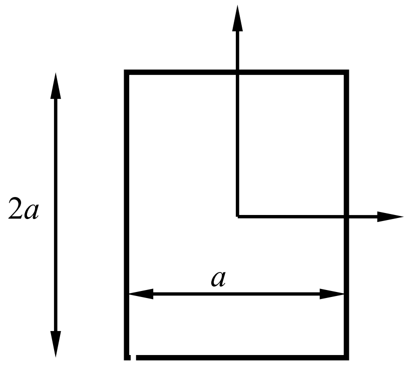
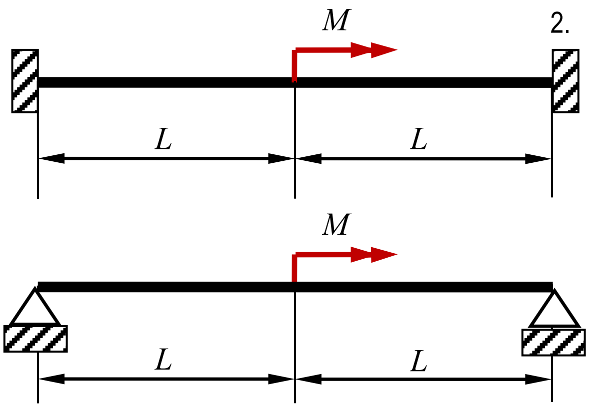


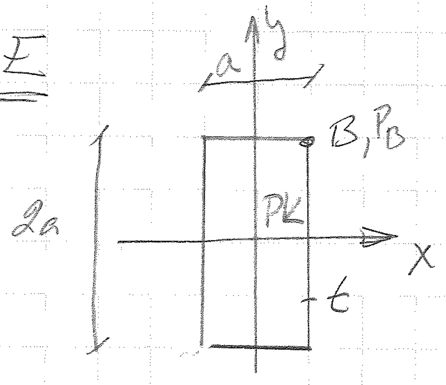
1. Määritä oheisen profiilin sektorineliömomentti (käyristymisjäyhyys) I_ω sekä sektoriaalinen staattinen momentti S_ω. Profiili on auki alanurkastaan. Profiilin aineenvahvuus on t.

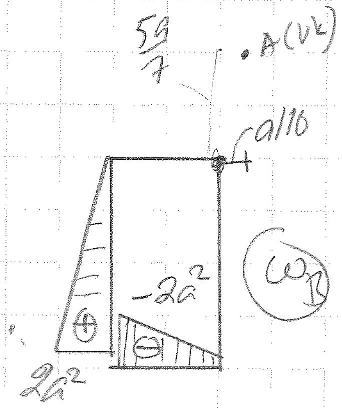


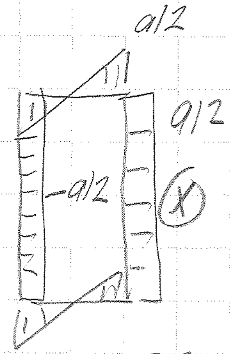
2. Määritä edellisen tehtävän profiilin päätyjen jäykän tuennan normaalijännitysjakauma $\sigma_z(s)$. Määritä vielä normaalijännitys $\sigma_z(s)$ keskiössä, kun jäykkä tuenta muuttuu haarukkatuennaksi.

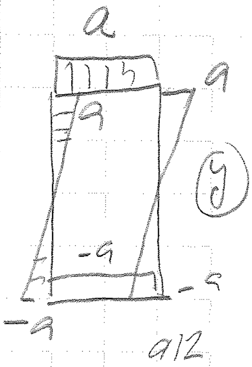
Kimmokerroin $E = 205\,\text{GPa}$, $\nu = 0{,}3$.

