ON THE STABILITY OF TRUSS BEAMS

J. Hartikainen, P. Hassinen, R. Kouhia, T. Manninen
TKK, Structural Mechanics

International Convention in Steel Construction
13–14 September 2006, HAMK, Hämeenlinna

HELSINKI UNIVERSITY OF TECHNOLOGY
Laboratory of Stuctural Mechanics

OUTLINE

SPECIFIC STRUCTURE

PROBLEM

STABILITY ANALYSIS

EIGENVALUE ANALYSIS

SIMPLE EXAMPLE CASE

CONCLUSIONS

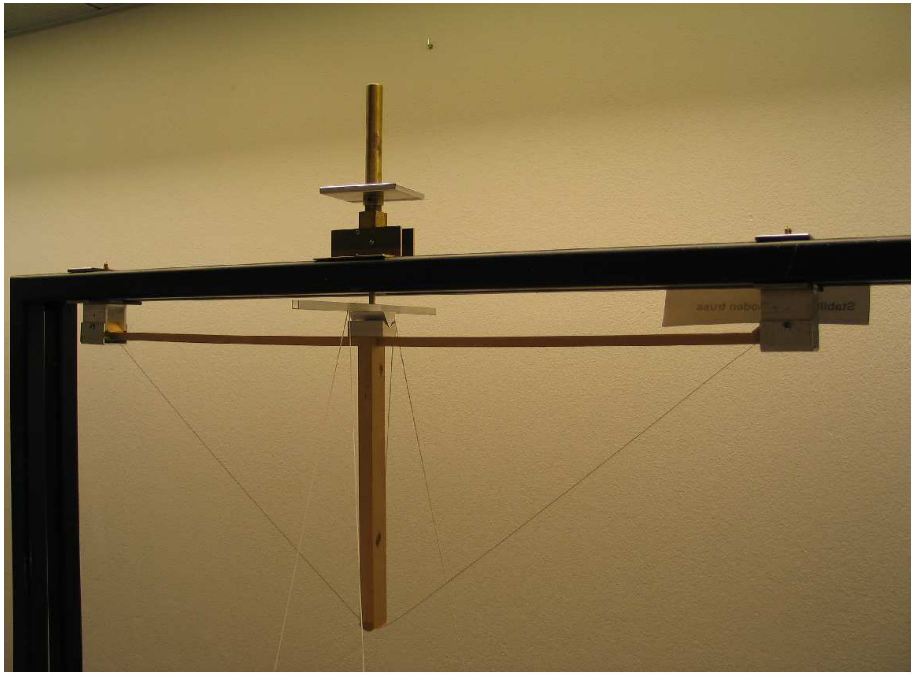
SPECIFIC STRUCTURE

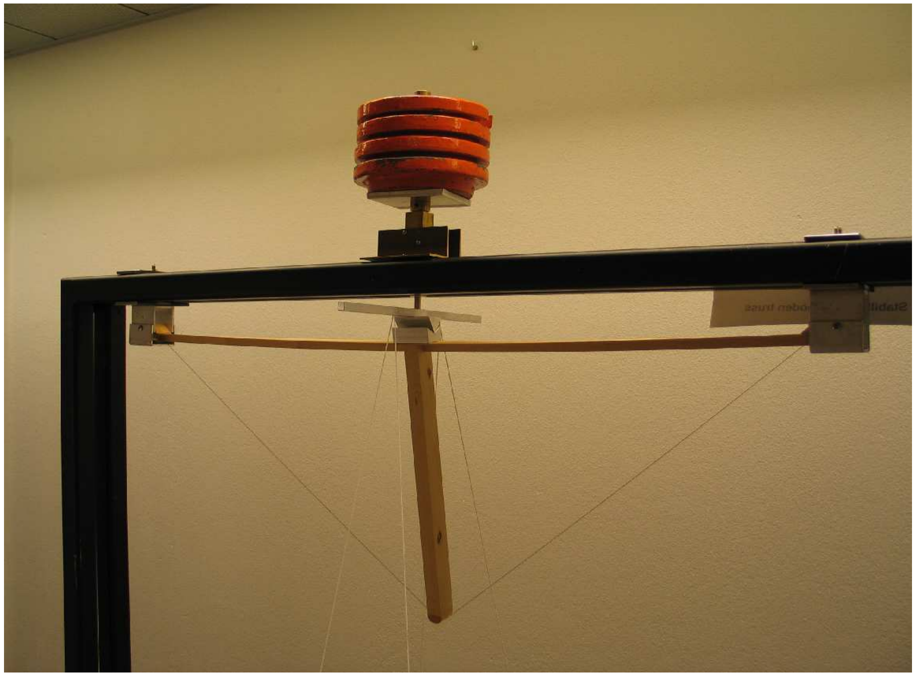
Source: www.ppth.fi

If the lower chord is unsupported in lateral direction, the compressed verticals/diagonals can buckle as a rigid body.

