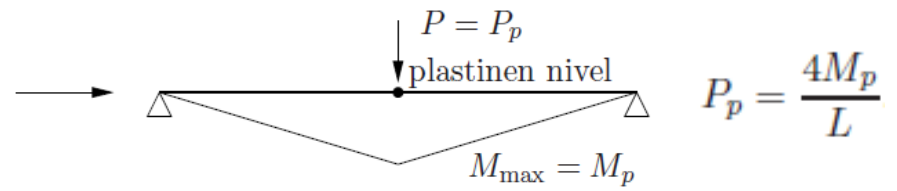
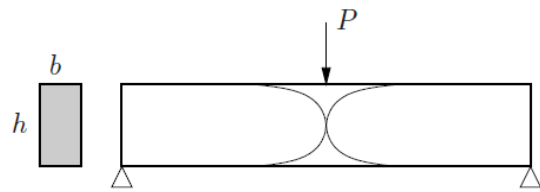
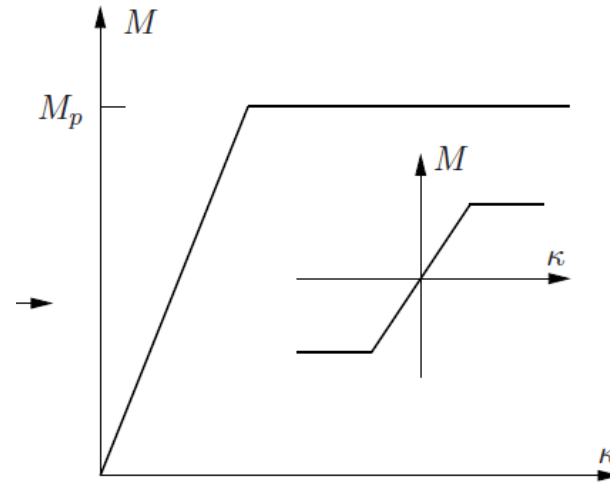
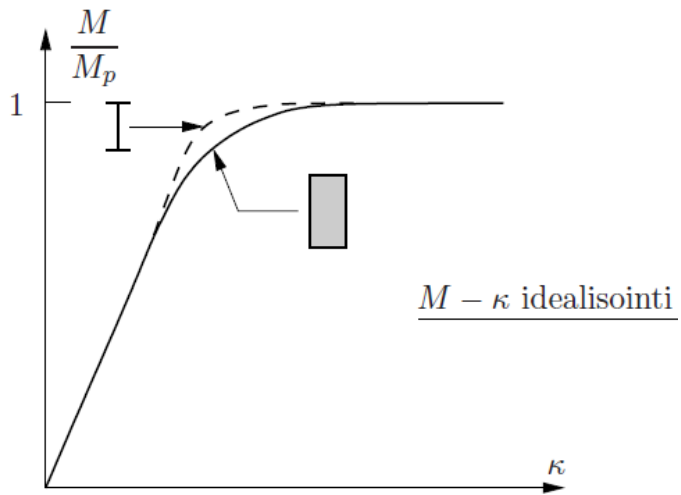
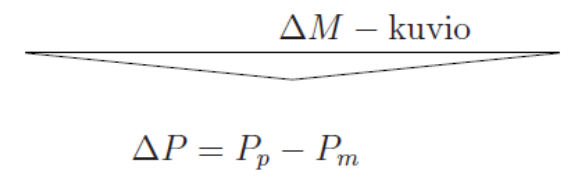
