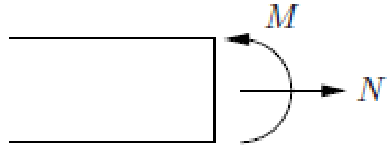
Luento 3:

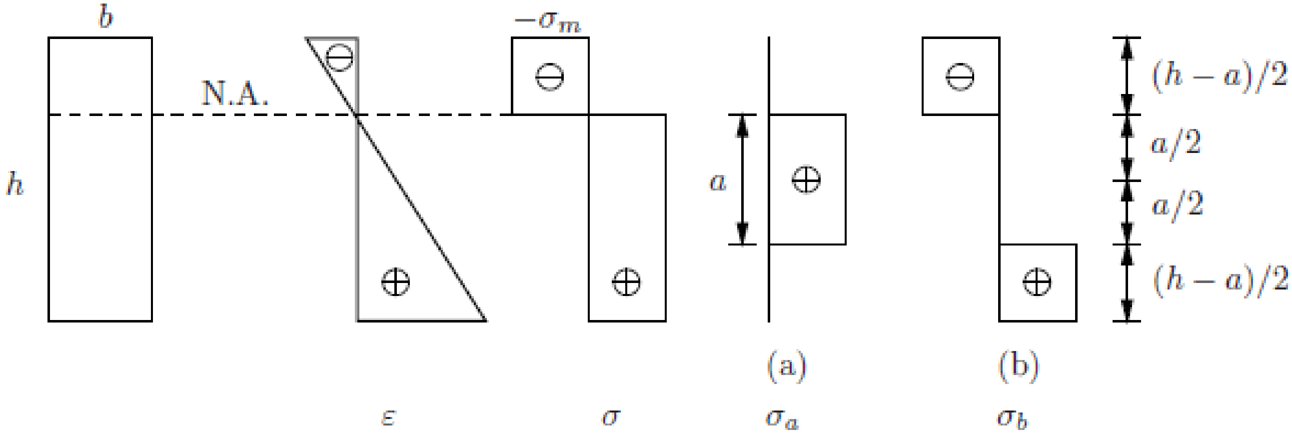
33001 Rakenteiden plastisuusmallit

Normaalivoiman ja leikkausvoiman vaikutus täysplastiseen momenttiin

Normaalivoima



$$\sigma = \sigma_a + \sigma_b$$

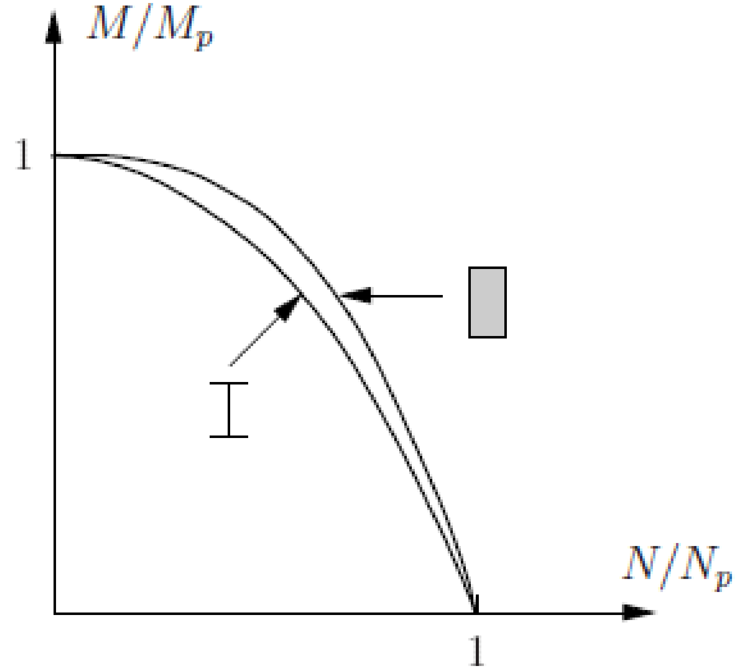


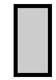
(a) $N = ab\sigma_m$

$M = 0.$

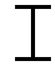
(b) $N = 0.$

$$M = M_p \left[1 - \left(\frac{a}{h}\right)^2\right]$$

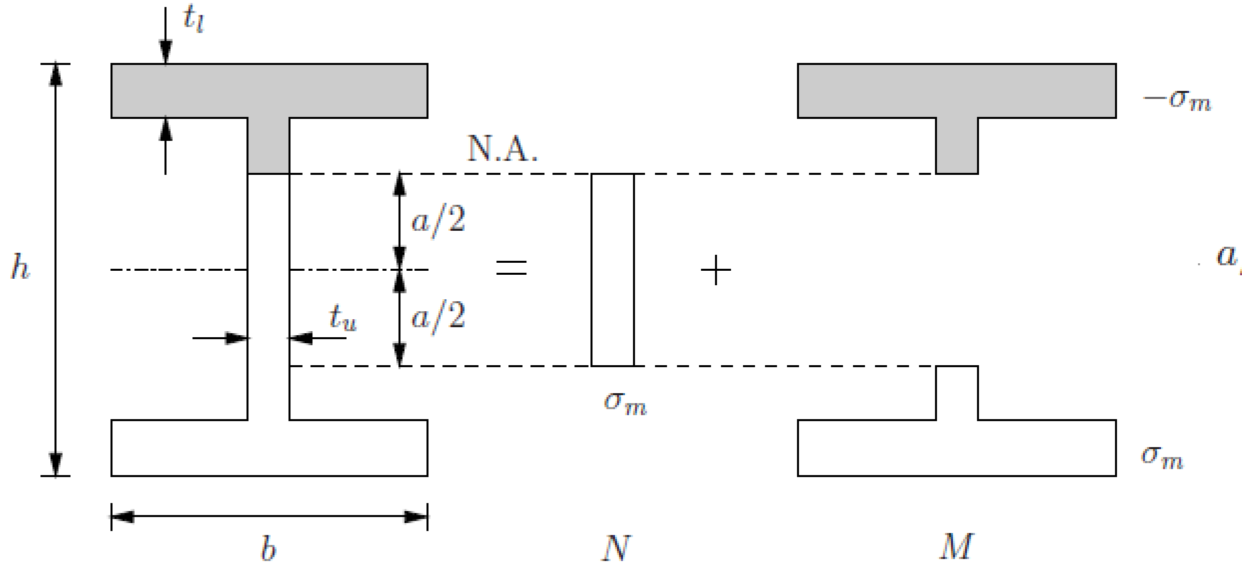




$$\frac{M}{M_p} = 1 - \left(\frac{N}{N_p}\right)^2$$



