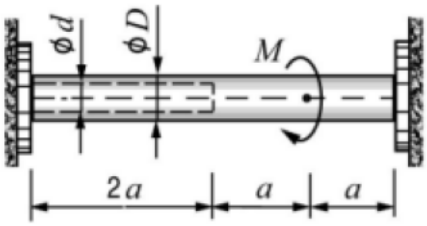
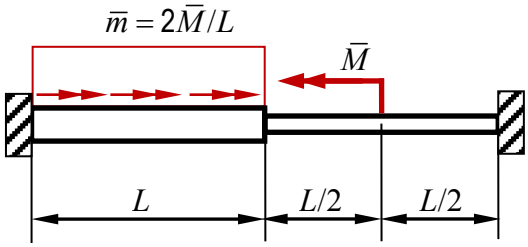


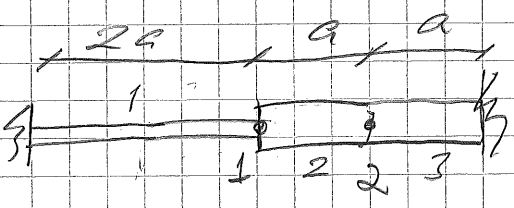
1. Ratkaise oheisen kuvan mukainen tehtävä elementtimenetelmällä. Määritä myös tukireaktiot. $d/D = 2/3$

   Määritä myös tehtävän vääntömomenttikuvio sekä leikkausjännitysten arvot rakenteen eri kohdissa.



2. Ratkaise oheinen vääntötehtävä, kun vääntösauvaa kuormittaa tasainen vääntömomenttikuorma $\bar{m}$ sekä vääntömomentti $\bar{M}$. Määritä tukireaktiot, vääntömomenttikuvio sekä leikkausjännitykset. Vääntösauvan paksumman osuuden halkaisija on D ja ohuemman halkaisija on $D/2$. Materiaalin liukukerroin on G.

1)



$$k^{(1)} = \frac{G I_1}{2a}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{matrix}1\\2\end{matrix}$$

$$k^{(2)} = \frac{G I_2}{a}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{matrix}2\\3\end{matrix}$$

$$k = \begin{bmatrix} \frac{G I_1}{2a} + \frac{G I_2}{a} & -\frac{G I_2}{a} \\ -\frac{G I_2}{a} & 2\frac{G I_2}{a} \end{bmatrix}\begin{matrix}1\\2\end{matrix}$$

$$= \frac{G I_2}{a}\begin{bmatrix} \overbrace{\frac{65}{81\cdot 2}}^{1{,}40123} + 1 & -1 \\ -1 & 2 \end{bmatrix}$$

$$k\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \begin{pmatrix} 0 \\ M \end{pmatrix}$$

$$\begin{pmatrix} e_1 \\ e_2 \end{pmatrix} = \frac{a}{G I_2}\frac{1}{1{,}40123\cdot 2 - 1}\begin{bmatrix} 2 & 1 \\ 1 & 1{,}40123 \end{bmatrix}\begin{pmatrix} 0 \\ M \end{pmatrix}$$

$$= \frac{a}{G I_2}\cdot 0{,}5546\begin{pmatrix} 1 \\ 1{,}40123 \end{pmatrix} M$$

$$= \frac{Ma}{G I_2}\begin{pmatrix} 0{,}5546 \\ 0{,}777 \end{pmatrix}$$

$$d = \frac{2}{3}D$$

(2.7)