E

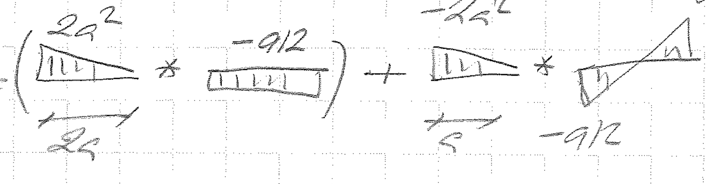








$I_{y\omega_B} = \int_A x\omega_B \, dA =$

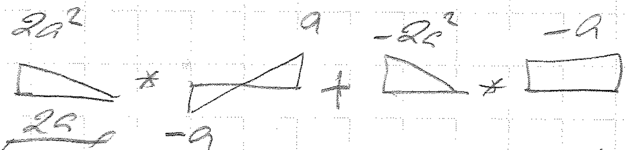


$dA = t\,ds$

$= \left(\frac{1}{2} 2a \cdot 2a^2 \left(-\frac{a}{2}\right) + \frac{1}{6} a (-2a^2)\left(-2\frac{a}{2} + \frac{a}{2}\right)\right) t$

$= -a^4 t + \frac{1}{6} a^4 t = -\frac{5}{6} a^4 t$

$I_{x\omega_B} = \int y \omega_B \, t\,ds =$



$= \frac{1}{6} 2a\, 2a^2 (-2a + a) + \frac{1}{2} a (-2a^2)(-a) = +\frac{1}{3} a^4 t$

$I_x = 2at\,a^2 + 2\frac{1}{12}(2a)^3 t = \frac{10}{3} a^3 t$

$I_y = 2\frac{1}{12} t a^3 + 2\, 2at \cdot \left(\frac{a}{2}\right)^2 = \frac{7}{6} a^3 t$

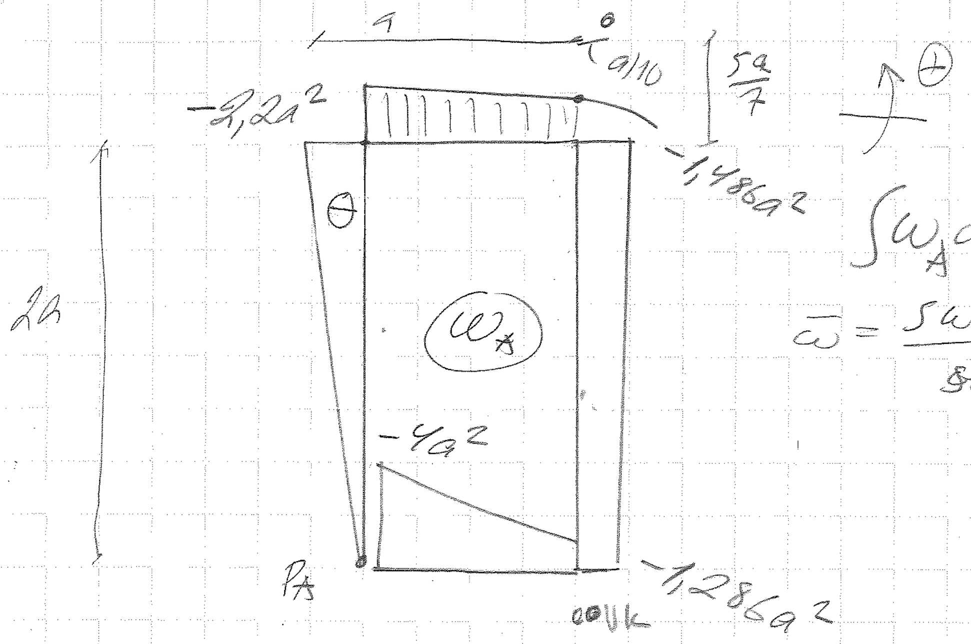
$I_{xy} = 0$

$y_A - y_B = \frac{-I_{y\omega_B}}{I_y} = \frac{\frac{5}{6} a^4 t}{\frac{7}{6} a^3 t} = \frac{5}{7} a$

$x_A - x_B = \frac{I_{x\omega_B}}{I_x} = \frac{+\frac{1}{3} a^4 t}{\frac{10}{3} a^3 t} = +\frac{1}{10} a$