$$\frac{M}{M_p} = 1 - \left(\frac{A^2}{4t_u W_p}\right)\left(\frac{N}{N_p}\right)^2,$$



$a/2 \leq h/2 - t_l,$

$$N = a t_u \sigma_m \qquad M = M_p - \frac{1}{4} t_u a^2 \sigma_m.$$

$$N_p = A\sigma_m \qquad \frac{M}{M_p} = 1 - \left(\frac{A^2}{4 t_u W_p}\right)\left(\frac{N}{N_p}\right)^2$$

$a/2 > h/2 - t_l.$ $$N = [A - (h-a)b]\sigma_m, \quad N_p = A\sigma_m$$

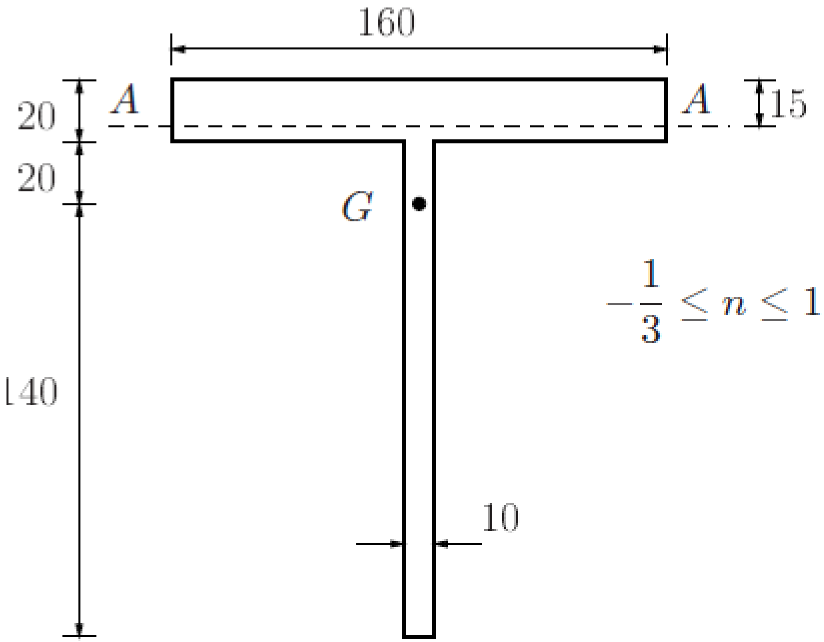
$$\frac{N}{N_p} \equiv n = 1 - \frac{b(h-a)}{A}$$

$$M = \left(\frac{h+a}{2}\right)\left(\frac{h-a}{2}\right) b\sigma_m.$$

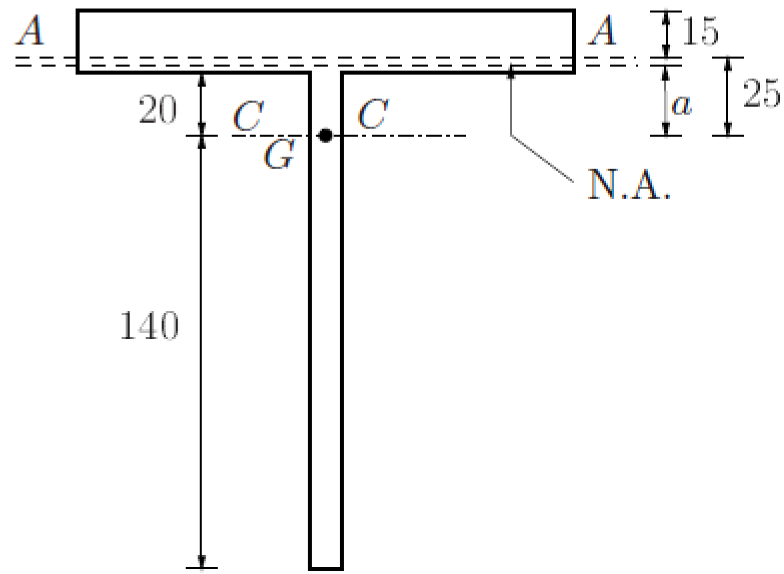
$$h - a = \frac{A}{b}(1-n), \quad h + a = 2h - \frac{A}{b}(1-n)$$