$$I_1 = \frac{\pi}{32}(D^4 - d^4) \quad \text{hohl.}$$

$$= \frac{65}{81}\frac{\pi}{32}D^4$$

$$I_2 = \frac{\pi}{32}D^4$$

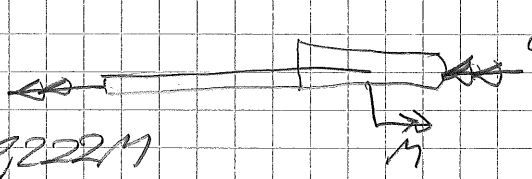
$$I_1 = \frac{65}{81}I_2$$

Elementsteifigkeit $\boxed{\underline{K}^e \underline{u}^e = \underline{f}^e}$

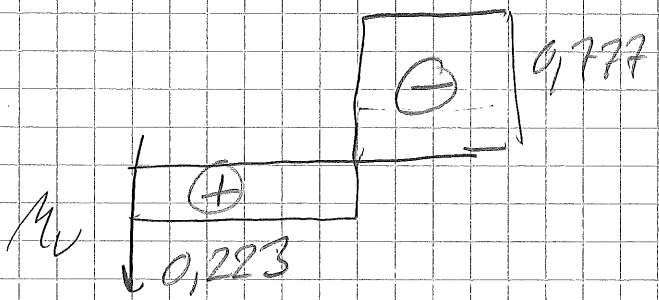
$$\underline{K}^{③}\begin{pmatrix} \varphi_2 \\ 0 \end{pmatrix} = \frac{6EI_2}{a}\frac{M_A}{6EI_2}\begin{pmatrix} 1 \\ -1 \end{pmatrix} 0{,}777 = 0{,}777\,M\begin{pmatrix} 1 \\ -1 \end{pmatrix} \leftarrow$$

$$I_1/I_2 = \frac{65}{91}$$

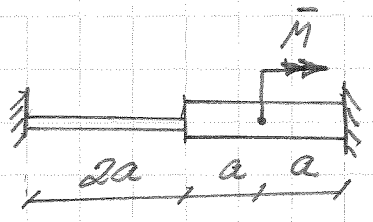
$$\underline{K}^{①}\begin{pmatrix} 0 \\ \varphi_1 \end{pmatrix} = \frac{6EI_1}{2a}\frac{M_A}{6EI_2}\begin{pmatrix} -0{,}5548 \\ +0{,}5548 \end{pmatrix} = 0{,}223\,M\begin{pmatrix} -1 \\ 1 \end{pmatrix}$$



$$\underline{K}^{②}\begin{pmatrix} \varphi_1 \\ \varphi_2 \end{pmatrix} = \frac{6EI_2}{a}\begin{pmatrix} \varphi_1 - \varphi_2 \\ -\varphi_1 + \varphi_2 \end{pmatrix}\frac{M_A}{6EI_2} = M\begin{pmatrix} -0{,}222 \\ +0{,}222 \end{pmatrix}$$



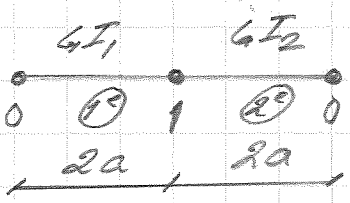
①



$$I_1 = \frac{\pi}{32}(D^4 - d^4)$$

$$= \frac{65}{81}\frac{\pi}{32}D^4$$

$$= \frac{65}{81} I_2$$



$$I_2 = \frac{\pi}{32}D^4$$

$$k^e = \frac{GI_e}{L^e}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{matrix}1\\2\end{matrix}$$

$$k^{①} = \frac{GI_1}{2a}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{matrix}0\\1\end{matrix} \qquad k^{②} = \frac{GI_2}{2a}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{matrix}1\\0\end{matrix}$$

$$K = \sum_i k^{(i)} = \frac{GI_1}{2a} + \frac{GI_2}{2a} = \frac{GI_2}{a}\left(\frac{65}{81\cdot 2} + \frac{1}{2}\right) = \frac{146}{162}\frac{GI_2}{a}$$

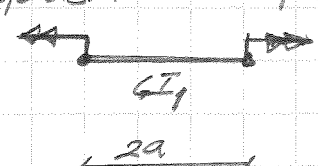
$$F^{②} = \begin{pmatrix} \bar{M}/2 \\ \bar{M}/2 \end{pmatrix}\begin{matrix}1\\0\end{matrix} \Rightarrow \varphi = \frac{\bar{M}}{2}$$

$$K\varphi = \frac{\bar{M}}{2} \Rightarrow \varphi = \frac{162}{146\cdot 2}\frac{Ma}{GI_2} = \underline{0{,}5548\,\frac{Ma}{GI_2}}$$

