$$\varepsilon = \frac{u(x+\mathrm{d}x)-u(x)}{\mathrm{d}x} = \frac{\mathrm{d}u}{\mathrm{d}x}$$

$$\sigma = E\,\varepsilon$$

$$\Delta L = u(L) - u_0 = \frac{N\,L}{EA}$$

$$\bar{y}_\mathrm{N} = \frac{\sum (EA)_\mathrm{i}\,\bar{y}_\mathrm{i}}{\sum (EA)_\mathrm{i}} \qquad \bar{z}_\mathrm{N} = \frac{\sum (EA)_\mathrm{i}\,\bar{z}_\mathrm{i}}{\sum (EA)_\mathrm{i}}$$

$$\frac{\mathrm{d}Q}{\mathrm{d}x} = -q(x) \qquad \frac{\mathrm{d}M_\mathrm{t}}{\mathrm{d}x} = Q(x) \qquad \frac{\mathrm{d}^2 M_\mathrm{t}}{\mathrm{d}x^2} = -q(x)$$

$$N = \iint \sigma_x \,\mathrm{d}A \qquad M_{\mathrm{tz}} = \iint y\,\sigma_x\,\mathrm{d}A \qquad M_{\mathrm{ty}} = \iint z\,\sigma\,\mathrm{d}A$$

$$\sigma_x = \frac{y\,I_\mathrm{y} - z\,I_\mathrm{yz}}{I_\mathrm{y}\,I_\mathrm{z} - I_\mathrm{yz}^2}\,M_\mathrm{tz} + \frac{z\,I_\mathrm{z} - y\,I_\mathrm{yz}}{I_\mathrm{y}\,I_\mathrm{z} - I_\mathrm{yz}^2}\,M_\mathrm{ty}$$

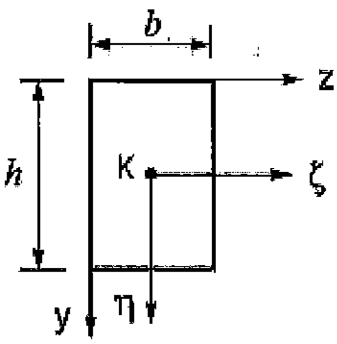
$$I_\mathrm{y} = \iint_A z^2\,\mathrm{d}A \qquad I_\mathrm{z} = \iint_A y^2\,\mathrm{d}A \qquad I_\mathrm{yz} = \iint_A yz\,\mathrm{d}A$$

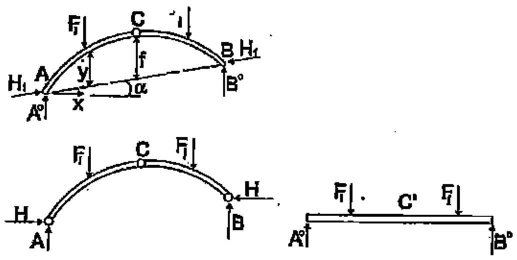
$$\widehat{EI}_\mathrm{z}\,v_{,xx} = -M_\mathrm{tz} \qquad \widehat{EI}_\mathrm{z} = \iint_A y^2\,E(y,z)\,\mathrm{d}A$$

$$\sigma_x(y,z) = E(y,z)\,\frac{M_\mathrm{tz}}{\widehat{EI}_\mathrm{z}}\,y \qquad \sigma_x(y,z) = E(y,z)\,\frac{N}{\widehat{EA}}$$

$$I_\zeta = \tfrac{1}{12} b h^3$$

$$I_\eta = \tfrac{1}{12} b^3 h$$





Kaaren rasitussuureet:

$$M(x) = M^{o}(x) - H_1 * y * \cos\alpha$$
$$Q(x) = Q^{o}(x) * \cos\varphi - H_1 * \sin(\varphi - \alpha)$$
$$N(x) = -Q^{o}(x) * \sin\varphi - H_1 * \cos(\varphi - \alpha)$$

$$\begin{cases} M(x) = M^{o}(x) - H * y \\ Q(x) = Q^{o}(x) * \cos\varphi - H_1 \sin(\varphi - \alpha) \\ N(x) = -Q^{o}(x) * \sin\varphi - H_1 * \cos(\varphi - \alpha) \end{cases}$$

Jos kulma α on nolla eli tuet A ja B ovat samalla tasolla saadaan:

$H = H_1 = M_c^{o} / f \qquad A = A^{o} \quad$ ja $\quad B = B^{o}$

$$\begin{cases} M(x) = M^{o}(x) - H * y \\ Q(x) = Q_{o}(x) * \cos\varphi - H * \sin\varphi. \\ N(x) = -Q^{o}(x) * \sin\varphi - H * \cos\varphi \end{cases}$$

