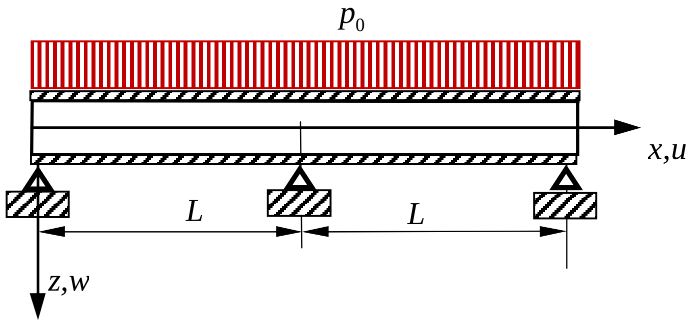


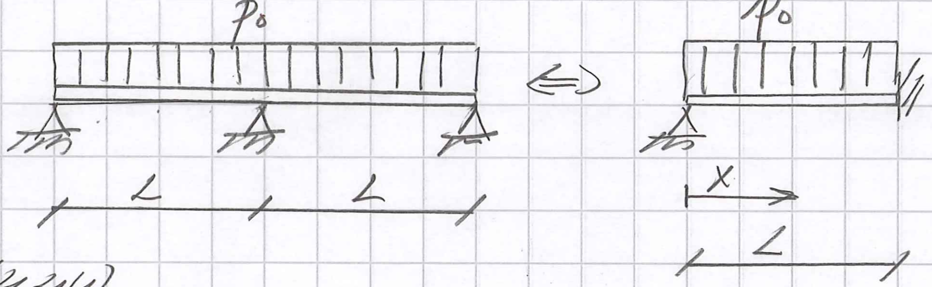
Tarkastellaan oheisen kuvan mukaista hyperstaattista kerrospalkkia, joka on päistään ja keskeltä niveltuettu ja jota kuormitetaan tasaisella viivakuormalla p_0. Kerros-palkki koostuu 'vaahtoytimestä' ja vanerikansilevyistä.

Vaahtoytimen paksuus on 130 mm ja kansivanerien paksuu on 20 mm.

Määritä rakenteen taipumaviiva, kun viivakuorma suuruus on $p_0 = 10$ N/mm. Miten laskisit yläpaarteen jännitykset?

Dataa. L = 2000 mm ja b =1200 mm (leveys).

E_{vaneri} = 10000 MPa, G_{vaneri} = 500 MPa
E_{ydin} = 0 MPa, G_{ydin} = 2 MPa



(4.44)

DY: $$w^{(6)}(x) - \alpha^2 w^{(4)}(x) = -\alpha^2 p(x)/EI + p^{(2)}(x)/EI_0$$

$$p(x) = p_0 \Rightarrow p^{(2)}(x) = 0$$

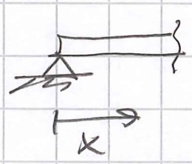
ratkaisu: $$w(x) = w_h(x) + w_p(x)$$

(4.46) $$w_h^o(x) = C_1 \sinh \alpha x + C_2 \cosh \alpha x + C_3 x^3 + C_4 x^2 + C_5 x + C_6$$

täydellisen yhtälön ratkaisu:
$$w_p^o(x) = \frac{p_0}{24EI} x^4$$

termit x^2, 1, $\cosh \alpha x$ sisältyvät w_h:ihon

Reunaehdot:



$$w(0) = 0 \quad (1)$$
$$w^{(2)}(0) = 0 \quad (2)$$
$$w^{(4)}(0) - p_0/EI_0 = 0 \quad (3)$$

(1) $w(0) = 0 \Rightarrow C_1 \sinh 0 + C_2 \cosh 0 + C_3 0^3 + C_4 \cdot 0^2 + C_5 \cdot 0 + C_6 + \frac{p_0}{24EI} 0^4 = 0$

$$\Rightarrow C_2 + C_6 = 0 \Leftrightarrow C_6 = -C_2$$

$$w^{(2)}(x) = C_1 \alpha^2 \sinh \alpha x + C_2 \alpha^2 \cosh \alpha x + 6 C_3 x + 2 C_4 + \frac{p}{6EI} x^2$$