Elementtivoimat

$$R^{(e)} = k^{(e)}u^{(e)} - f^{(e)} \Rightarrow R^{①} = k^{①}u^{①} = \frac{GI_1}{2a}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{pmatrix} 0 \\ 0{,}5548\,Ma/(GI_2) \end{pmatrix}$$

$$= \frac{65}{81}\frac{GI_2}{2a}\cdot\frac{0{,}5548\,Ma}{GI_2}\begin{pmatrix}-1\\1\end{pmatrix}$$



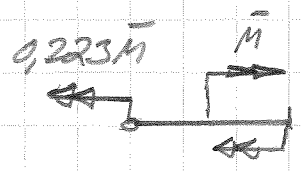
$$= 0{,}223\bar{M}\begin{pmatrix}-1\\1\end{pmatrix}$$

$$\frac{81-L}{81}$$

$$81 - L = 65\left(1 - \frac{L}{81}\right)$$

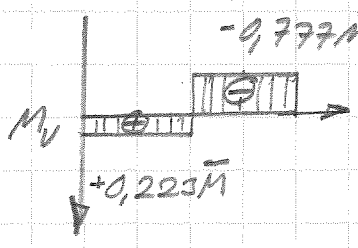
$$\tau = \frac{MR}{I_1} = \frac{0{,}223\bar{M}\cdot 32\cdot D}{\pi(D^4 - d^4)\cdot 2} = 1{,}415\,\frac{\bar{M}}{D^3}$$

$$z^{②} = L^{②} u^{②} - f^{②} = \frac{6I_2}{2a}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\left(\begin{matrix} 0{,}5548\,\bar{M}a/6I_2 \\ 0 \end{matrix}\right) - \begin{pmatrix} \bar{M}/2 \\ \bar{M}/2 \end{pmatrix}$$



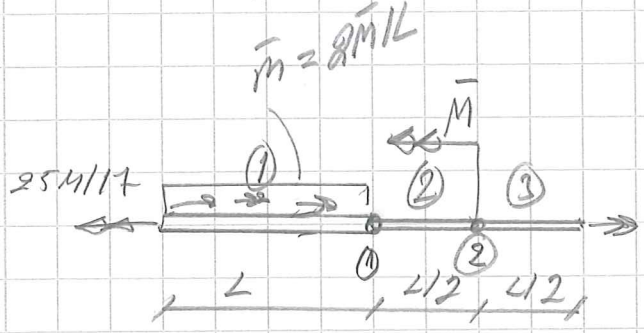
$$= \frac{0{,}5548}{2}\bar{M}\begin{pmatrix} 1 \\ -1 \end{pmatrix} - \begin{pmatrix} \bar{M}/2 \\ \bar{M}/2 \end{pmatrix} = \begin{pmatrix} -0{,}223 \\ -0{,}777 \end{pmatrix}\bar{M}$$

$$\tau_1 = \frac{MR}{I_2} = \frac{0{,}223\bar{M}\cdot D\cdot 32}{2\cdot\pi D^4} = 1{,}136\ \bar{M}/D^3$$



$$\tau_2 = \frac{MR}{I_2} = \frac{-0{,}777\bar{M}\cdot D\cdot 32}{2\cdot\pi D^4} = 3{,}96\,\bar{M}/D^3$$

UK1



$$q^{(0)} = \begin{bmatrix} +\bar{M} \\ +\bar{M} \end{bmatrix} \begin{matrix} 0 \\ 1 \end{matrix}$$

$$I_{L1} = \frac{\pi}{32} D^4$$

$$I_{L2} = \frac{\pi}{512} D^4$$

$$k^{(0)} = \frac{G I_{L1}}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{matrix} 0 \\ 1 \end{matrix} \qquad k^{(2)} = k^{(1)} = \frac{G I_{L2}}{L/2} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{matrix} 1 & 2 \\ 2 & 3 \end{matrix}$$

$$K = \frac{\pi D^4 G}{256 L} \begin{bmatrix} 8+1 & -1 \\ -1 & 1+1 \end{bmatrix} = \frac{\pi D^4 G}{256 L} \begin{bmatrix} 9 & -1 \\ -1 & 2 \end{bmatrix}$$

$$q^e = \begin{bmatrix} +M \\ -M \end{bmatrix}$$

$$\underline{\varphi} = K^{-1} \underline{f} = \frac{256 L}{\pi D^4 G} \frac{1}{17} \begin{bmatrix} 2 & 1 \\ 1 & 9 \end{bmatrix} \begin{pmatrix} M \\ -M \end{pmatrix} =$$

