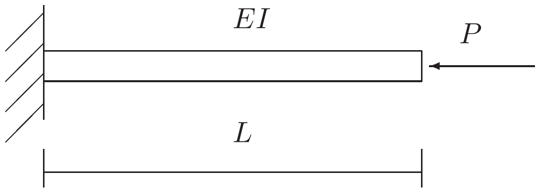
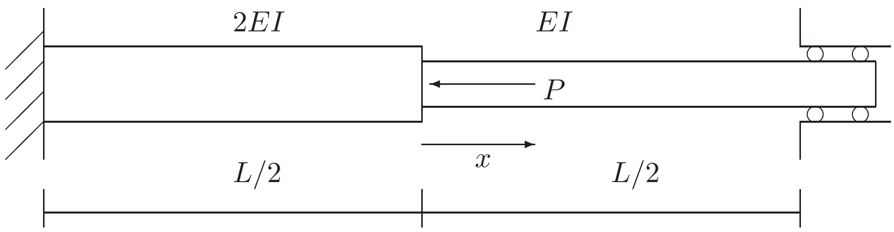
Stability of structures

4. exercise – continuous systems, column buckling

1. Derive the Euler equations of the cantilever beam shown below. Assume inextensible beam and small deflections. Solve the equations and determine the eigenmodes and show that the eigenmodes are orthogonal.



2. For the structure shown below, determine P_{cr} starting from the differential equation.



Home exercises 3 and 4

Home exercise 3. Determine the asymptotic post-buckling behaviour of the column in exercise 1.
Hint. Start from the exact expression of the curvature (in Lagrangian formulation)

$$\kappa = \frac{v''}{\sqrt{1-(v')^2}}$$

resulting in the potential energy expression

$$\Pi(v) = \frac{1}{2}\int_0^L EI\kappa^2 \, \mathrm{d}x - P\int_0^L \left(1-\sqrt{1-(v')^2}\right) \mathrm{d}x.$$

Use series eqpansion up to fourth order and the displacement field in the form $v(x) = av_1(x)$ where a is the unknown amplitude and $v_1(x)$ is the buckling mode corresponding to the lowest buckling load.

Home exercise 4. For the structure shown below determine P_{cr} starting from the differential equation. You can use the following values for the non-dimensional coefficients: $\alpha = 2,\ \beta = 1$. Compare the results to exercise 2, what are your conclusions?