(2) $w^{(2)}(0) = 0 \Rightarrow C_2 \alpha^2 + 2 C_4 = 0 \Leftrightarrow C_4 = -\frac{1}{2} \alpha^2 C_2$

$$w^{(4)}(x) = C_1\alpha^4 \sinh\alpha x + C_2\alpha^4 \cosh\alpha x + \frac{P_0}{EI}$$

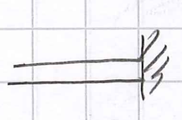
$$(3)\quad w^{(4)}(0) - P_0/EI_0 = 0 \Rightarrow \quad C_2\alpha^4 + \frac{P_0}{EI} - \frac{P_0}{EI_0} = 0$$

$$C_2 = \frac{P_0}{\alpha^4}\left(\frac{1}{EI_0} - \frac{1}{EI}\right)$$

$$(2) \Rightarrow \quad C_4 = -\frac{1}{2}\alpha^2 C_2 = -\frac{P_0}{2\alpha^2}\left(\frac{1}{EI_0} - \frac{1}{EI}\right)$$

$$(1) \Rightarrow \quad C_6 = -C_2 = \frac{P_0}{\alpha^4}\left(\frac{1}{EI} - \frac{1}{EI_0}\right)$$

Reunaehdot:



$x=L$

$$w(L) = 0 \quad (4)$$

$$w'(L) = 0 \quad (5)$$

$$w^{(5)}(L) - \alpha^2\left(1 - \frac{EI_0}{EI}\right) w^{(3)}(L) = 0 \quad (6)$$

$$(4)\quad w(L)=0 \Rightarrow \quad C_1 \sinh\alpha L + C_2 \cosh\alpha L + C_3 L^3 + C_4 L^2 +$$

$$+ C_5 L + C_6 + \frac{P_0}{24EI} L^4 = 0$$

$$w'(x) = C_1\alpha \cosh\alpha x + C_2\alpha \sinh\alpha x + 3C_3 x^2 + 2C_4 x + C_5 + \frac{P_0}{6EI}x^3$$

$$(5)\quad w'(L)=0 \Rightarrow C_1\alpha\cosh\alpha L + C_2\alpha\sinh\alpha L + 3C_3L^2 + 2C_4L + C_5 + \frac{P_0}{6EI}L^3 = 0$$

$$w^{(5)}(L) = C_1\alpha^5\cosh\alpha L + C_2\alpha^5\sinh\alpha L$$

$$w^{(3)}(L) = C_1\alpha^3\cosh\alpha L + C_2\alpha^3\sinh\alpha L + 6C_3 + \frac{PL}{EI}$$

$$(6)\quad w^{(5)}(L) - \alpha^2\left(1 - \frac{EI_0}{EI}\right)w^{(3)}(L) = 0$$

$$C_1\alpha^5\cosh\alpha L + C_2\alpha^5\sinh\alpha L - \alpha^2\left(1 - \frac{EI_0}{EI}\right)\left(C_1\alpha^3\cos\alpha L + \right.$$

$$\left. + C_2\alpha^3\sinh\alpha L + 6C_3 + \frac{PL}{EI}\right) = 0$$

C_1, C_3 ja C_5 ovat tuntemattomat kertoimet

C_2, C_4 ja C_6 tunnetaan

(4) $$\begin{bmatrix} \sinh\alpha L & L^3 & L \\ \alpha\cosh\alpha L & 3L^2 & 1 \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\begin{pmatrix} C_1 \\ C_3 \\ C_5 \end{pmatrix} = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix}$$

$$a_{31} = \alpha^5\cosh\alpha L - \alpha^5\left(1-\frac{EI_0}{EI}\right)\cosh\alpha L = \alpha^5\frac{EI_0}{EI}\cosh\alpha L$$

$$a_{32} = -6\alpha^2\left(1-\frac{EI_0}{EI}\right)$$

$$a_{33} = 0$$

$$d_1 = -C_2\cosh\alpha L - C_4L^2 - C_6 - \frac{p_0}{24EI}L^4$$

$$d_2 = -C_2\alpha\sinh\alpha L - 2C_4L - \frac{p_0}{6EI}L^3$$

$$d_3 = -C_2\alpha^5\sinh\alpha L + \alpha^2\left(1-\frac{EI_0}{EI}\right)\left(C_2\alpha^3\sinh\alpha L + \frac{pL}{EI}\right)$$