Muista kuitenkin, että aina pärjää vapaakappale-kuvilla ja tasapainoehdoilla.

$$\frac{\mathrm{d}^2 y}{\mathrm{d} x^2} = \frac{q(x)}{H}$$

$$W = \mathbf{F} \bullet \mathbf{u} = F_x\, u + F_y\, v + F_z\, w$$

$$y(x) = \frac{q_0}{2H} x^2$$

$$\delta W = \mathbf{F} \bullet \delta \mathbf{u}$$

$$\delta W_{\mathrm{q}} = \int_0^L \mathbf{q} \bullet \delta \mathbf{u}\, \mathrm{d}x = \int_0^L (q_x\, \delta u + q_y\, \delta v)\, \mathrm{d}x$$

$$\frac{\mathrm{d}^2 y}{\mathrm{d} x^2} = \frac{q_0}{H} \sqrt{1 + (\mathrm{d}y/\mathrm{d}x)^2}$$

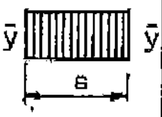
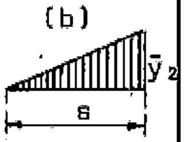
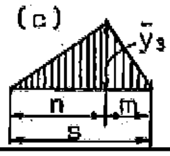
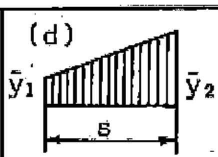
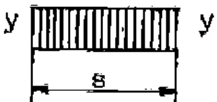
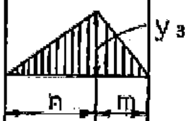
$$\delta W_{\text{ulk}} + \delta W_{\text{sis}} = 0, \ \forall \delta v$$

$$\widehat{EI}_{y} = \sum_{i=1}^{n} E_i (I_{y_i} + \bar{z}_i^2 A_i) \qquad \widehat{EI}_{yz} = \sum_{i=1}^{n} E_i (I_{y_i z_i} + \bar{y}_i \bar{z}_i A_i); \qquad \widehat{EI}_{z} = \sum_{i=1}^{n} E_i (I_{z_i} + \bar{y}_i^2 A_i);$$

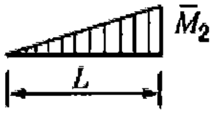
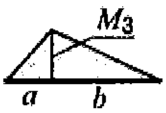
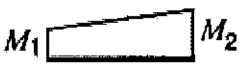
$$\sigma_x(y,z) = E_i \frac{N}{\widehat{EA}} + E_i \frac{\widehat{EI}_y M_{tz} - \widehat{EI}_{yz} M_{ty}}{\widehat{EI}_y \widehat{EI}_z - \widehat{EI}_{yz}^{\;2}} y_i + E_i \frac{\widehat{EI}_z M_{ty} - \widehat{EI}_{yz} M_{tz}}{\widehat{EI}_y \widehat{EI}_z - \widehat{EI}_{yz}^{\;2}} z_i \qquad \widehat{EA} = \sum_{i=1}^{n} E_i A_i$$

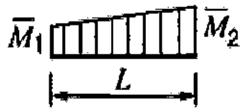
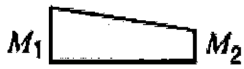
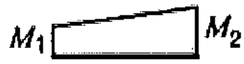
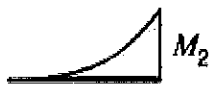
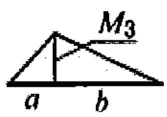
$\Delta L \approx \alpha L \Delta T$

<u>Taulukko 7.1</u> $\int_0^s M\bar{M}ds$

M̄-pinta / M-pinta	(a)	(b)	(c)	(d)
				