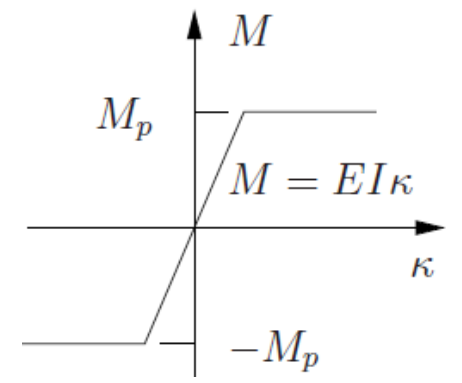
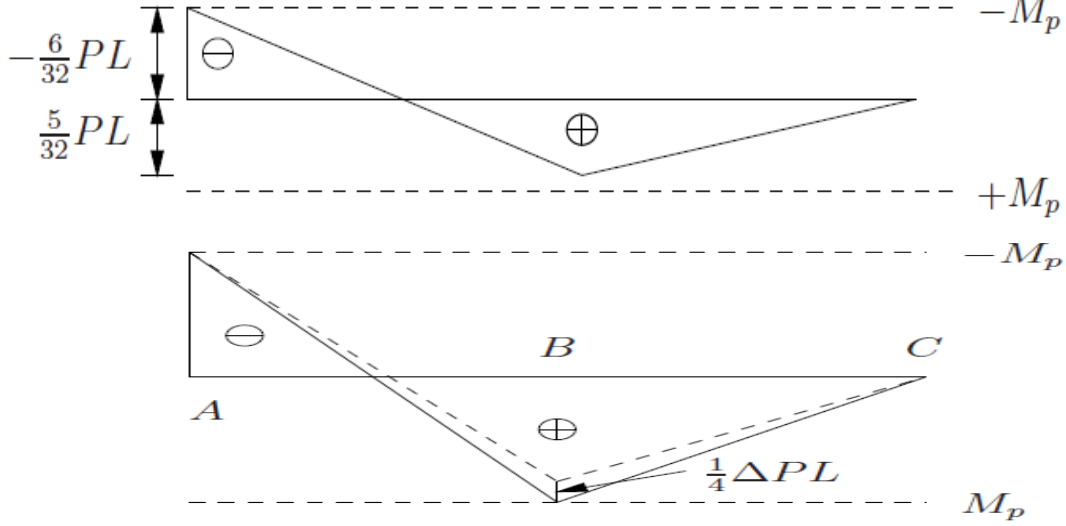
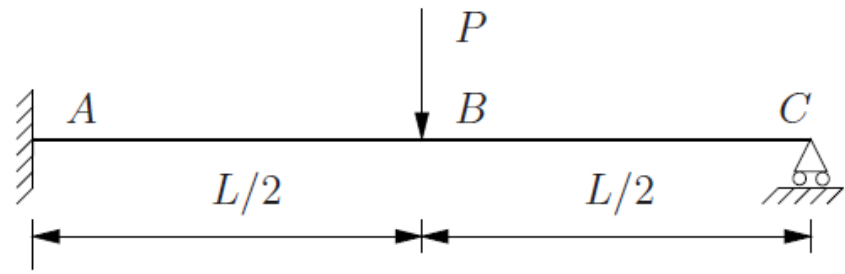
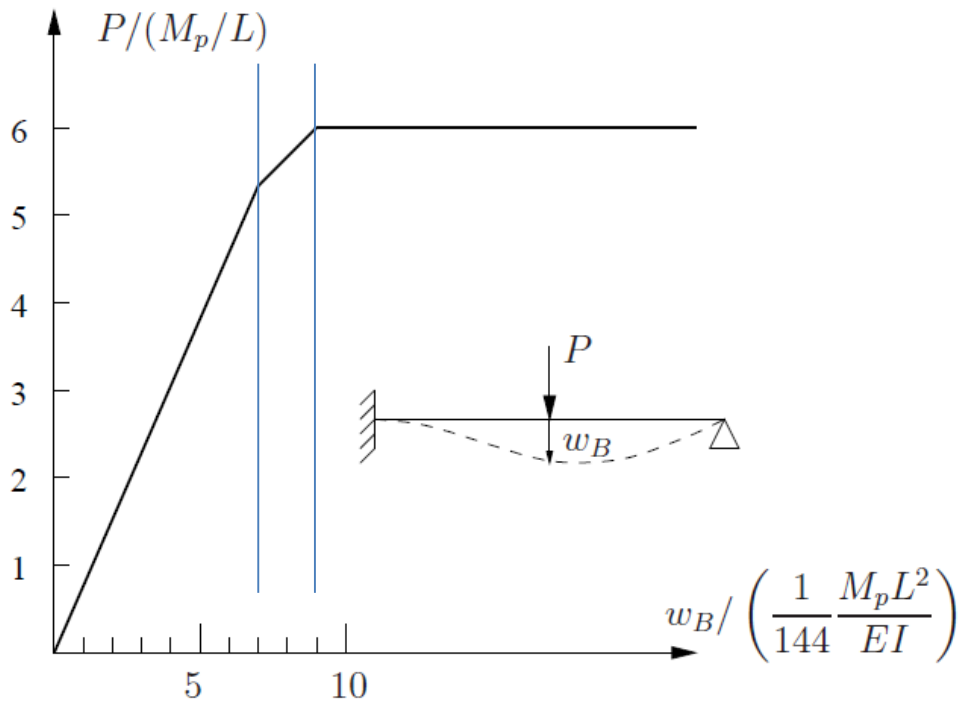