$L = 2000\,mm \qquad \alpha^2 = 26{,}11\cdot10^{-6}\,\frac{1}{mm^2} \qquad c = 150\,mm$

$EI_0 = EI_1 + EI_2 = 1{,}60\cdot10^{10}\,Nmm^2 \qquad A_1 = A_2$

$$EI = EI_0 + \frac{EA_1 EA_2}{EA_1+EA_2}c^2 = EI_0 + \frac{EA}{2}c^2 = 2{,}716\cdot10^{12}\,Nmm^2$$

$$\begin{bmatrix} 1{,}3732\cdot10^4 & 8\cdot10^9 & 2000 \\ 70{,}176 & 1{,}2\cdot10^7 & 1 \\ 2{,}8135\cdot10^{-10} & -1{,}5577\cdot10^{-4} & 0 \end{bmatrix}\begin{pmatrix} C_1 \\ C_3 \\ C_5 \end{pmatrix} = \begin{pmatrix} -1{,}2464\cdot10^4 \\ -63{,}888 \\ -2{,}5667\cdot10^{-10} \end{pmatrix}$$

$$\begin{pmatrix} C_1 \\ C_3 \\ C_5 \end{pmatrix} = \begin{pmatrix} -0{,}91063 & mm \\ -5{,}4153\cdot10^{-10} & 1/mm^2 \\ 2{,}2584\cdot10^{-2} & \end{pmatrix}$$

$C_2 = 0{,}31101\,mm$

$C_4 = -1{,}1836\cdot10^{-5}\,1/mm$

$C_6 = -0{,}31101\,mm$

Taivutusmomentti, kun $w(x)$ tunnetaan

$$M(x) = \frac{EI}{\alpha^2} w^{(4)}(x) - EI\, w^{(2)}(x) - \frac{EI}{\alpha^2 E I_0} p \qquad (4.58)$$

Paarrevoima ja -momentti

$$N_1(x) = -\frac{1}{e}\left[M(x) + E I_0\, w^{(2)}(x)\right] \qquad (4.60)$$

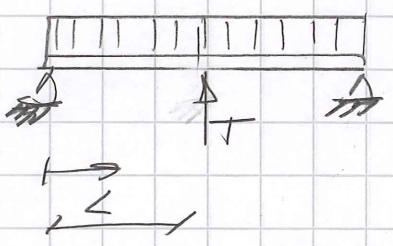
$$M_1(x) = -E I_1\, w^{(2)}(x) \qquad (4.62)$$

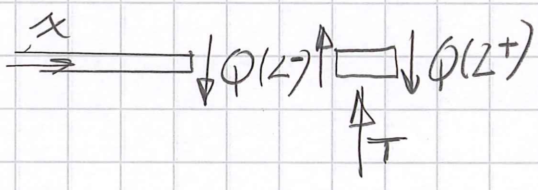
Jännitykset paarteessa

$$\sigma_x = \frac{N_1(x)}{A_1} + \frac{M_1(x)}{I_1} z^{0}$$

tukireaktio T

$$Q(L^-) = -Q(L^+)$$





$$Q(x=L^-) = \frac{dM(x=L)}{dx} = M'(x=L)$$

$$T = -2Q(L)$$

```
> restart;
> 
```

Määritellään kuudennen kertaluvun differentiaaliyhtälö (4.44), nyt viivakuormitus on vakio

```
> ode:=diff(w(x),x$6)-alpha^2*diff(w(x),x,x,x,x)=-alpha^2*p/EI;
```

$$ode := \left(\frac{d^6}{dx^6} \mathrm{w}(x)\right) - \alpha^2 \left(\frac{d^4}{dx^4} \mathrm{w}(x)\right) = -\frac{\alpha^2 p}{EI}$$