(1) 	$s\cdot y\cdot\bar{y}$	$\frac{1}{2}s\cdot y\cdot\bar{y}_2$	$\frac{1}{2}\cdot s\cdot y\cdot\bar{y}_3$	$\frac{1}{2}sy(\bar{y}_1+\bar{y}_2)$
(2) 	$\frac{1}{2}\cdot s\cdot y_2\cdot\bar{y}$	$\frac{1}{3}s\cdot y_2\cdot\bar{y}_2$	$\frac{1}{6}(s+n)y_2\bar{y}_3$	$\frac{1}{6}sy_2\cdot(\bar{y}_1+2\bar{y}_2)$
(3) 	$\frac{1}{2}\cdot s\cdot y_1\cdot\bar{y}$	$\frac{1}{6}\cdot s\cdot y_1\cdot\bar{y}_2$	$\frac{1}{6}(s+m)y_1\bar{y}_3$	$\frac{1}{6}\cdot s\cdot y_1\cdot(2\bar{y}_1+\bar{y}_2)$
(4) 	$\frac{1}{2}\cdot s\cdot y_3\cdot\bar{y}$	$\frac{1}{6}(s+n)y_3\bar{y}_2$	$\frac{1}{3}\cdot s\cdot y_3\cdot\bar{y}_3$	$\frac{1}{6}y_3\cdot[(s+m)\bar{y}_1+(s+n)\bar{y}_2]$
(5) 	$\frac{s}{2}(y_1+y_2)\bar{y}$	$\frac{s}{6}(y_1+2y_2)\bar{y}_2$	$\frac{\bar{y}_3}{6}[(s+m)y_1+(s+n)y_2]$	$\frac{s}{6}[y_1(2\bar{y}_1+\bar{y}_2)+y_2(\bar{y}_1+2\bar{y}_2)]$
(6) 	$\frac{2}{3}\cdot s\cdot y_3\cdot\bar{y}$	$\frac{1}{3}sy_3\bar{y}_2$	$\frac{y_3\bar{y}_3}{3s}(s^2+nm)$	$\frac{sy_3}{3}(\bar{y}_1+\bar{y}_2)$
(7) 	$\frac{2}{3}\cdot s\cdot y_2\cdot\bar{y}$	$\frac{5}{12}\cdot s\cdot y_2\cdot\bar{y}_2$	$\frac{y_2\bar{y}_3}{12\cdot s}\cdot(5s^2-ms-m^2)$	$\frac{sy_2}{12}(3\bar{y}_1+5\bar{y}_2)$
(8) 	$\frac{2}{3}\cdot s\cdot y_1\cdot\bar{y}$	$\frac{1}{4}\cdot s\cdot y_1\cdot\bar{y}_2$	$\frac{y_1\bar{y}_3}{12\cdot s}\cdot(5s^2-ns-n^2)$	$\frac{sy_1}{12}(5\bar{y}_1+3\bar{y}_2)$
(9) 	$\frac{1}{3}\cdot s\cdot y_2\cdot\bar{y}$	$\frac{1}{4}\cdot s\cdot y_2\cdot\bar{y}_2$	$\frac{y_2\bar{y}_3}{12s}\cdot(s^2+ns+n^2)$	$\frac{sy_2}{12}(\bar{y}_1+3\bar{y}_2)$
(10) 	$\frac{1}{3}\cdot s\cdot y_1\cdot\bar{y}$	$\frac{1}{12}\cdot s\ y_1\cdot\bar{y}_2$	$\frac{y_1\bar{y}_3}{12\cdot s}\cdot(s^2+ms+m^2)$	$\frac{sy_1}{12}\cdot(3\bar{y}_1+\bar{y}_2)$
(11) $\int\bar{M}^2ds$	$s\cdot\bar{y}^2$	$\frac{1}{3}\cdot s\cdot\bar{y}_2^2$	$\frac{1}{3}\cdot s\cdot\bar{y}_3^2$	$\frac{1}{3}s(\bar{y}_1^2+\bar{y}_1\bar{y}_2+\bar{y}_2^2)$

Taulukko 1 MOHRin integraalitaulukot

		$\int_0^L \overline{M}\, M\, \mathrm{d}x$
1		$\frac{1}{3} L \overline{M}_2 M_2$
2		$\frac{1}{6} L \overline{M}_2 M_1$
3		A
4		$\frac{1}{6} L \overline{M}_2 (M_1 + 2 M_2)$
5		$\frac{1}{2} L \overline{M}_2 M_1$
6		$\frac{5}{12} L \overline{M}_2 M_2$
7		$\frac{1}{4} L \overline{M}_2 M_1$
8		$\frac{1}{4} L \overline{M}_2 M_2$
9		$\frac{1}{12} L \overline{M}_2 M_1$
10		$\frac{1}{3} L \overline{M}_2 M_3$