# **33001 Rakenteiden plastisuusmallit**

Rajakuorman määrittäminen yksinkertaiselle rakenteille





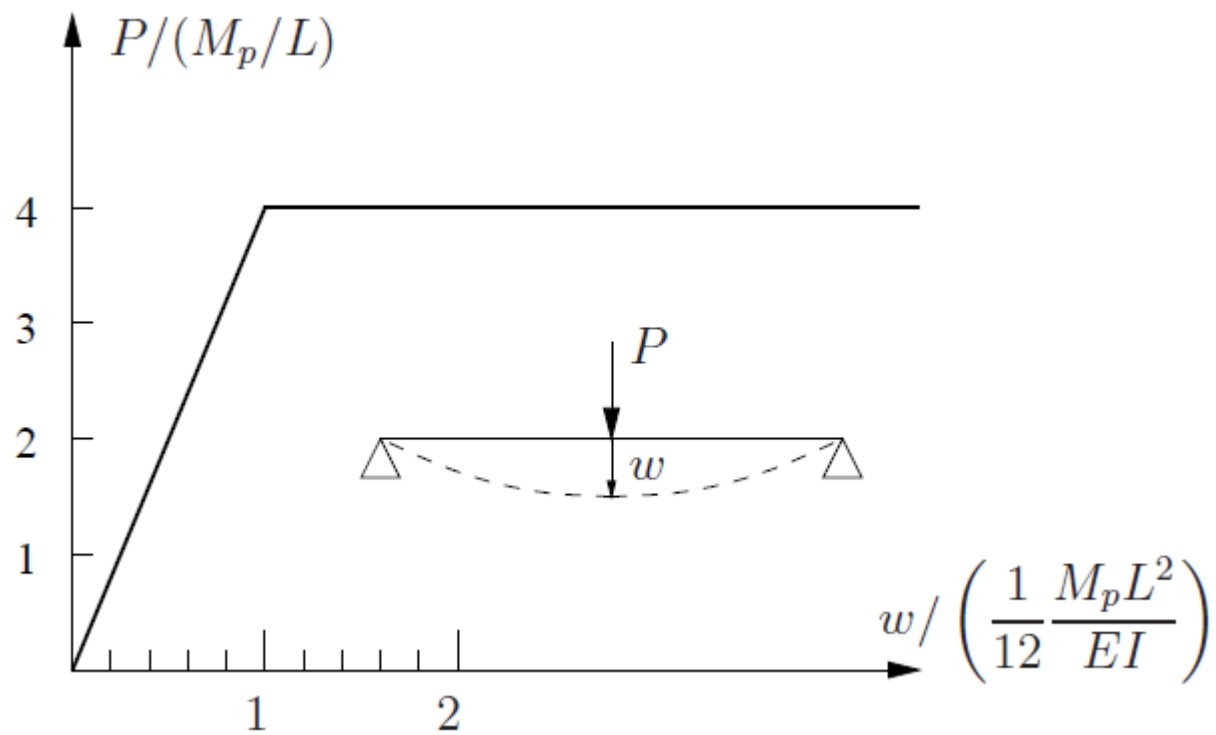


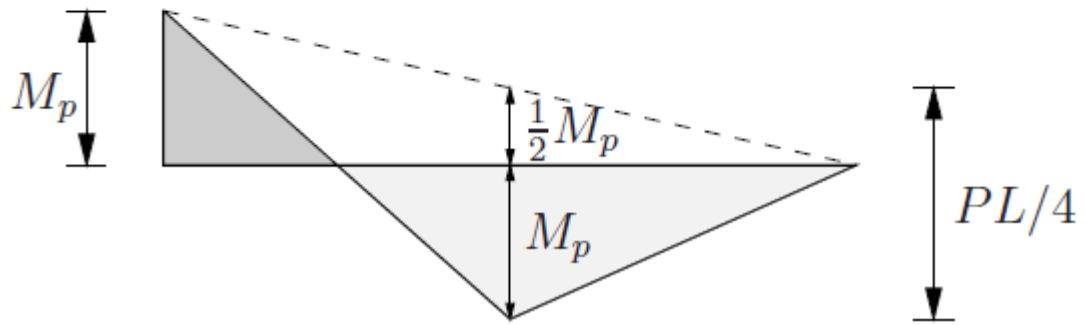
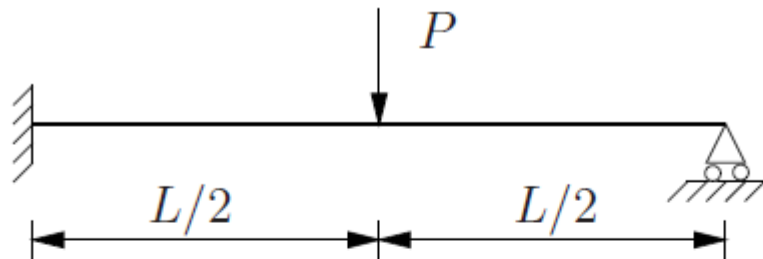
$$w_{B1} = \frac{7}{144} \frac{M_p L^2}{EI}$$