Ratkaistaan DY kuudella reunaehdolla ja muutetaan ratkaisu trigonometriseen muotoon 'trig' ja vielä yksinkertaistetaan. Saatu ratkaisu on aika pitkä

```
> vast:=simplify(convert(dsolve({ode,w(0)=0, D(D(w))(0)=0,
  D(D(D(D(w))))(0)-p/EI0, w(L)=0, D(w)(L)=0,
  D(D(D(D(D(w)))))(L)-alpha^2*(1-EI0/EI)*D(D(D(w)))(L)=0},
  w(x)),'trig'));
```

$$\begin{aligned}
vast := \mathrm{w}(x) = -\frac{1}{48} p\, (& -288\, EI0\, EI \sinh(\alpha x) - 24\, x^3\, EI0\, \alpha^4\, L\, EI \sinh(\alpha L) \\
& - 144\, \alpha\, x\, EI^2 \cosh(\alpha L) - 144\, EI0^2 \cosh(\alpha L) \sinh(\alpha x) - 144\, x^2\, \alpha^3\, L\, EI\, EI0 \cosh(\alpha L) \\
& - 24\, x^3\, EI0\, \alpha^3\, EI - 288\, EI0\, EI \sinh(\alpha L) \cosh(\alpha x) + 288\, EI0\, EI \cosh(\alpha L) \sinh(\alpha x) \\
& - 72\, EI^2\, L^2\, \alpha^2 \sinh(\alpha x) + 30\, EI0^2\, L^4\, \alpha^4 \sinh(\alpha x) - 72\, EI0^2\, L^2\, \alpha^2 \sinh(\alpha x) \\
& - 6\, x^4\, EI0^2\, \alpha^4 \sinh(\alpha L) - 24\, x^3\, EI0^2\, \alpha^3 \cosh(\alpha L) - 72\, x^2\, \alpha^2\, EI^2 \sinh(\alpha L) \\
& - 72\, x^2\, \alpha^2\, EI0^2 \sinh(\alpha L) + 288\, EI0\, EI \sinh(\alpha L) - 144\, EI^2 \cosh(\alpha L) \sinh(\alpha x) \\
& + 144\, EI^2 \sinh(\alpha L) \cosh(\alpha x) + 288\, \alpha\, L\, EI\, EI0 \cosh(\alpha L) \cosh(\alpha x) \\
& - 144\, \alpha\, L\, EI0^2 \cosh(\alpha L) \cosh(\alpha x) - 48\, \alpha^3\, EI0^2\, L^3 \sinh(\alpha L) \sinh(\alpha x) \\
& - 144\, \alpha\, L\, EI^2 \cosh(\alpha L) \cosh(\alpha x) + 144\, \alpha\, L\, EI^2 \sinh(\alpha L) \sinh(\alpha x) \\
& + 144\, \alpha\, L\, EI0^2 \sinh(\alpha L) \sinh(\alpha x) + 48\, \alpha^3\, EI0\, L^3\, EI \sinh(\alpha L) \sinh(\alpha x) \\
& + 6\, x^4\, EI0\, \alpha^4\, EI \sinh(\alpha L) - 2\, x^4\, EI0^2\, \alpha^7\, L^3 \cosh(\alpha L) - 6\, x^4\, EI0\, \alpha^5\, L\, EI \cosh(\alpha L) \\
& + 6\, x^4\, EI0^2\, \alpha^5\, L \cosh(\alpha L) + 24\, x^3\, EI0^2\, \alpha^4\, L \sinh(\alpha L) + 3\, x^3\, EI0^2\, \alpha^7\, L^4 \cosh(\alpha L) \\
& - 12\, x^3\, EI0^2\, \alpha^5\, L^2 \cosh(\alpha L) + 144\, x^2\, \alpha^2\, EI0\, EI \sinh(\alpha L) \\
& + 24\, x^2\, \alpha^5\, EI0\, L^3\, EI \cosh(\alpha L) + 72\, x^2\, \alpha^3\, L\, EI^2 \cosh(\alpha L) - 24\, x^2\, \alpha^5\, EI0^2\, L^3 \cosh(\alpha L) \\
& + 72\, x^2\, \alpha^3\, L\, EI0^2 \cosh(\alpha L) + 288\, \alpha\, x\, EI0\, EI \cosh(\alpha L) - \alpha^7\, x\, L^6\, EI0^2 \cosh(\alpha L) \\
& - 48\, \alpha^4\, x\, EI0^2\, L^3 \sinh(\alpha L) + 144\, \alpha^2\, x\, L\, EI^2 \sinh(\alpha L) + 144\, \alpha^2\, x\, L\, EI0^2 \sinh(\alpha L) \\
& + 30\, \alpha^5\, x\, EI0^2\, L^4 \cosh(\alpha L) - 288\, \alpha^2\, x\, L\, EI\, EI0 \sinh(\alpha L) + 48\, \alpha^4\, x\, EI0\, L^3\, EI \sinh(\alpha L)
\end{aligned}$$