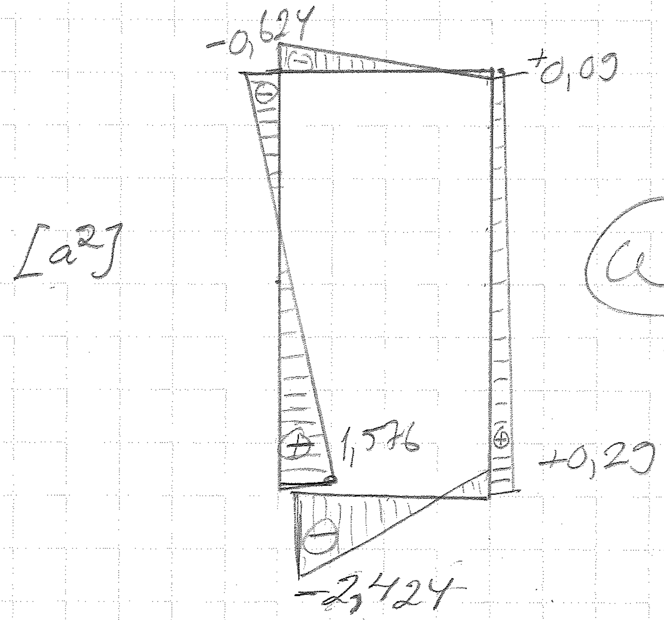
Ehdo:

$I_{x\omega_A} = 0 \qquad I_{y\omega_A} = 0 \qquad S_{\omega_A} = 0$



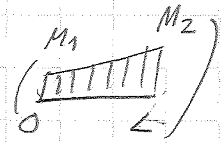
$$\int \omega_A \, dA = 9{,}46 a^3 t$$

$$\bar{\omega} = \frac{\int \omega_A dA}{\int t\, da} = 1{,}576 a^2$$



$$I_\omega = \int_A \omega_A^2 \, dA$$

$$\int_0^L M^2 dx =$$



$$\frac{1}{3} L \left(M_1^2 + M_1 M_2 + M_2^2 \right)$$

$$I_\omega = \frac{1}{3} 2at \left(1{,}576^2 - 0{,}624 \cdot 1{,}576 + 0{,}624^2\right) a^4 +$$

$$\frac{1}{3} at \left(0{,}624^2 - 0{,}624 \cdot 0{,}05 + 0{,}05^2\right) a^4 +$$

$$\frac{1}{3} 2at \left(0{,}05^2 + 0{,}09 \cdot 0{,}29 + 0{,}29^2\right) +$$

$$+ \frac{1}{3} at \left(2{,}424^2 - 2{,}424 \cdot 0{,}29 + 0{,}29^2\right)$$

$$= 1{,}2598 a^5 t + 0{,}1138 a^5 t + 0{,}0789 a^5 t + 1{,}7523 a^5 t$$

$$= 3{,}205 a^5 t$$

$$\sigma_z = \frac{B(z)}{I_\omega} \omega_A(M)$$

$$S_\omega = \int_0^s \omega_s(s)\, t\, ds$$

$$\tau_\omega = -\frac{M_\omega S_\omega(s)}{I_\omega t}$$

$$S_{\omega_1} = \int_0^{2a} (1{,}576a^2 - 1{,}1as)\, t\, ds = \Big|_0^{2a} \left(1{,}576a^2 t s - 1{,}1 \cdot \tfrac{1}{2} a s^2 t\right)$$

$$= 1{,}576a^2 t \cdot 2a - 1{,}1 \cdot \tfrac{1}{2} a\, 4a^2 t = 0{,}952 a^3 t$$

$$S_{\omega_2} = 0{,}952a^3 t + \int_0^a (-0{,}624a^2 + 0{,}714as)\, t\, ds$$

$$= 0{,}952a^3 t + \Big|_0^a \left(-0{,}624a^2 t s + 0{,}714 \cdot \tfrac{1}{2} a s^2 t\right)$$