		$\int_0^L \overline{M}\, M\, \mathrm{d}x$
1		$\frac{1}{2} L (\overline{M}_1 + \overline{M}_2) M_1$
2		B
3		C
4		D
5		E
6		$\frac{1}{3} L (\overline{M}_1 + \overline{M}_2) M_3$
7		F

$$A = \tfrac{1}{6} L \overline{M}_2 M_3 (1 + a/L)$$

$$B = \tfrac{1}{6} L \left[\overline{M}_1 (2 M_1 + M_2) + \overline{M}_2 (M_1 + 2 M_2) \right]$$

$$C = \tfrac{1}{3} L (M_1^2 + M_1 M_2 + M_2^2)\,, \quad M = \overline{M}$$

$$D = \tfrac{1}{12} L (3 \overline{M}_1 + 5 \overline{M}_2) M_2$$

$$E = \tfrac{1}{12} L (\overline{M}_1 + 3 \overline{M}_2) M_2$$

$$F = \tfrac{1}{6} \left[\overline{M}_1 (a + 2b) + \overline{M}_2 (2a + b) \right] M_3$$

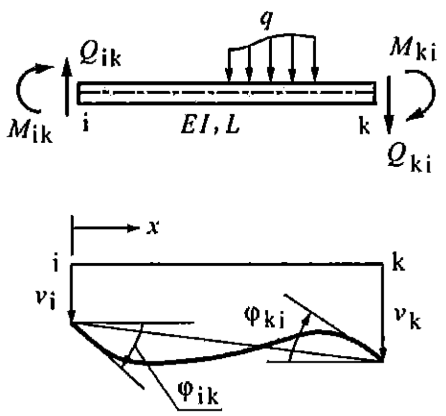
5.2.2 Joustokertoimien menetelmä

$$\{\hat{u}\} = [a]\{X\} + \{u_0\}$$

$$a_{ij} = \int_s \frac{\overline{M}_{ti}\,\overline{M}_{tj}}{EI}\,ds$$

$$u_{i0} = \int_s \frac{M_{t0}\,\overline{M}_{ti}}{EI}\,ds$$

5.3 Momentti-siirtymämenetelmä



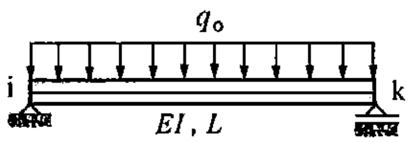
$$\varphi_{ik} = \alpha_{ik} M_{ik} - \beta_{ik} M_{ki} + (v_k - v_i)/L + \overline{\alpha}_{ik}$$

$$\varphi_{ki} = -\beta_{ki} M_{ik} + \alpha_{ki} M_{ki} + (v_k - v_i)/L + \overline{\alpha}_{ki}$$

$$Q_{ik} = -(M_{ik} + M_{ki})/L + \overline{Q}_{ik}$$

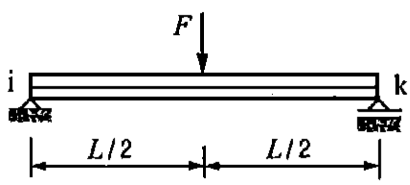
$$Q_{ki} = -(M_{ik} + M_{ki})/L + \overline{Q}_{ki}$$

$$\alpha_{ik} = \alpha_{ki} = \frac{L}{3EI} \qquad \beta_{ik} = \beta_{ki} = \frac{L}{6EI}$$



$$\overline{\alpha}_{ik} = -\overline{\alpha}_{ki} = \frac{q_0 L^3}{24\,EI}$$

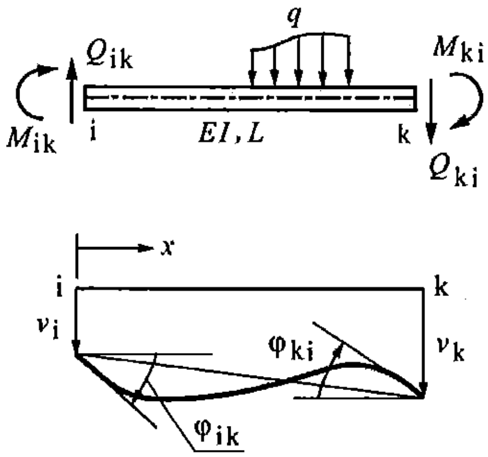
(a)



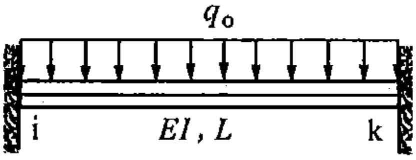
$$\overline{\alpha}_{ik} = -\overline{\alpha}_{ki} = \frac{F L^2}{16\,EI}$$

(b)