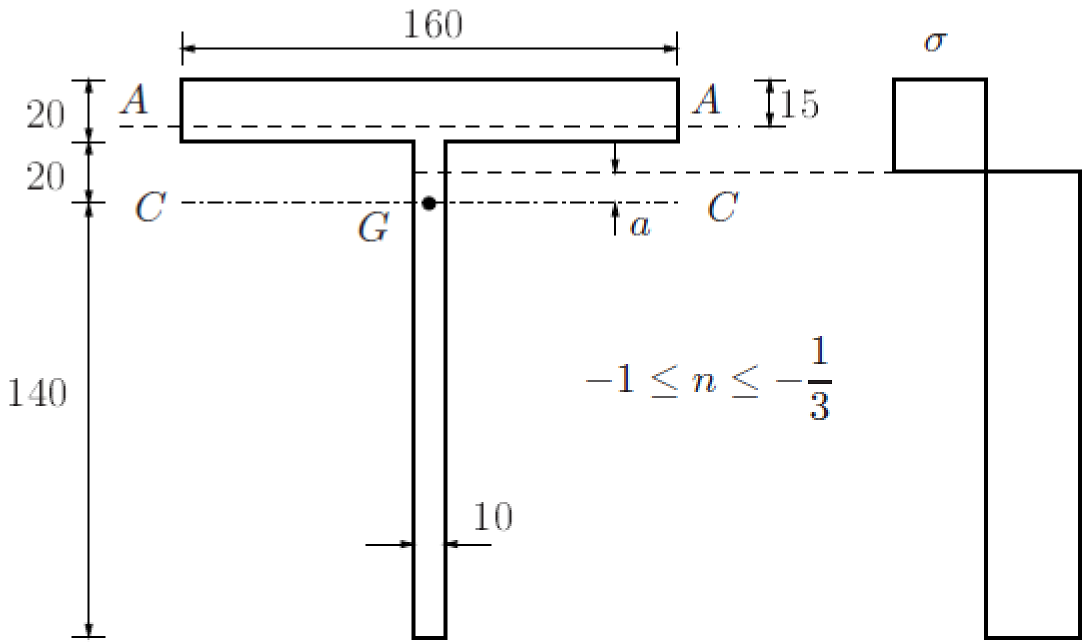
$$m = \frac{A^2}{4bW_p}(1-n)\left(\frac{2bh}{A} - 1 + n\right)$$