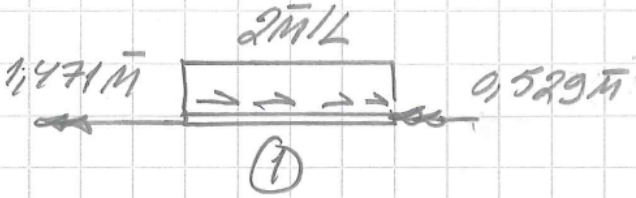
$$= \frac{256 L \bar{M}}{17 \pi D^4 G} \begin{pmatrix} 1 \\ -8 \end{pmatrix} = \frac{L \bar{M}}{\pi D^4 G} \begin{pmatrix} 15{,}059 \\ -120{,}47 \end{pmatrix}$$

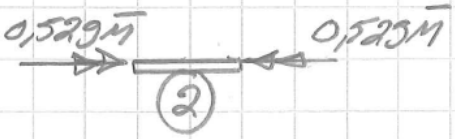
$$R^{(1)} = K^1 \varphi^{(1)} - q^{(1)} = \frac{\pi G D^4}{32} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{pmatrix} 0 \\ \frac{256 L \bar{M}}{17 \pi D^4 G} \end{pmatrix} - \begin{pmatrix} +\bar{M} \\ +\bar{M} \end{pmatrix}$$

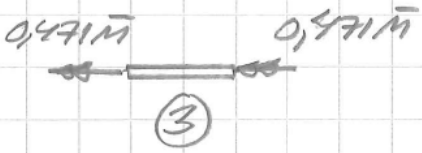
$$\frac{\bar{M}}{17} \begin{pmatrix} -8-17 \\ +8+17 \end{pmatrix} = \bar{M} \begin{pmatrix} -25/17 \\ -9/17 \end{pmatrix} = \bar{M} \begin{pmatrix} -1{,}4706 \\ -0{,}529 \end{pmatrix}$$

$$R^{(2)} = \frac{\pi D^4 G}{256 L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{pmatrix} 1 \\ -8 \end{pmatrix} \frac{256 L \bar{M}}{17 \pi D^4 G} = \bar{M} \begin{pmatrix} 9/17 \\ -9/17 \end{pmatrix}$$

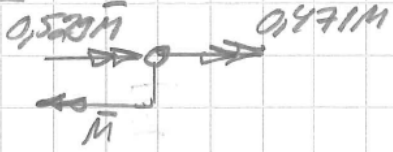
$$\varphi^{(3)} = \frac{\pi D^4 G}{256 L}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{pmatrix}-8\\ 0\end{pmatrix}\frac{256 L \bar{M}}{17\pi D^4 G} = \bar{M}\begin{pmatrix}-8/17\\ +8/17\end{pmatrix}$$



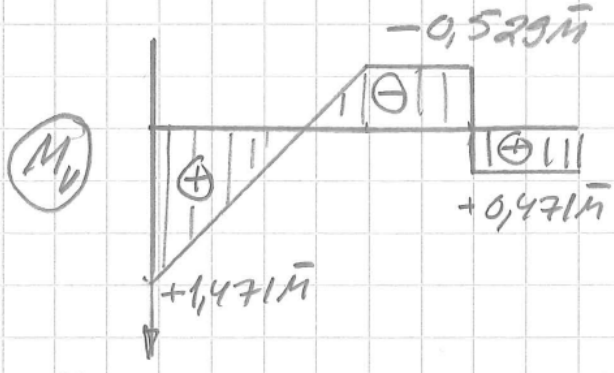




Solmu 2



Vääntömomentikuvio



Leikkausjännitykset

$$\tau_{max} = \frac{M_v R}{I_p} = \frac{M_v}{W_v}$$

$$I_p = \frac{\pi}{32} D^4$$

$$W_v = \frac{\pi D^3}{16}$$

$$D_1 = D, \quad D_2 = D_3 = D/2$$