$$P_m = \frac{16}{3} \frac{M_p}{L} \approx 5.333 \frac{M_p}{L}$$

$$w_{B1} = \frac{9}{144} \frac{M_p L^2}{EI}$$

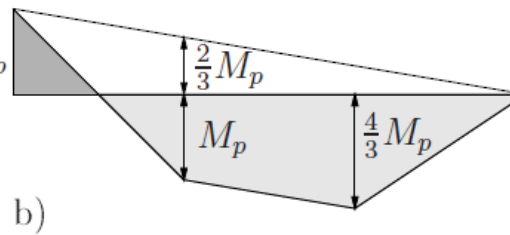
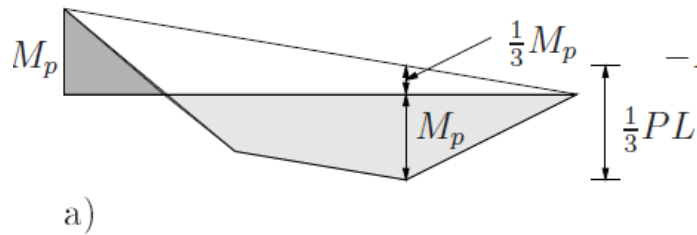
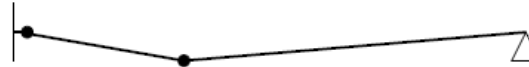
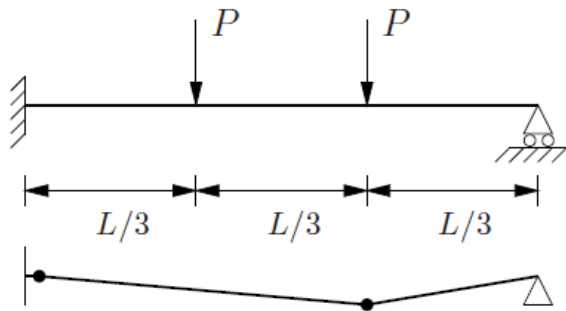
$$P_2 = 6 \frac{M_p}{L}$$