$$= 0{,}952a^3 t - 0{,}624a^3 t + 0{,}357a^3 t = 0{,}685a^3 t$$

$$S_{\omega_3} = 0{,}685a^3 t + \int_0^{2a} (0{,}09a^2 + 0{,}10as)\, t\, ds$$

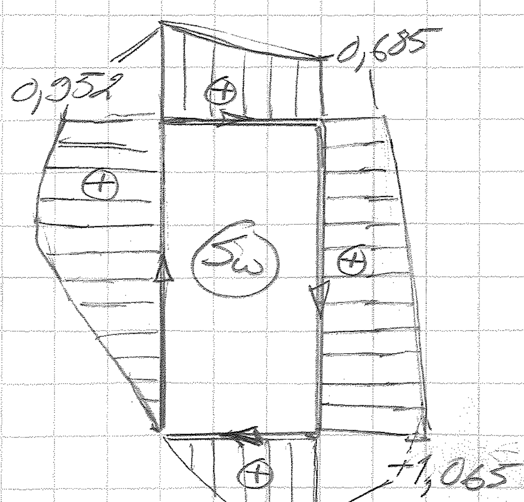
$$= 0{,}685a^3 t + \Big|_0^{2a} \left(0{,}09a^2 t s + 0{,}05 a t s^2\right)$$

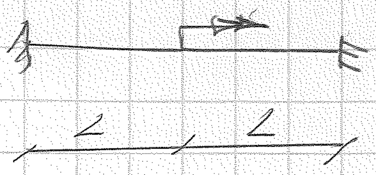
$$= 0{,}685a^3 t + 0{,}18a^3 t + 0{,}05at(2a)^2 = 1{,}065a^3 t$$

$$S_{\omega_4} = 1{,}065a^3 t + \int_0^a (0{,}29a^2 - 2{,}714as)\, t\, ds$$

$$= 1{,}065a^3 t + 0{,}29a^3 t - 1{,}357a^3 t = 0{,}002a^3 t \quad \underline{\text{ok}}$$

$[a^3 t]$





$$\varphi(z) = C_1 + C_2 z + C_3 \sinh kz + C_4 \cosh kz$$

$$k^2 = \frac{GI_v}{EI_\omega}$$

$$\varphi'(z) = C_2 + C_3 k \cosh kz + C_4 k \sinh kz$$

$M(0) = -M/2 \qquad \varphi(L) = 0$

$\varphi'(0) = 0 \qquad \varphi'(L) = 0$

$$M(0) = -\frac{M}{2} = GI_v\left(C_2 + \cancel{\varphi_0'}^{\,0} - \frac{1}{k^2}\cancel{\varphi_0'''}^{\,0}\right) = GI_v C_2$$

$$C_2 = -\frac{M}{2GI_v}$$

$$\varphi'(0) = 0 \Rightarrow \quad C_2 + C_3 k \underbrace{\cosh 0}_{=1} + C_4 k \cancel{\sinh 0}^{\,0} = 0$$

$$C_3 = -\frac{C_2}{k} = +\frac{M}{2GI_v k}$$

*) $\varphi(L) = 0 \Rightarrow \quad C_1 + C_2 L + C_3 \sinh kL + C_4 \cosh kL = 0$

$\varphi'(L) = 0 \Rightarrow \quad C_2 + C_3 k \cosh kL + C_4 k \sinh kL = 0$

$$C_4 = \frac{-C_2 - C_3 k \cosh kL}{k \sinh kL} = \frac{M}{2GI_v}\left(\frac{1 - \cosh kL}{k \sinh kL}\right)$$

$$C_1 = -C_2 L - C_3 \sinh kL - C_4 \cosh kL$$

$$= \frac{ML}{2GI_v} - \frac{M}{2GI_v k}\sinh kL - \frac{M}{2GI_v k}\frac{1 - \cosh kL}{\sinh kL}\cosh kL$$