$$+ 72 \alpha^3 x\, EI\, L^2\, EI0 \cosh(\alpha L) - 30 \alpha^5 x\, EI\, L^4\, EI0 \cosh(\alpha L)$$
$$- 48 \alpha^3\, EI0\, L^3\, EI \cosh(\alpha L) \cosh(\alpha x) - 288 \alpha L\, EI\, EI0 \sinh(\alpha L) \sinh(\alpha x)$$
$$- 48 \alpha^3\, EI0^2\, L^3 \cosh(\alpha L) + 144 \alpha L\, EI^2 \cosh(\alpha L) + 144 \alpha L\, EI0^2 \cosh(\alpha L)$$
$$- 288 \alpha L\, EI\, EI0 \cosh(\alpha L) + 48 \alpha^3\, EI0\, L^3\, EI \cosh(\alpha L) - 288 \alpha x\, EI\, EI0$$
$$- 30\, EI\, L^4 \alpha^4\, EI0 \sinh(\alpha x) + 144\, EI0^2 \sinh(\alpha x) + 144 \alpha x\, EI^2 + 144 \alpha x\, EI0^2$$
$$+ 24 x^3\, EI0^2 \alpha^3 + 72 \alpha^3 x\, EI\, L^2\, EI0 - 72 \alpha^3 x\, L^2\, EI0^2 + 12 x^3\, EI0\, \alpha^5 L^2\, EI \cosh(\alpha L)$$
$$+ 24 x^3\, EI0\, \alpha^3\, EI \cosh(\alpha L) + 144\, EI^2 \sinh(\alpha x) - 72 \alpha^3 x\, EI^2 L^2 \cosh(\alpha L)$$
$$- 144\, EI0^2 \sinh(\alpha L) - 144 \alpha x\, EI0^2 \cosh(\alpha L) + 144\, EI0^2 \sinh(\alpha L) \cosh(\alpha x)$$
$$+ 144\, EI\, L^2 \alpha^2\, EI0 \sinh(\alpha x) + 48 \alpha^3\, EI0^2 L^3 \cosh(\alpha L) \cosh(\alpha x) - 144\, EI^2 \sinh(\alpha L))$$
$$\Big/ (EI\, (-3\, EI \sinh(\alpha L) + 3\, EI0 \sinh(\alpha L) + \alpha^3\, EI0\, L^3 \cosh(\alpha L) + 3 \alpha L\, EI \cosh(\alpha L)$$
$$- 3 \alpha L\, EI0 \cosh(\alpha L))\, EI0\, \alpha^4)$$

