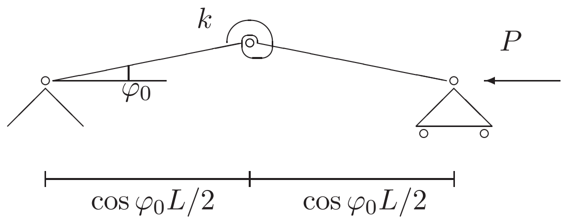
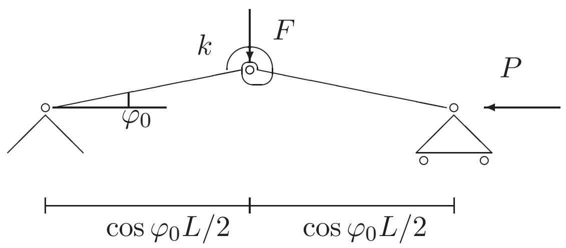
RAK-33030 Stability of structures

2. exercise – equilibrium paths of discrete structural models

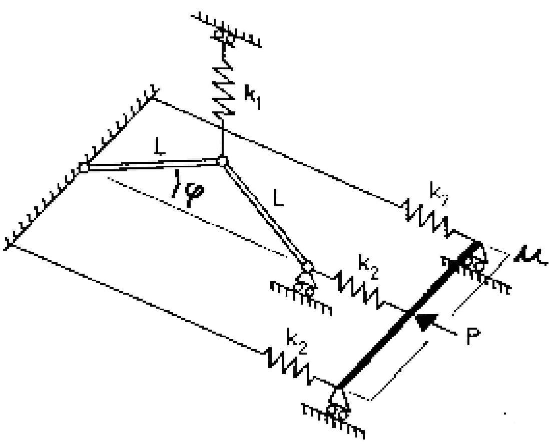
1. Determine all equilibrium paths of the structure consisting of two rigid bars and a linear elastic rotational spring. The structure has a geometrical imperfection ϕ_0 in its unloaded state. Investigate also the stability of all paths. Are there critical points on the paths?



2. Determine all equilibrium paths starting from the unloaded state of the structure consisting of two rigid bars (length $L/2$) and a linear elastic rotational spring. The structure has a geometrical imperfection ϕ_0 in its unloaded state ($P = F = 0$). Investigate also the stability of all paths. The perturbation load $F = \epsilon 4k/L$, where ϵ is a dimensionless (second) perturbation parameter.



3. Determine the equilibrium paths of the simple structure shown, consisting of rigid bars and elastic springs. Investigate also the stability of the equilibrium paths. Investigate especially cases $k_1 = k_2$ and $k_1 = 5k_2$. What kind of real structures these models imitate?

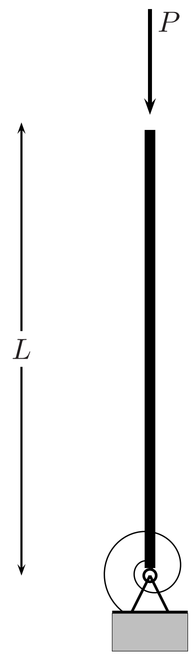


Home exercises 1 and 2

Home exercise 1. Investigate the behaviour of the column model shown beside. In this idealisation, the column is assumed to be rigid and the material behaviour is taken into account by the non-linear rotational spring having the following elastic constitutive equation

$$M = c_1 \tanh(c_0 \varphi / c_1), \tag{1}$$

which tries to imitate elasto-plastic behaviour. Determine the coefficients c_0 and c_1 such that the ultimate bending moment is the fully plastic moment M_p and the initial stiffness results in the classical Euler buckling load of a column $P_\mathrm{E} = \frac{1}{4}\pi^2 EI/L^2$. Assume that the cross-section is a square with height h. Investigate the post-buckling behaviour as a function of the slenderness ratio $\lambda = L/i > \lambda_0$, where $i = \sqrt{I/A}$ is the radius of gyration and λ_0 is the slenderness ratio at which the buckling load equals to the compressive yield load $P_\mathrm{p} = \sigma_\mathrm{y} A$. In your solution use the non-dimensional load parameter μ, such that $P = \mu \frac{1}{4}\pi^2 EI/L^2$. We use now μ for the load parameter since λ is a common symbol for the slenderness ratio.



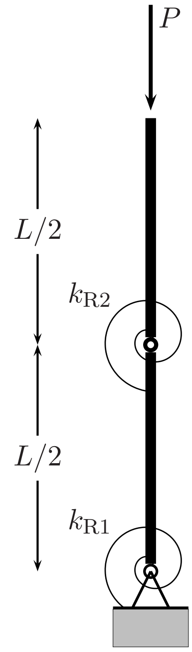
Analyse also the imperfect behaviour where the vertical load is not concentric but displaced by an amount ϵL from the center line of the column, however, remaining strictly vertical. Is there a limit point in the equilibrium path of the imperfect structure? If so, plot the limit load as a function of the imperfection parameter $\epsilon \in (0, 1/100)$.

Use the Young's modulus vs. yield stress σ_y ratio from the following table, where FL is the first letter of your family name.

FL	E/σ_y
A-F	700
H-I	600
J-K	500
L-M	450
N-P	400
Q-S	350
T-U	300
V-Ö	250

What are your conclusions of this problem?

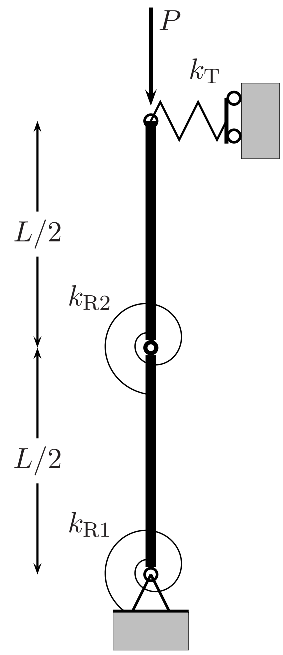
Home exercise 2a. Investigate the behaviour of a two degree of freedom column model shown beside. In this idealisation, the column is assumed to be composed of two rigid bars and the material behaviour is taken into account by the linear rotational springs with stiffnesses k_{R1} and k_{R2}.



1. Determine the rotational spring stiffnesses k_{R} and k_{R2} such that the critical load of the model coincides with the two lowest buckling loads of the continuous elastic column.

2. Solve, at least in an asymptotic sense all equilibrium paths and investigate their stability.

Home exercise 2b. A designer tries to increase the load carrying capacity of the column over the first buckling load by adding to the top a horizontal spring with stiffness k_{T}.



1. Determine the minimum spring stiffness k_{T} which prevents the buckling mode corresponding to the first buckling load.

2. Determine the load to which the horizontal buckling support has to be designed. Use an imperfect structure to analyse the force in the buckling support.

Solutions to the home exercises should be returned as pdf-files in the Moodle area of the course at latest 1.2.2019.