PROBLEM

- What are the relevant buckling modes for truss beams?
- Can they be adequately analysed by common computational tools?
- If not, how to analyse?





STABILITY ANALYSIS

Definition for critical state: Find displacements $\boldsymbol{q}_{\mathrm{cr}}$, critical load λ_{cr} and the corresponding eigenmode ϕ such, that

$$\boldsymbol{f}'(\boldsymbol{q}_{\mathrm{cr}}, \lambda_{\mathrm{cr}})\boldsymbol{\phi} = \boldsymbol{0} \quad \text{and} \quad \boldsymbol{f}(\boldsymbol{q}_{\mathrm{cr}}, \lambda_{\mathrm{cr}}) = \boldsymbol{0}, \tag{1}$$

where $\boldsymbol{f}' = \partial \boldsymbol{f} / \partial \boldsymbol{q}$. *The non-linear* mapping $\boldsymbol{f}$ defines the equilibrium path in the displacement $\boldsymbol{q}$ and load parameter λ space:

$$\boldsymbol{f}(\boldsymbol{q}, \lambda) \equiv \boldsymbol{r}(\boldsymbol{q}) - \lambda \boldsymbol{p}_r(\boldsymbol{q}) = \boldsymbol{0} \tag{2}$$

and constitutes the balance between internal- and external forces.

System (1) is a non-linear eigenvalue problem, which is HARD TO SOLVE!

EIGENVALUE ANALYSIS

Expanding the nl-ev-problem into Taylor's series wrt the state $(\boldsymbol{q}_*, \lambda_*)$

$$\boldsymbol{q} = \boldsymbol{q}_* + \Delta\lambda \boldsymbol{q}_1 + \frac{1}{2}(\Delta\lambda)^2 \boldsymbol{q}_2 + \cdots$$

results in a polynomial ev-problem:

$$\left(\boldsymbol{K}_{0|*} + \Delta\lambda \boldsymbol{K}_{1|*} + \frac{1}{2}(\Delta\lambda)^2 \boldsymbol{K}_{2|*} + \cdots\right) \boldsymbol{\phi} = \boldsymbol{0}$$

$$\boldsymbol{K}_{0|*} = \boldsymbol{f}'_*,$$

$$\boldsymbol{K}_{1|*} = \left.\frac{\mathrm{d}\boldsymbol{f}}{\mathrm{d}\lambda}\right|_* = \boldsymbol{f}''_* \boldsymbol{q}_1 + \dot{\boldsymbol{f}}'_*, \qquad \dot{\boldsymbol{f}} = \frac{\partial \boldsymbol{f}}{\partial \lambda},$$

$$\boldsymbol{K}_{2|*} = \left.\frac{\mathrm{d}^2\boldsymbol{f}}{\mathrm{d}\lambda^2}\right|_* = \boldsymbol{f}''_* \boldsymbol{q}_2 + \boldsymbol{f}'''_* \boldsymbol{q}_1 \boldsymbol{q}_1 + 2\dot{\boldsymbol{f}}''_* \boldsymbol{q}_1 + \ddot{\boldsymbol{f}}'_*$$

EIGENVALUE ANALYSIS (cont.)

In the linear stability eigenvalue analysis the reference state is usually the initial state: $(\boldsymbol{q}_*, \lambda_*) = (\mathbf{0}, 0)$.[1] The eigenvalue problem to be solved is

$$\left(\boldsymbol{K}_{0|0} + \lambda \boldsymbol{K}_{1|0}\right) \boldsymbol{\phi} = \mathbf{0}$$

