force.




3. The buckling length of a uniform straight column is $L_n$. The stress-strain curve of the material is quadratic ($\sigma = A\varepsilon^2 + B\varepsilon + C$), which has an apex at $\sigma_0 = 392$ MPa, $\varepsilon_0 = 0.002$. Determine the expression for the tangent modulus $E_{\mathrm{t}}(\sigma)$ and show that the critical load according to the tangent modulus theory is $P_{\mathrm{cr}} = 2\sigma_0 A(\sqrt{K+1})/K$, where $K = (\varepsilon_0 L_n^2 A/\pi^2 I)^2$.
   Calculate the value of the critical load for the two columns shown below. Measures shown in mm.




4. Determine the dependence of the critical stress $\sigma_{\mathrm{cr}}$ on the slenderness $\lambda = L_n/i$ (where $L_n$ is the buckling length and $i = \sqrt{I/A}$ is the radius of gyration of the cross-section) for a uniform centrally compressed straight column. The tangent modulus $E_{\mathrm{t}}$ has the form

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\epsilon} = E_{\mathrm{t}} = E\frac{\sigma_{\mathrm{y}} - \sigma}{\sigma_{\mathrm{y}} - c\sigma},$$

   where $\sigma_{\mathrm{y}}$ is the yield stress and $c$ is an additional material constant. Draw the figure showing the critical buckling stress as a function of the slenderness in a $(\sigma_{\mathrm{cr}}/\sigma_{\mathrm{y}}) - \lambda$ -coordinate system with $(\sigma_{\mathrm{cr}}/\sigma_{\mathrm{y}}) \in [0,1], \lambda \in [0,200]$ Use the value $c = 0,9$ and ratios $E/\sigma_{\mathrm{y}} = 500$ (steel) and $E/\sigma_{\mathrm{y}} = 200$ (aluminium, pinewood). Draw also in the same figure the elastic buckling stress.