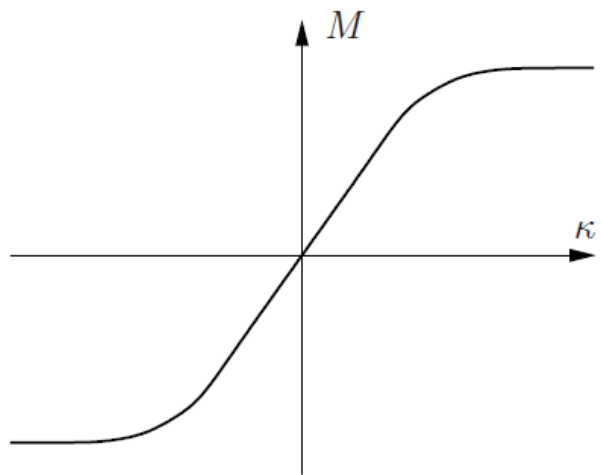
$$\frac{1}{2}M_p + M_p = \frac{PL}{4}$$

$$\Rightarrow P_p = \frac{6M_p}{L}$$

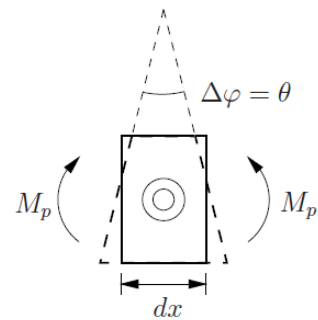
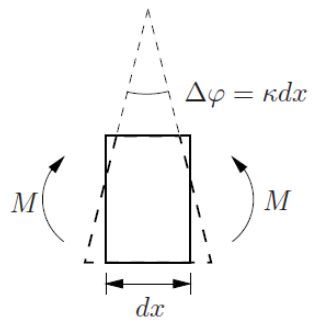
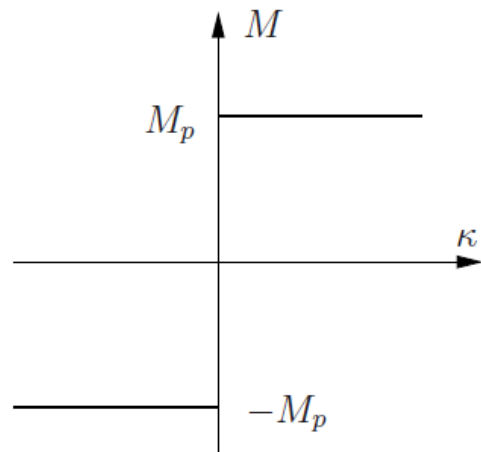


$$M_p + \frac{1}{3}M_p = \frac{1}{3}PL \Rightarrow P_p^+ = 4M_p/L.$$

$$\frac{2}{3}M_p + M_p = \frac{1}{3}PL \Rightarrow P_p^+ = 5M_p/L.$$



idealisointi





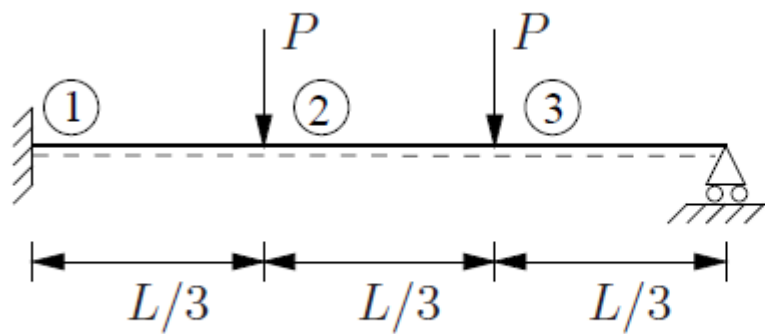
Virtuaalisen työn yhtälö:  $W_u + W_s = 0$