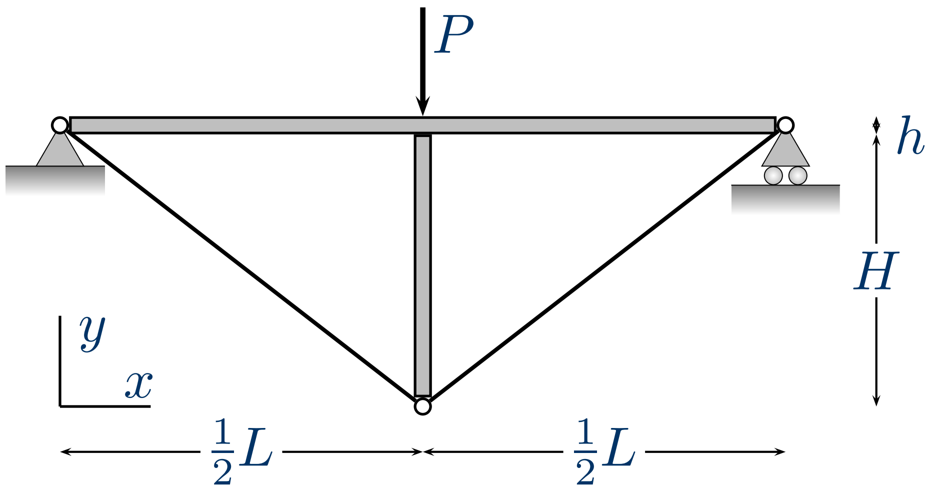
where the matrices are (assuming dead weight loading, i.e. $\dot{\boldsymbol{f}}' \equiv \mathbf{0}$)

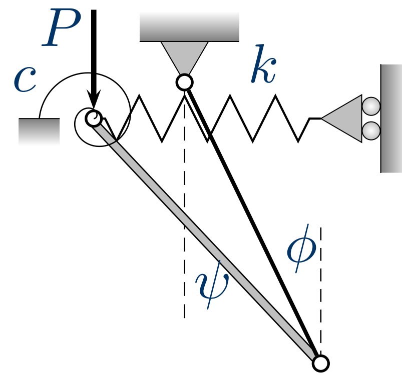
$$\begin{aligned} \boldsymbol{K}_{0|0} &= \boldsymbol{f}'(\mathbf{0}, 0) \\ \boldsymbol{K}_{1|0} &= \boldsymbol{f}''(\mathbf{0}, 0)\boldsymbol{q}_1, \end{aligned}$$

and the pre-buckling displacement field, $\boldsymbol{q}_1$, is solved from $\boldsymbol{K}_{0|0}\boldsymbol{q}_1 = \boldsymbol{p}_r$.

[1] This is usually the case in commercial FE software.

SIMPLE EXAMPLE CASE





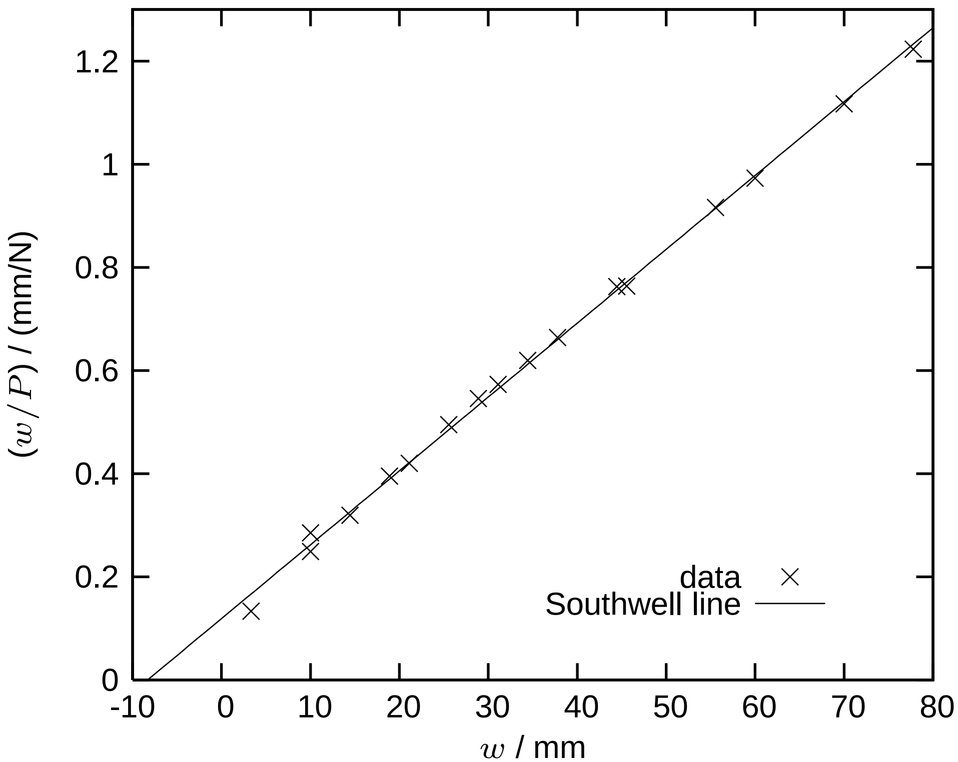
$c = 1310\,\mathrm{Nmm}$
$k = 3.66\,\mathrm{N/mm}^2$
$L = 930\,\mathrm{mm}$
$H = 360\,\mathrm{mm}$
$h = 10\,\mathrm{mm}$