$$n = \frac{N}{N_p}, \quad m = \frac{M}{M_p}.$$

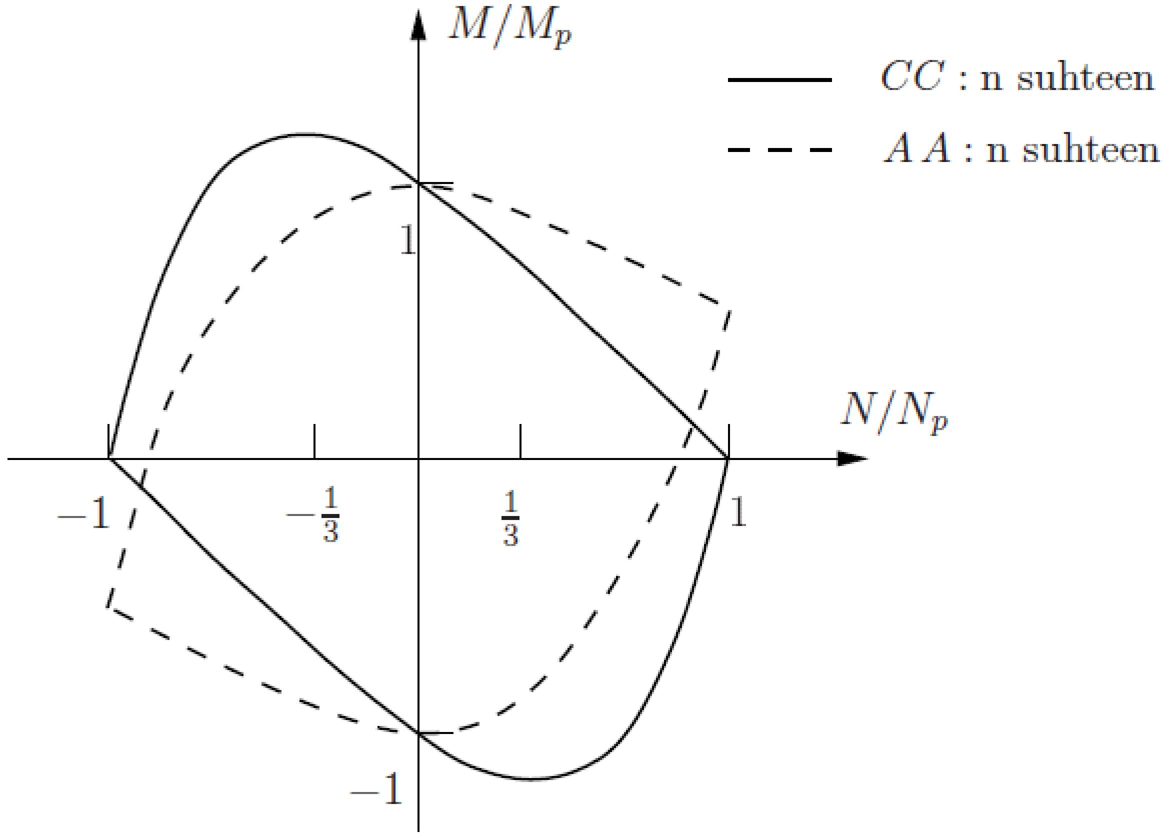


$$-\frac{1}{3} \leq n \leq 1$$



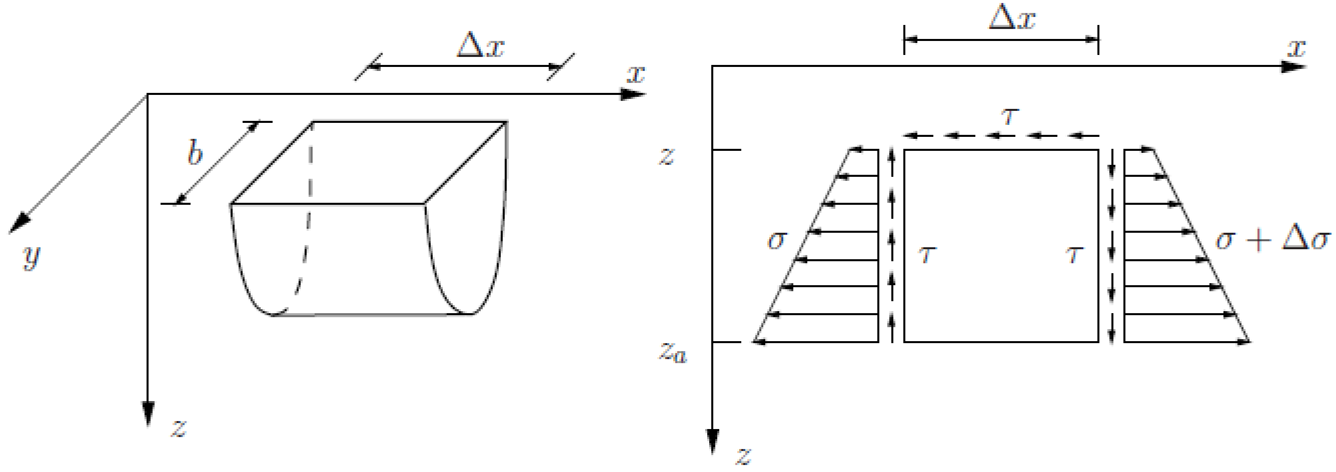


160
σ
20
A
A
15
20
C
C
G
a
140
$-1 \leq n \leq -\frac{1}{3}$
10



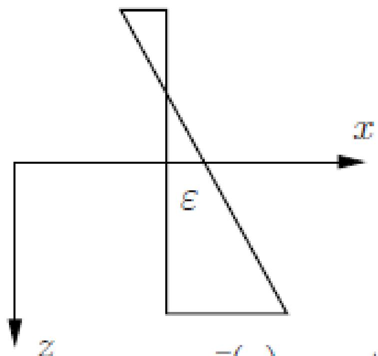
M/M_p
CC : n suhteen
$A\,A$: n suhteen
1
N/N_p
−1
$-\frac{1}{3}$
$\frac{1}{3}$
1
−1

Leikkausvoima

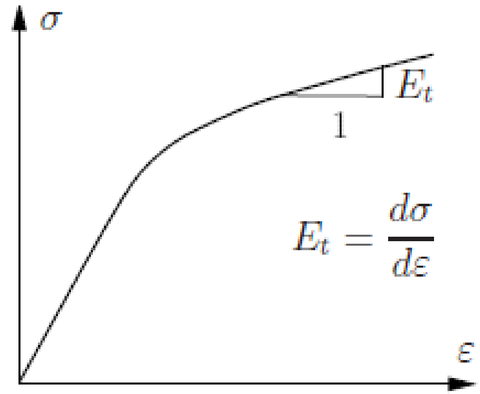


$$-\tau(z)b(z)\Delta x + \int_z^{z_a} \Delta\sigma b(z)\,dz = 0,$$

$$\tau(z) = \frac{1}{b(z)} \int_z^{z_a} \frac{d\sigma}{dx} b(z)\,dz.$$



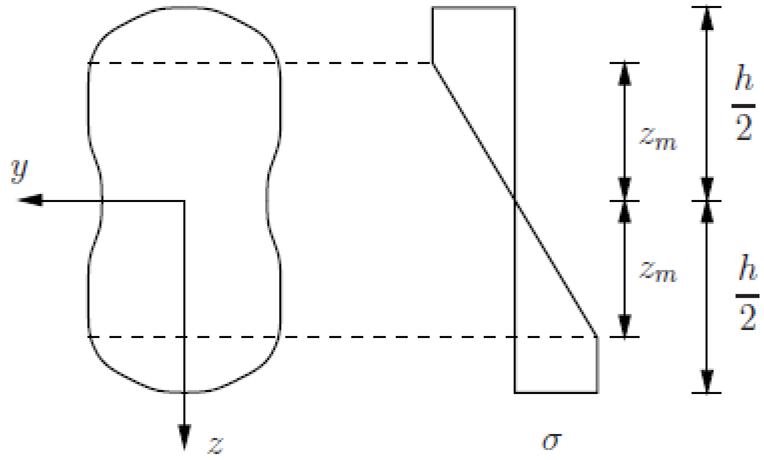
$$\bar{\varepsilon}(z) = \varepsilon + z\kappa$$



$$\tau(z) = \frac{1}{b(z)}\left[\frac{d\varepsilon}{dx}\int_{z}^{z_a} E_t(z)b(z)\,dz + \frac{d\kappa}{dx}\int_{z}^{z_a} E_t(z)b(z)\,dz\right]$$

ideaaliplastinen

$$E_t = E, \quad \text{kun} \quad |\varepsilon| \leq \frac{\sigma_m}{E},$$
$$E_t = 0, \quad \text{kun} \quad |\varepsilon| > \frac{\sigma_m}{E}.$$