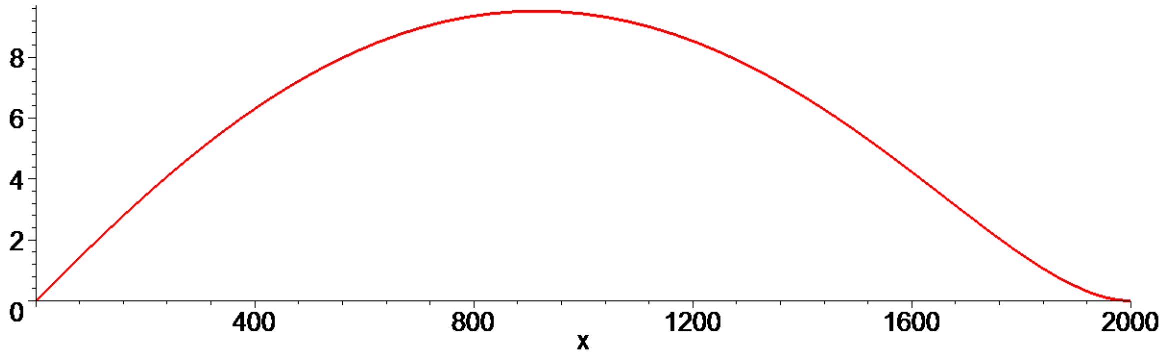
Sijoitetaan saatuun ratkaisuun arvot

```
> ? matlab
> vast2:=sort(evalf((subs({EI=2.716e12,alpha=sqrt(26.11e-6),L=2000
  ,EI0=1.60e10,p=10},vast))),x);
```

$$vast2 := \mathrm{w}(x) = 0.1534118801\ 10^{-12} x^4 - 0.5415243518\ 10^{-9} x^3 - 0.00001189808723\, x^2$$
$$+ 0.02258847651\, x - 0.9110022429 \sinh(0.005109794516\, x)$$
$$+ 0.9113816330 \cosh(0.005109794516\, x) - 0.9113816330$$

Plotataan saatu ratkaisu x=0..2000, rhs (right hand side) poimii ratkaisun oikeanpuolen

```
> plot(rhs(vast2),x=0..2000,thickness=3);
```



Haetaan välilta x e [0, 2000] taipuman maksimi

```
> maximize(rhs(vast2), x=0..2000,location);
```

$$9.517239657,\ \{[\{x = 914.4085557\},\ 9.517239657]\}$$

```
> w:=rhs(vast2);
```

$$w := 0.1534118801\ 10^{-12} x^4 - 0.5415243518\ 10^{-9} x^3 - 0.00001189808723\, x^2$$

$+ 0.02258847651\,x - 0.9110022429 \sinh(0.005109794516\,x)$

$+ 0.9113816330 \cosh(0.005109794516\,x) - 0.9113816330$

Taivutusmomentti M(x)

```
> M:=subs({EI=2.716e12,alpha=sqrt(26.11e-6),L=2000,EI0=1.60e10,p=10}, EI/alpha^2*diff(w,x$4) - EI*diff(w,x,x)-EI/alpha^2/EI0*p);
```