Upper chord unsupported in lateral direction

Critical load P_{cr} in N		
2 dof model	FEM	experiment
61.4	60.6	69.8

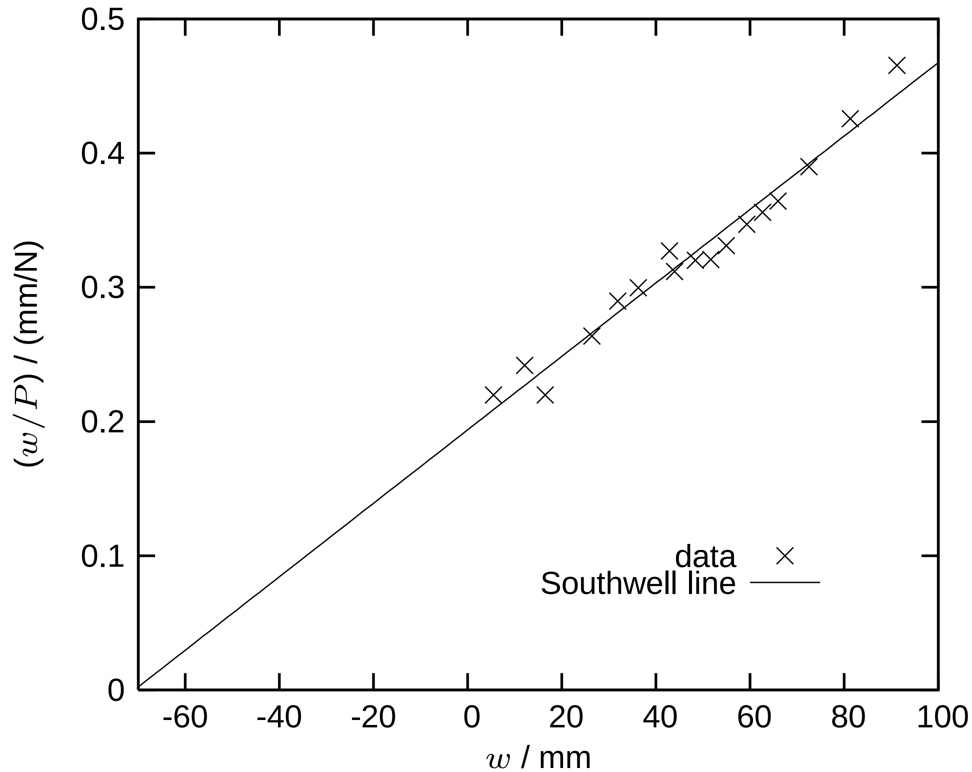
Southwell plot

Upper chord free in lateral direction



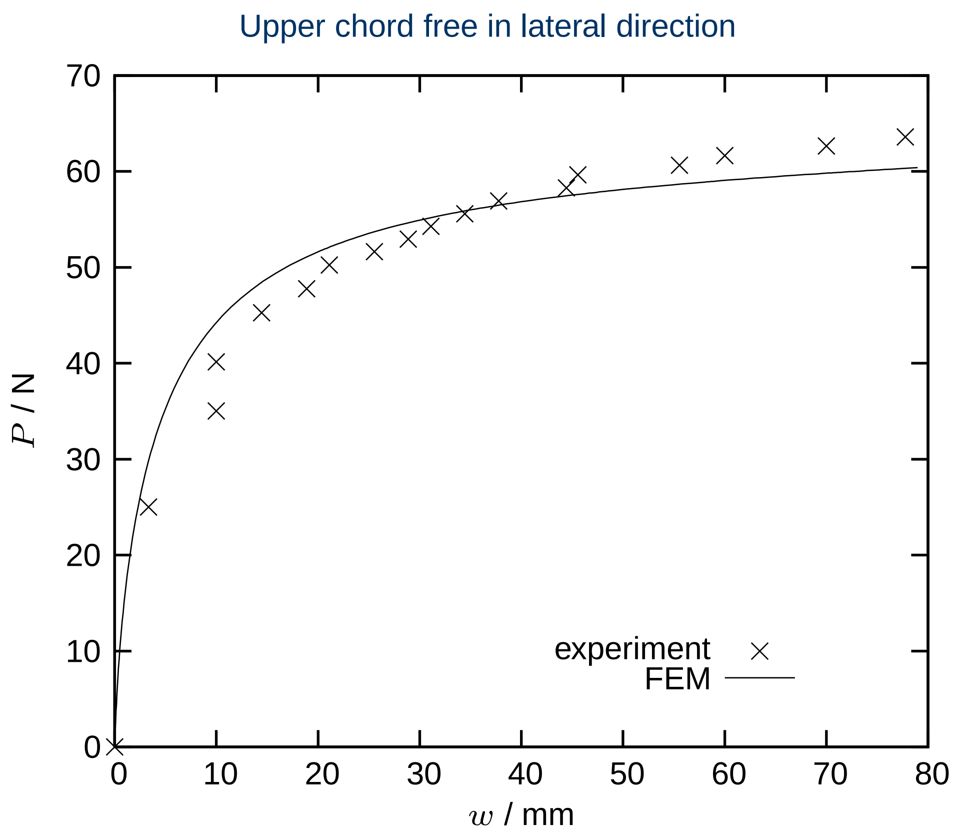
$P_{\text{cr}} = 69.8$ N, $w_0 = 8$ mm

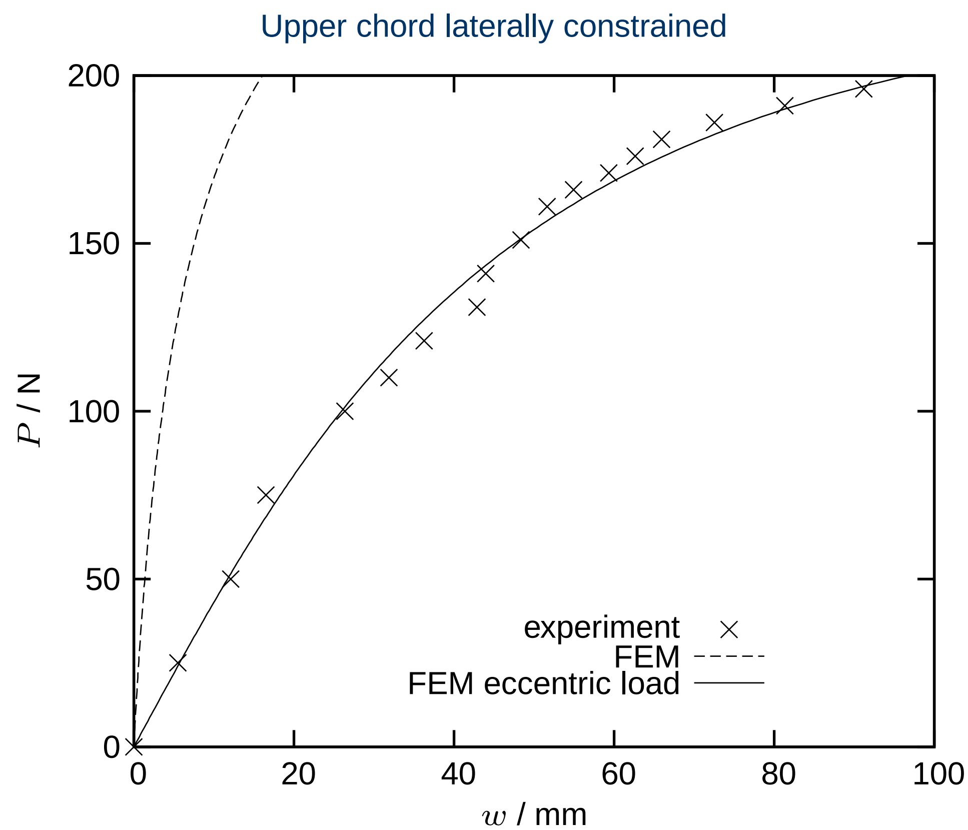
Upper chord laterally constrained



$P_{\text{cr}} = 365.4$ N, $w_0 = 70$ mm

Load-displacement curves





CONCLUSIONS

- Vertical/diagonal compression members can buckle as a rigid body if the torsional rigidity of the upper chord is low and the lower chord is unsupported in lateral direction.

- If the upper chord is laterally supported, such a stability phenomenon cannot be analysed by the linear buckling eigenvalue problem linearised wrt the initial state.

- Since commercial FE programs do not have quadratic buckling eigenvalue analysis routines, the problem has to be analysed by using full non-linear analysis.

- In the full non-linear analysis special emphasis has to be paid on selecting the proper imperfection. The “eigenmode injection” from the linearised buckling analysis do not provide the correct mode.