Ideaaliplastinen + ”$N = 0$”

$$\sigma = \sigma_m \frac{z}{z_m}, \quad \text{kun} \quad |z| \leq z_m$$

$$\sigma = \sigma_m \mathrm{sgn}(z), \quad \text{kun} \quad |z| > z_m$$

$$\tau = \frac{1}{b(z)} \int\limits_{z}^{h/2} \frac{d\sigma}{dx} b(z)\, dz$$

$$M = 2\sigma_m \int\limits_{z_m}^{h/2} z b(z)\, dz + 2\frac{\sigma_m}{z_m} \int\limits_{0}^{z_m} z^2 b(z)\, dz$$

$$Q = \frac{dM}{dx}$$

$$\frac{dM}{dx} = -2\frac{\sigma_m}{z_m^2} \left(\int\limits_{0}^{z_m} z^2 b(z)\, dz \right) \frac{dz_m}{dx}$$

$$\sigma = \sigma_m \frac{z}{z_m}, \quad \text{kun} \quad |z| \leq z_m,$$

$$\frac{d\sigma}{dx} = -\sigma_m z \frac{1}{z_m^2} \frac{dz_m}{dx} = -\sigma_m z \left(-\frac{Q}{\sigma_m I_m} \right) = \frac{Qz}{I_m}$$

$$I_m = \int_{-z_m}^{z_m} z^2 b(z)\, dz.$$

$$\tau = \frac{1}{b} \frac{Q}{I_m} \int_{z}^{z_m} z b(z)\, dz = \frac{Q S_m(z)}{b I_m}$$

$$S_m = \int_{z}^{z_m} z b(z)\, dz$$

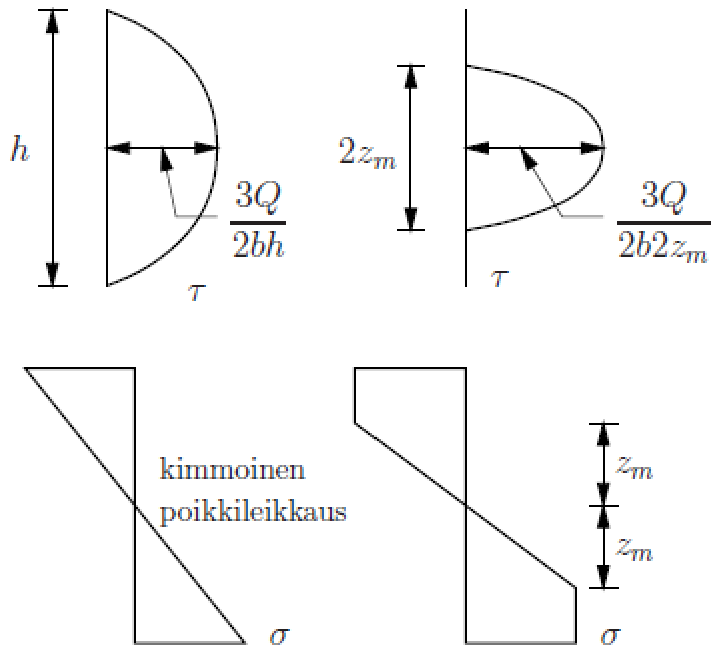
suorakaide, $N = 0$, $\varepsilon = 0$.

$$
\begin{aligned}
\tau(z) &= \frac{1}{b}\int_{z}^{z_m} \frac{d\sigma}{dx} b\, dz \\
&= \int_{z}^{z_m} E\frac{d\kappa}{dx} z\, dz \\
&= E\frac{d\kappa}{dx}\int_{z}^{z_m} z\, dz \\
&= E\frac{d\kappa}{dx}\frac{1}{2}\left(z_m^2 - z^2\right)
\end{aligned}
$$

$$
Q = \int_A \tau\, dA = E\frac{d\kappa}{dx}\frac{b}{2}\int_{z}^{z_m}\left(z_m^2 - z^2\right)\, dz = \frac{2}{3}E\frac{d\kappa}{dx} b z_m^3 \quad \Longrightarrow \quad E\frac{d\kappa}{dx} = \frac{3}{2}\frac{Q}{b z_m^3}
$$

$$\tau(z) = \frac{3}{4}\frac{Q}{bz_m^3}\left(z_m^2 - z^2\right) = \frac{3Q}{4bz_m}\left[1 - \left(\frac{z}{z_m}\right)^2\right]$$