sisäinen työ:  $W_s = - \int_0^L M \kappa dx$        $W_s = -M_p \theta$

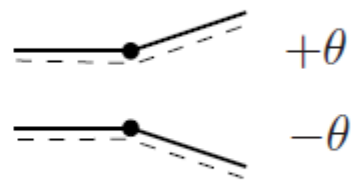
ulkoinen työ:  $W_u = \sum F_k v_k$

$$\sum_{n=1}^N |M_{pn} \theta_n| + \int_0^L M \kappa dx = \sum_{k=1}^K F_k v_k$$

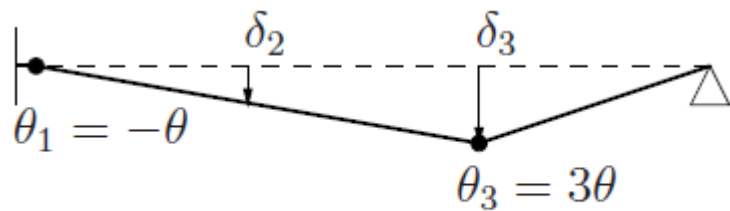
$\kappa = 0$        $\sum_{n=1}^N |M_{pn} \theta_n| = \sum_{k=1}^K F_k v_k$



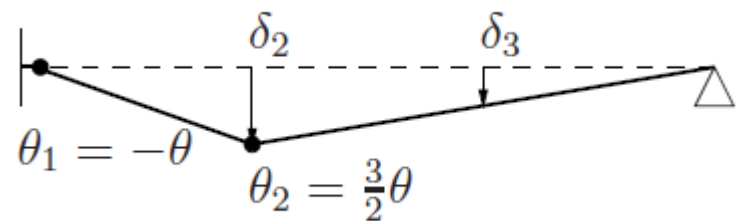
a)



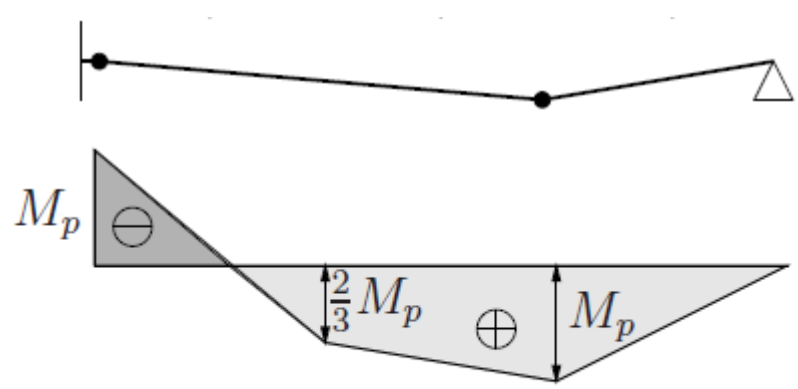
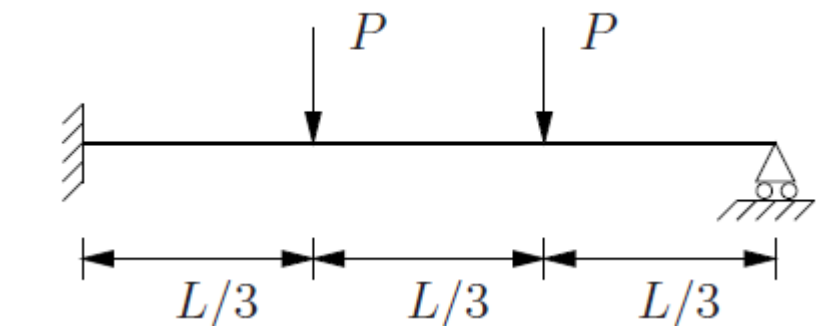
b)