5.4 Siirtymämenetelmä

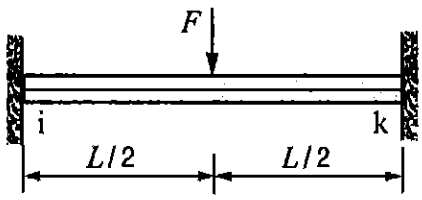


$M_{ik} = a_{ik}\,\varphi_{ik} + b_{ik}\,\varphi_{ki} - c_{ik}\,(v_k - v_i) + \overline{M}_{ik}$ $M_{ki} = b_{ki}\,\varphi_{ik} + a_{ki}\,\varphi_{ki} - c_{ki}\,(v_k - v_i) + \overline{M}_{ki}$	
$Q_{ik} = -c_{ik}\,(\varphi_{ik} + \varphi_{ki}) + d_{ik}\,(v_k - v_i) + \overline{Q}_{ik}$ $Q_{ki} = -c_{ki}\,(\varphi_{ik} + \varphi_{ki}) + d_{ki}\,(v_k - v_i) + \overline{Q}_{ki}$	
$a_{ik} = a_{ki} = 4\,EI/L$	$b_{ik} = b_{ki} = 2\,EI/L$
$c_{ik} = c_{ki} = 6\,EI/L^2$	$d_{ik} = d_{ki} = 12\,EI/L^3$



$$\overline{M}_{ik} = -\overline{M}_{ki} = -\frac{q_0 L^2}{12}$$

$$\overline{Q}_{ik} = -\overline{Q}_{ki} = \frac{q_0 L}{2}$$

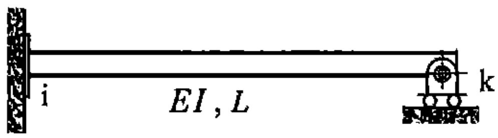


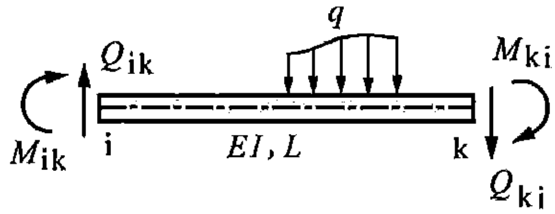
(b)

$$\overline{M}_{ik} = -\overline{M}_{ki} = -\frac{F L}{8}$$

$$\overline{Q}_{ik} = -\overline{Q}_{ki} = \frac{F}{2}$$

5.4.3 Toisesta päästään nivelkiinnitetty palkki





Kuva 1 Jatkuva palkki

<table>
<tr><td colspan="2">$M_{ik} = a_{ik}^{o}\,\varphi_{ik} - c_{ik}^{o}\,(v_k - v_i) + \overline{M}_{ik}^{o}$</td></tr>
<tr><td colspan="2">$Q_{ik} = -c_{ik}^{o}\,\varphi_{ik} + d_{ik}^{o}\,(v_k - v_i) + \overline{Q}_{ik}^{o}$
$Q_{ki} = -c_{ki}^{o}\,\varphi_{ki} + d_{ki}^{o}\,(v_k - v_i) + \overline{Q}_{ki}^{o}$</td></tr>
<tr><td>$a_{ik}^{o} = 3\,EI/L$</td><td>$c_{ik}^{o} = c_{ki}^{o} = 3\,EI/L^2$</td></tr>
<tr><td>$d_{ik}^{o} = d_{ki}^{o} = 3\,EI/L^3$</td><td></td></tr>
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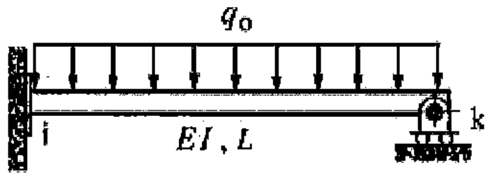
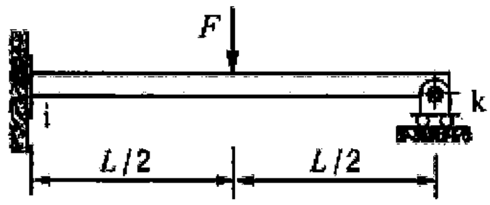


(a)

$$\overline{M}_{ik}^{o} = -\frac{q_0 L^2}{8}$$

$$\overline{Q}_{ik}^{o} = \frac{5 q_0 L}{8}\ , \quad \overline{Q}_{ki}^{o} = -\frac{3 q_0 L}{8}$$



(b)

$$\overline{M}_{ik}^{o} = -\frac{3FL}{16}$$

$$\overline{Q}_{ik}^{o} = \tfrac{11}{16} F\ , \quad \overline{Q}_{ki}^{o} = -\tfrac{5}{16} F$$