$$\tau_{\max} = \tau(0) = \frac{3Q}{4bz_m}$$



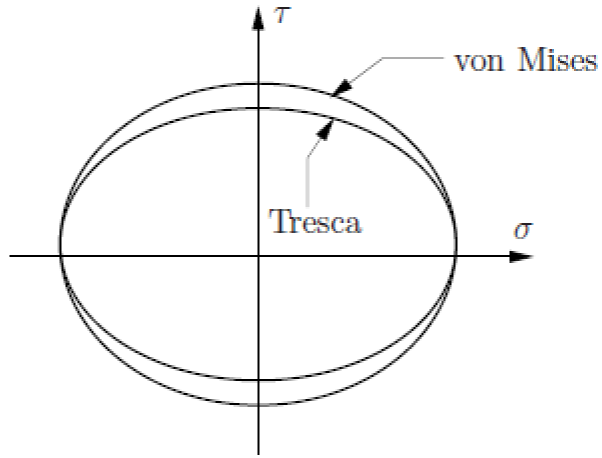
$$\tau_{\max} = \frac{3Q}{2bh}$$

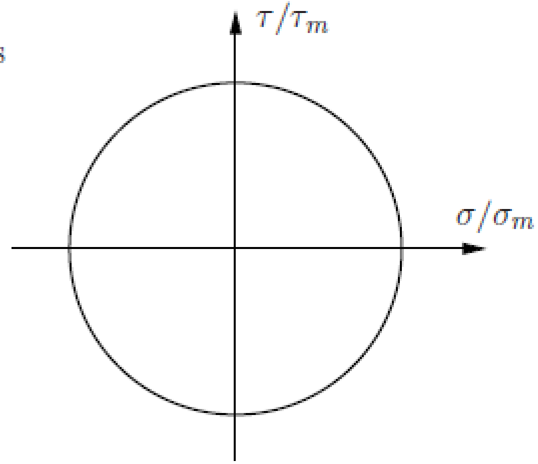
$$\tau_{\max} = \tau(0) = \frac{3Q}{4bz_m}$$

von Mises myötöehto: $\sqrt{\sigma^2 + 3\tau^2} = \sigma_m$

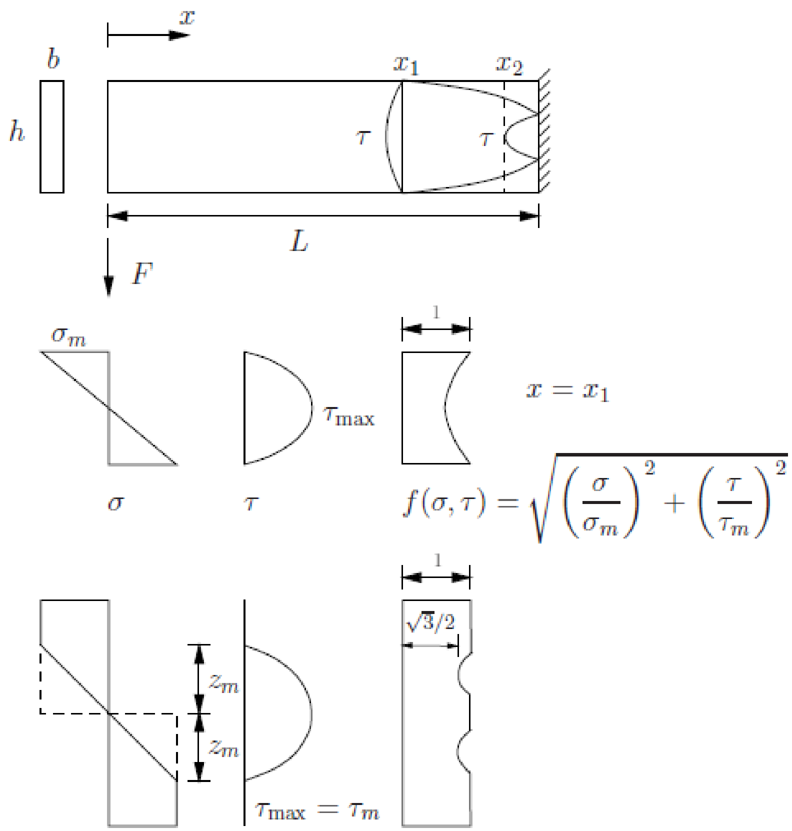
Trescan myötöehto: $\sqrt{\sigma^2 + 4\tau^2} = \sigma_m$

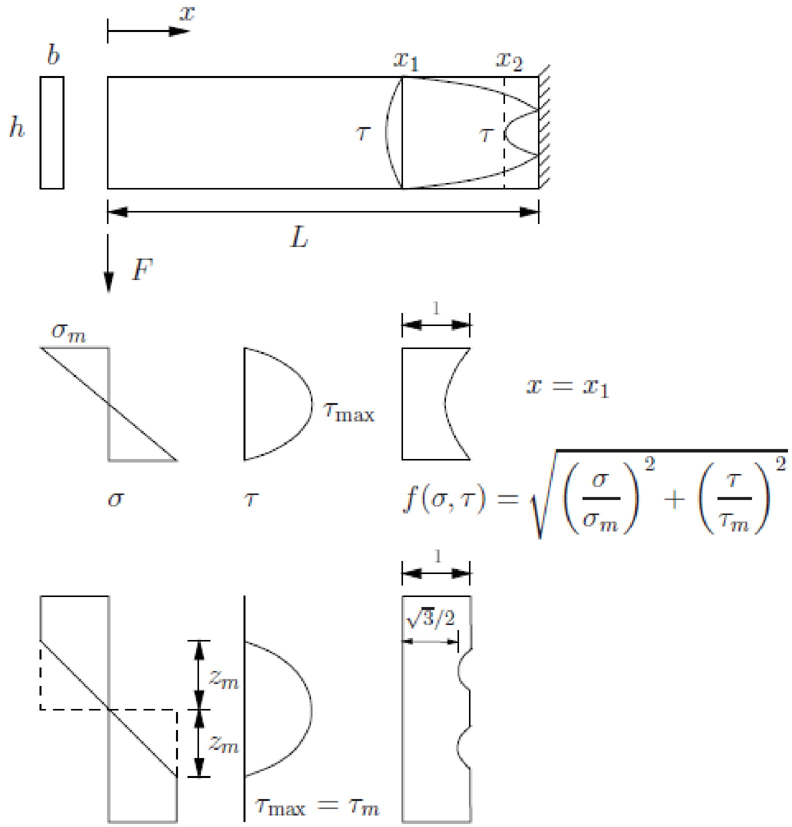
$$\left(\frac{\sigma}{\sigma_m}\right)^2 + \left(\frac{\tau}{\tau_m}\right)^2 = 1$$