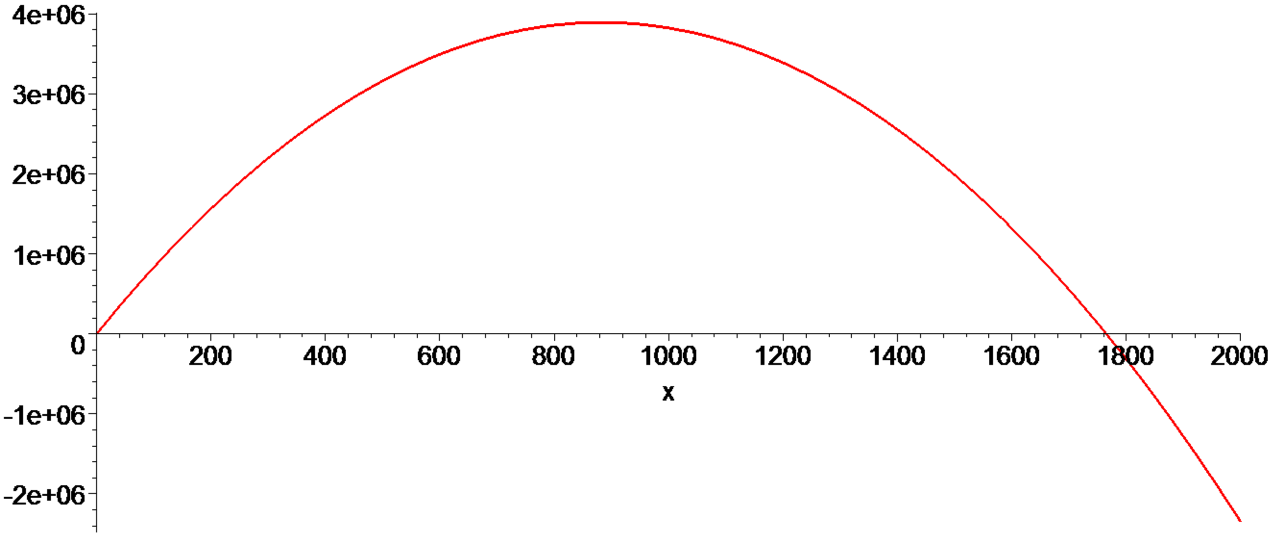
$$M := 0.04 + 0.05 \sinh(0.005109794516\,x) - 0.03 \cosh(0.005109794516\,x) - 4.999999996\,x^2 + 8824.680835\,x$$

```
> with(plots): with(plottools):
Warning, the name changecoords has been redefined

Warning, the assigned name arrow now has a global binding
```

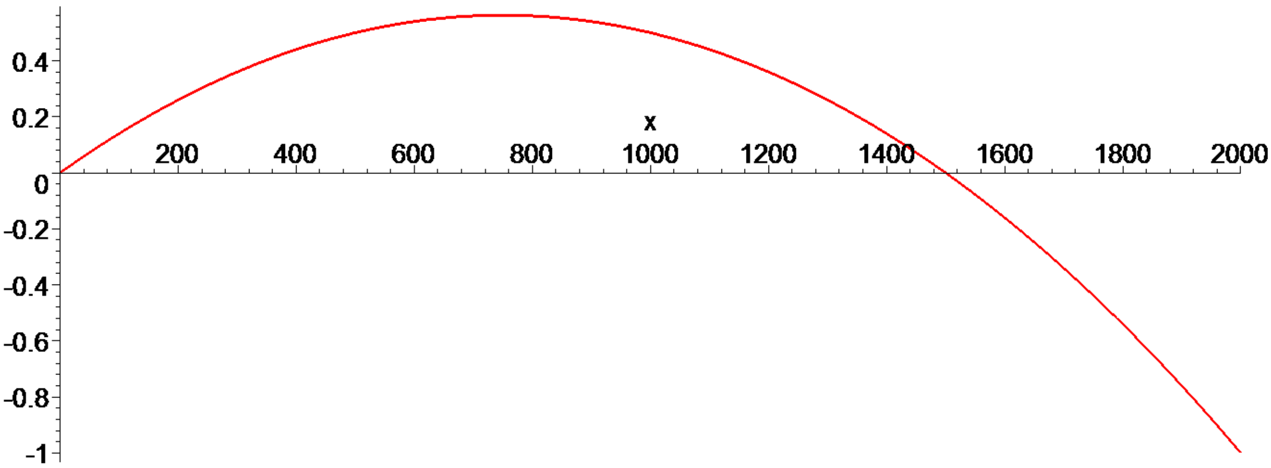
Taivutusmomenttikuvaaja M(x)

```
> plot(M,x=0..2000,thickness=3);
```



Tavallisen palkin taivutusmomenttikuvaaja

```
> plot(3*x/2000-4*(x/2000)^2,x=0..2000,thickness=3);
```