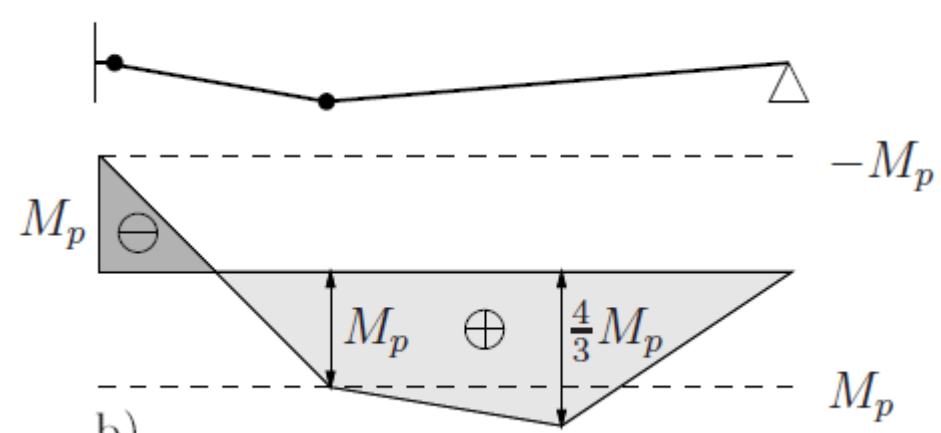
c)



d)



a)

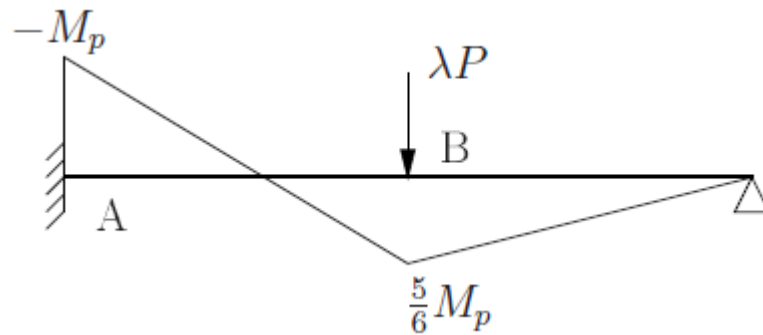


b)

Todellinen rajakuorma toteuttaa seuraavat ehdot:

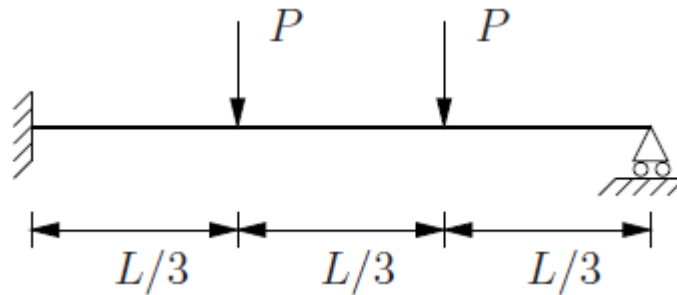
1. Myötöehtoa  $|M| \leq M_p$  ei rikota missään rakenteen osassa (myötöehto). alaraja
2. Momentti  $M(x)$  on tasapainossa kuormien kanssa (tasapainoehto).
3. Rakenteeseen syntyy riittävä määrä myötöniveliä mekanismin muodostumista varten (mekanismiehto). yläraja

1. Myötöehtoa  $|M| \leq M_p$  ei rikota missään rakenteen osassa (myötöehto).



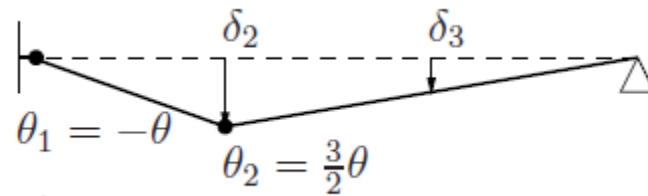
$$\lambda M_A = M_p \quad \Rightarrow \quad \lambda \left( -\frac{3}{16} PL \right) = -M_p \quad \Rightarrow \quad \lambda P = \frac{16}{3} \frac{M_p}{L}. \quad \text{alaraja}$$