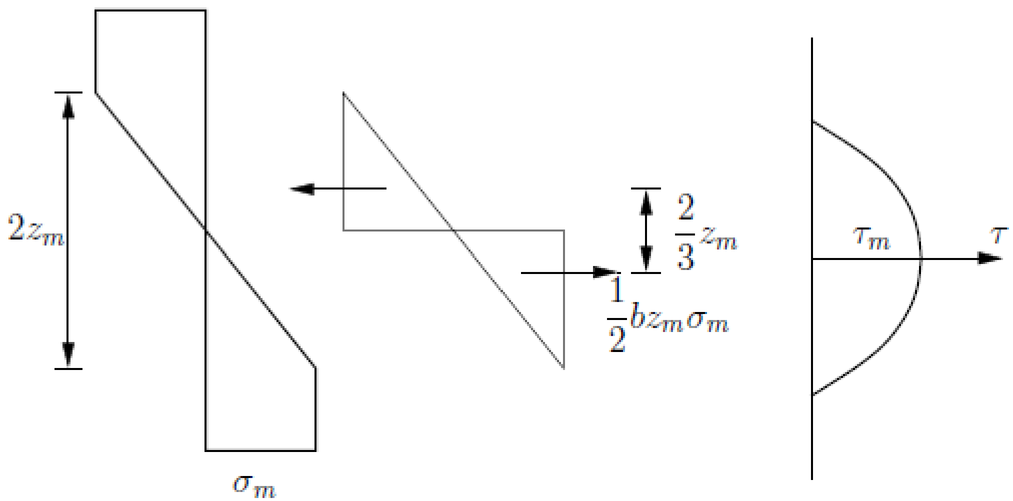


$$\left(\frac{\sigma}{\sigma_m}\right)^2 + \left(\frac{\tau}{\tau_m}\right)^2 = 1$$

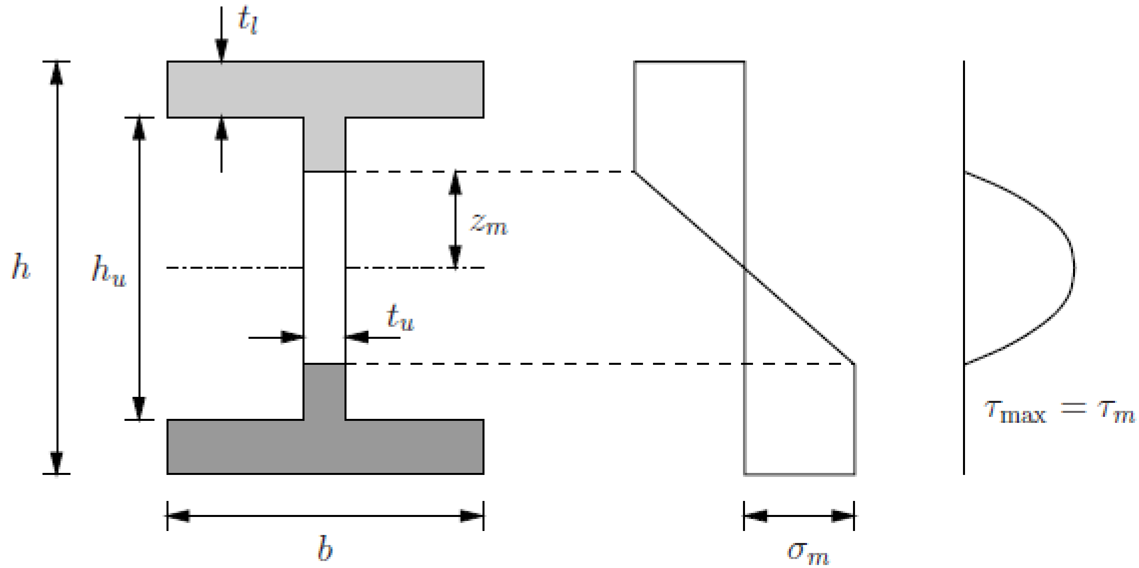

x
b
x1
x2
h
τ
τ
L
F
σm
1
τmax
x = x1
σ
τ
f(σ, τ) = √((σ/σm)² + (τ/τm)²)
1
√3/2
zm
zm
τmax = τm