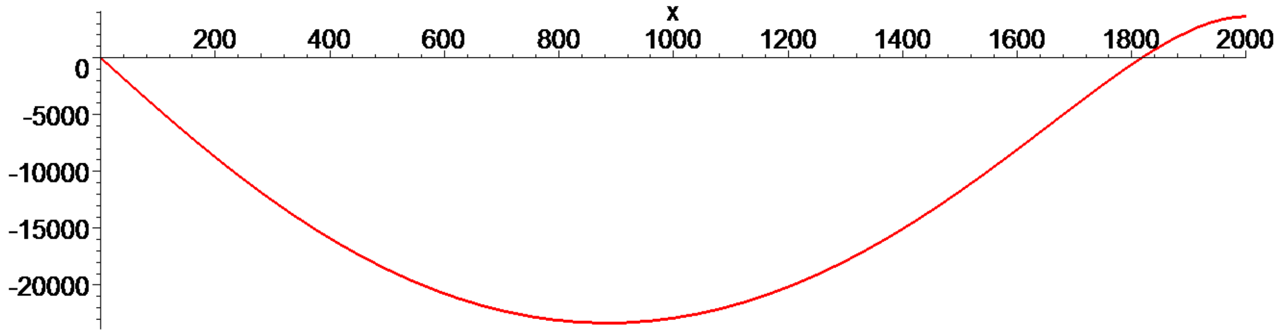
Yläpaarteen normaalivoimakuvuuja N1(x)

```
> N1:=-1/150*(M+1.60e10*diff(w,x,x));
```

$$N1 := 2538.258343 + 2537.201648 \sinh(0.005109794516\,x)$$

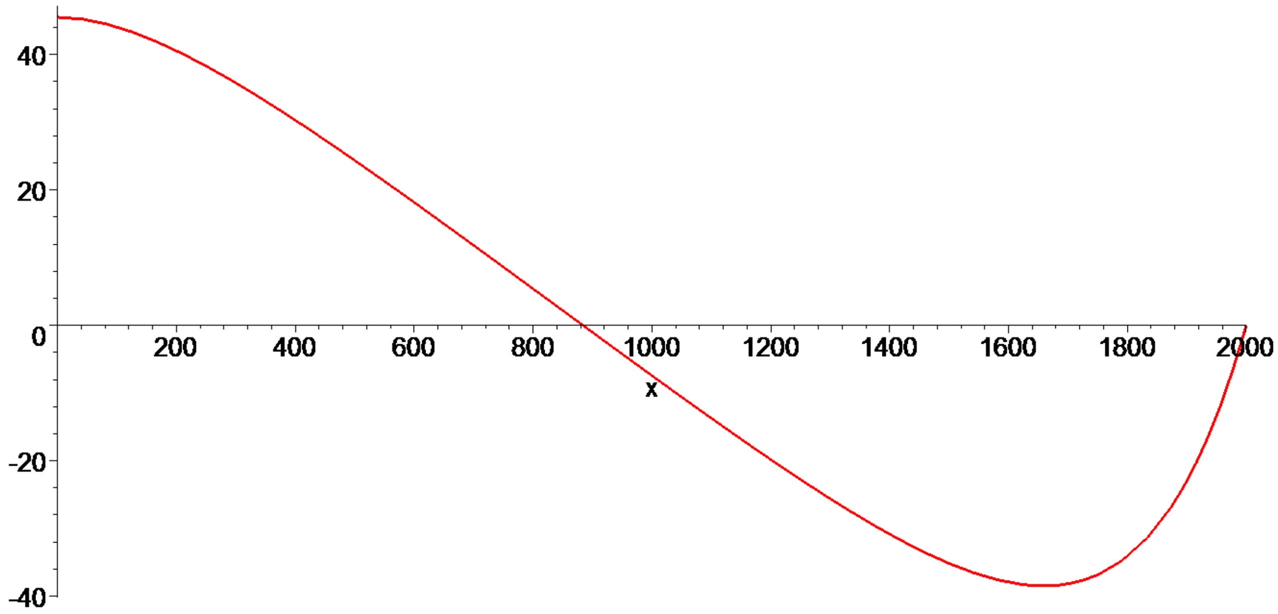
$- 2538.258407 \cosh(0.005109794516\, x) + 0.03313696610\, x^2 - 58.48462998\, x$

```
> plot(N1,x=0..2000,thickness=3);
```



Leikkausvuo q(x)

```
> plot(-diff(N1,x),x=0..2000,thickness=3);
```




```
>
```