3. Rakenteeseen syntyy riittävä määrä myötöniveliä mekanismin muodostumista varten (mekanismiehto).



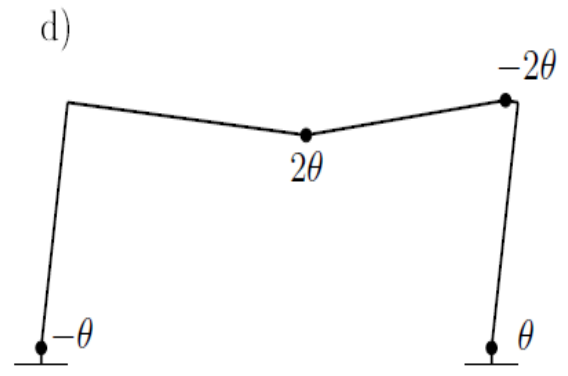
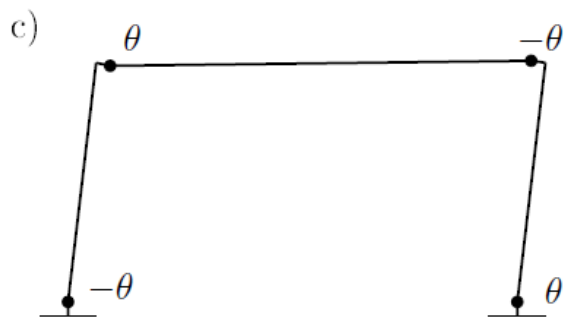
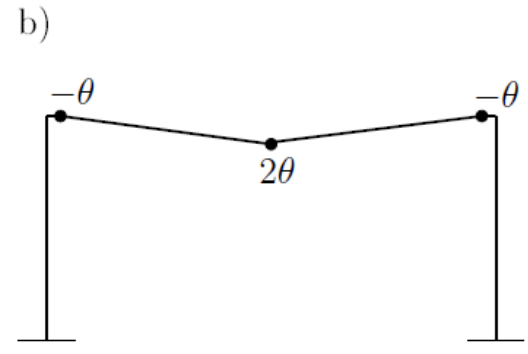
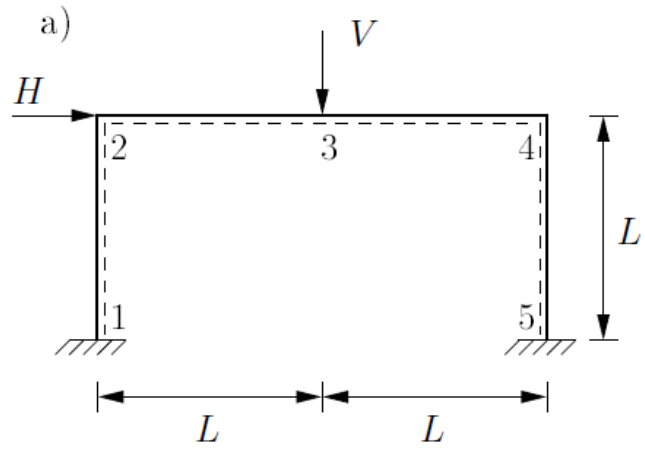
$$M_p\theta + M_p\frac{3}{2}\theta = P\frac{\theta L}{3} + P\frac{1}{2}\theta\frac{L}{3}$$

$$\frac{5}{2}M_p\theta = \frac{3}{6}PL\theta \Rightarrow P_p = \frac{5M_p}{L}$$



yläraja

d)



## Rajamekanismi

$$m = s - n_s$$

$s$  = kriittisten leikkausten lukumäärä

$n_s$  = staattisen määräämättömyyden kertaluku

$$\text{Nyt } s = 5 \text{ ja } n_s = 3 \quad m = 5 - 3 = 2$$

$a$ -mekanismi

$$M_p\theta + M_p2\theta + M_p\theta = VL\theta \quad \Rightarrow \quad V = 4\frac{M_p}{L}$$

$b$ -mekanismi

$$4M_p\theta = HL\theta \quad \Rightarrow \quad H = 4\frac{M_p}{L}$$

$a+b$ -mekanismi

$$6M_p\theta = HL\theta + VL\theta \quad \Rightarrow \quad H + V = 6\frac{M_p}{L}$$