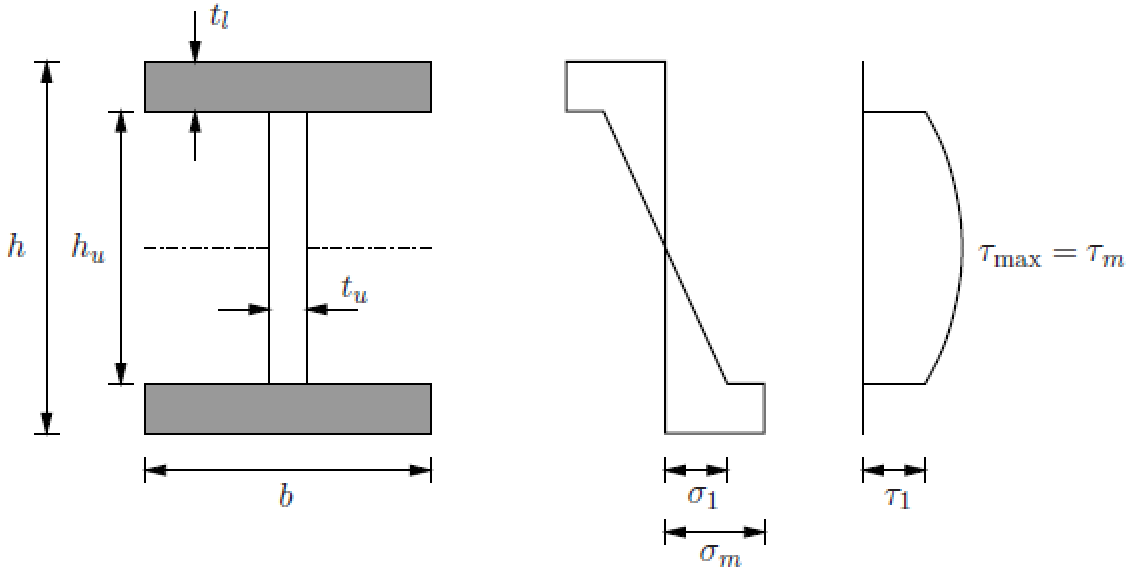


x
b
h
x_1
x_2
τ
τ
L
F
σ_m
$\tau_{\max}$
1
$x = x_1$
σ
τ
$f(\sigma,\tau) = \sqrt{\left(\frac{\sigma}{\sigma_m}\right)^2 + \left(\frac{\tau}{\tau_m}\right)^2}$
1
$\sqrt{3}/2$
z_m
z_m
$\tau_{\max} = \tau_m$

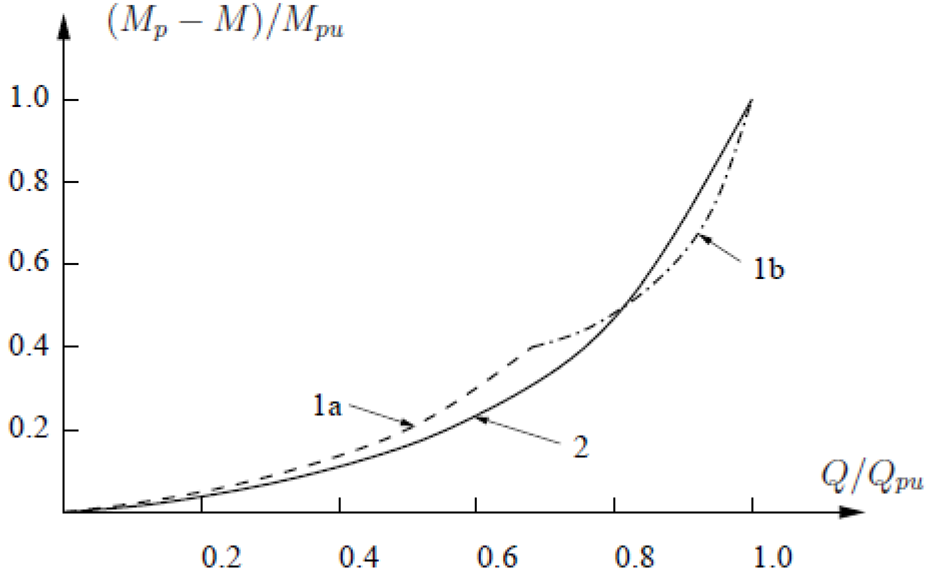


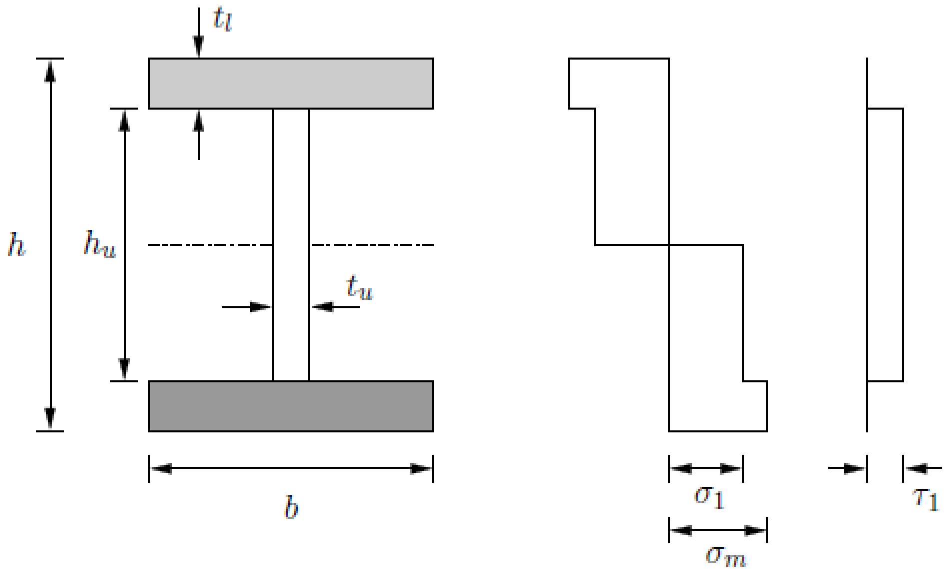
$2z_m$
σ_m
$\frac{2}{3}z_m$
$\frac{1}{2}bz_m\sigma_m$
τ_m
τ


t_l
h
h_u
z_m
t_u
b
σ_m
$\tau_{\max} = \tau_m$



t_l
h
h_u
t_u
b
$\tau_{\max} = \tau_m$
σ_1
σ_m
τ_1


$(M_p - M)/M_{pu}$
1.0
0.8
0.6
0.4
0.2
1b
1a
2
Q/Q_{pu}
0.2
0.4
0.6
0.8
1.0


t_l
h
h_u
t_u
b
σ_1
σ_m
τ_1