$$= \frac{M}{2GI_v}\left(L - \frac{\sinh kL}{k} - \frac{1 - \cosh kL}{k \sinh kL}\cosh kL\right)$$

$$\varphi(z) = C_1 + C_2 z + C_3 \cosh kz + C_4 \sinh kz$$

$$= \frac{M}{2GI_v}\left(L - \frac{\sinh kL}{k} - \frac{1-\cosh kL}{k \sinh kL}\cosh kL\right) +$$

$$- \frac{M}{2GI_v} z + \frac{M}{2GI_v k} \cosh kz + \frac{M}{2GI_v} \frac{1-\cosh kL}{k \sinh kL} \sinh kz$$

Tyren jännitys σ_z

$$\sigma_z = \frac{B}{I_\omega} \omega_A(s) \Rightarrow B = -EI_\omega \varphi''(z)$$

$$\sigma_z = \frac{-EI_\omega \varphi''(z)}{I_\omega} \omega_A(s) = -E\varphi''(z)\omega_A(s)$$

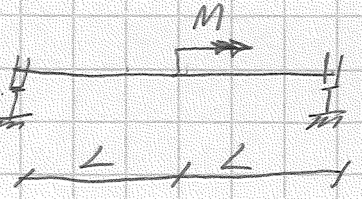
$$\varphi''(z) = \frac{M}{2GI_v k} k^2 \cosh kz + \frac{M}{2GI_v k} \frac{1-\cosh kL}{\sinh kL} k^2 \sinh kz$$

$$= \frac{Mk}{2GI_v}\left(\cosh kz + \frac{1-\cosh kL}{\sinh kL} \sinh kL\right)$$

$$\sigma_z = -\frac{EMk}{2GI_v}\left(\cosh kz + \frac{1-\cosh kL}{\sinh kL} \sinh kz\right)\omega_A(s)$$

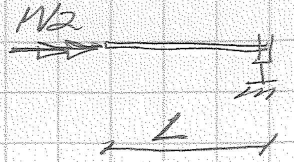
$$E = 2(1+\nu)G \qquad \frac{E}{G} = 2(1+\nu)$$

b)



$$\varphi(z) = C_1 + C_2 z + C_3 \sinh kz + C_4 \cosh kz$$

$$k^2 = \frac{GI_v}{EI_\omega}$$



$M(0) = -M/2 \qquad \varphi(L) = 0$

$\varphi'(0) = 0 \qquad B(L) = 0$

$$M(0) = -M/2 = GI_v C_2 \iff C_2 = -\frac{M}{2GI_v}$$

$$\varphi'(0) = 0 \Rightarrow \varphi'(z) = C_2 + C_3 k \cosh kz + C_4 k \sinh kz$$

$$\varphi'(0) = C_2 + C_3 k \underbrace{\cosh 0}_{=1} + C_4 \underbrace{\sinh 0}_{0}$$

$$C_3 = -\frac{C_2}{k} = +\frac{M}{2GI_v k}$$

*) $\varphi(L) = 0 \Rightarrow C_1 + C_2 L + C_3 \sinh kL + C_4 \cosh kL = 0$

$$B(L) = 0 \Rightarrow -GI_v(C_3 \sinh kL + C_4 \cosh kL) = 0$$

$$C_4 = \frac{-C_3 \sinh kL}{\cosh kL} = -\frac{M \sinh kL}{2GI_v k \cosh kL}$$

*) $C_1 = -C_2 L - C_3 \sinh kL - C_4 \cosh kL$

$$= \frac{ML}{2GI_v} - \frac{M}{2GI_v k} \sinh kL + \frac{M \sinh kL}{2GI_v k \cosh kL} \cdot \cosh kL$$

$$= \frac{ML}{2GI_v} (L -$$

$$\varphi(z) = \frac{M}{2GI_v}\left(L - z + \frac{1}{k} \sinh kz - \frac{\sinh kL}{k \cosh kL} \cosh kz\right)$$