Implementation of a continuum damage model for creep fracture and fatigue analyses to ANSYS

Petteri Kauppila[1], Reijo Kouhia[2], Juha Ojanperä[1], Timo Saksala[2], Timo Sorjonen[1]

[1] Valmet Technologies Oy, P.O. Box 109, FI 33101 Tampere, Finland
[2] Tampere University, Civil Engineering, Structural Mechanics, P.O. Box 600, FI-33014 Tampere University

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Introduction

- Sustainable energy system is a combination of wide variety of energy resources.
- Result in flexible power generation.
- New requirements for boiler creep fatigue design due to intermittent power demand.

Thermodynamic formulation

Developed models are completely defined by two potential functions:
the **specific Helmholtz free energy** $\psi = \psi(T, \boldsymbol{\varepsilon}_{\mathrm{te}}, \omega)$,
(linear kinematics assumed $\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_{\mathrm{e}} + \boldsymbol{\varepsilon}_{\mathrm{c}} + \boldsymbol{\varepsilon}_{\mathrm{th}}$, $\boldsymbol{\varepsilon}_{\mathrm{te}} = \boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_{\mathrm{c}}$, $\omega = 1 - D$)
and the **complementary dissipation potential** $\varphi(Y, \boldsymbol{q}, \boldsymbol{\sigma}; T, \omega)$ defined as

$$\gamma = \frac{\partial \varphi}{\partial \boldsymbol{q}} \cdot \boldsymbol{q} + \frac{\partial \varphi}{\partial \boldsymbol{\sigma}} : \boldsymbol{\sigma} + \frac{\partial \varphi}{\partial Y} Y.$$

Together with the Clausius-Duhem inequality

$$\gamma = -\rho(\dot{\psi} + s\dot{T}) + \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}} - T^{-1}\, \mathrm{grad} T \cdot \boldsymbol{q} \geq 0$$

results the constitutive equations

$$-\rho\left(s + \frac{\partial \psi}{\partial T}\right)\dot{T} + \left(\boldsymbol{\sigma} - \rho\frac{\partial \psi}{\partial \boldsymbol{\varepsilon}_{\mathrm{te}}}\right) : \dot{\boldsymbol{\varepsilon}}_{\mathrm{te}} + \left(\dot{\boldsymbol{\varepsilon}}_{\mathrm{c}} - \frac{\partial \varphi}{\partial \boldsymbol{\sigma}}\right) : \boldsymbol{\sigma}$$
$$- \left(\dot{\omega} + \frac{\partial \varphi}{\partial Y}\right) Y - \left(\frac{\mathrm{grad} T}{T} + \frac{\partial \varphi}{\partial \boldsymbol{q}}\right) \cdot \boldsymbol{q} = 0.$$

Consistently coupled heat equation

From the energy balance equation (TD1):

$$\rho c_\varepsilon \dot{T} = -\operatorname{div}\boldsymbol{q} + \rho r + \boldsymbol{\sigma}\boldsymbol{:}\dot{\boldsymbol{\varepsilon}}_{\mathrm{c}} + \rho \frac{\partial^2 \psi}{\partial \boldsymbol{\varepsilon}_{\mathrm{te}} \partial T} \dot{\boldsymbol{\varepsilon}}_{\mathrm{te}} + \left(\rho \frac{\partial^2 \psi}{\partial \omega \partial T} - Y \right) \dot{\omega},$$

where the specific heat capacity c_ε is defined as

$$c_\varepsilon = -T \frac{\partial^2 \psi}{\partial T^2}.$$

Specific models

The specific Helmholtz free energy

$$\rho\psi = \rho c_\varepsilon \left(T - T \ln \frac{T}{T_\mathrm{r}} \right) + \frac{1}{2} (\boldsymbol{\varepsilon}_\mathrm{te} - \boldsymbol{\varepsilon}_\mathrm{th}) \mathbin{:} \omega \boldsymbol{C}_\mathrm{e} \mathbin{:} (\boldsymbol{\varepsilon}_\mathrm{te} - \boldsymbol{\varepsilon}_\mathrm{th}),$$

$\boldsymbol{\varepsilon}_\mathrm{th} = \boldsymbol{\alpha}(T - T_\mathrm{r})$, thermal strain, $\boldsymbol{C}_\mathrm{e}$ elasticity tensor, $\boldsymbol{\alpha}$ thermal expansion coefficients, T_r stress free reference temperature.

The complementary dissipation potential

$$\varphi(Y, \boldsymbol{q}, \boldsymbol{\sigma}; T, \omega) = \varphi_\mathrm{th}(\boldsymbol{q}; T) + \varphi_\mathrm{d}(Y; T, \omega) + \varphi_\mathrm{c}(\boldsymbol{\sigma}; T, \omega),$$

where the thermal part is

$$\varphi_\mathrm{th}(\boldsymbol{q}; T) = \frac{1}{2} T^{-1} \boldsymbol{q} \cdot \boldsymbol{\lambda}^{-1} \boldsymbol{q}.$$

For creep the following Norton type potential function is adopted

$$\varphi_\mathrm{c}(\boldsymbol{\sigma}; T, \omega) = \frac{h_\mathrm{c}(T)}{p+1} \frac{\omega \sigma_\mathrm{rc}}{t_\mathrm{c}} \left(\frac{\bar{\sigma}}{\omega \sigma_\mathrm{rc}} \right)^{p+1},$$

$\bar{\sigma} = \sqrt{3J_2}, \quad h_\mathrm{c}(T) = \exp(-Q_\mathrm{c}/RT).$

Damage potential

Kachanov-Rabotnov type

$$\varphi_{\mathrm{d}}(Y;T,\omega) = \frac{h_{\mathrm{d}}(T)}{r+1}\frac{Y_{\mathrm{r}}}{t_{\mathrm{d}}\,\omega^k}\left(\frac{Y}{Y_{\mathrm{r}}}\right)^{r+1}, \qquad \text{model 1}$$

$$\varphi_{\mathrm{d}}(Y;T,\omega) = \frac{h_{\mathrm{c}}(T)}{(\frac{1}{2}p+1)(1+k+p)}\frac{Y_{\mathrm{r}}}{t_{\mathrm{d}}\,\omega^k}\left(\frac{Y}{Y_{\mathrm{r}}}\right)^{\frac{1}{2}p+1}, \qquad \text{model 2}$$

t_{d} is a characteristic time for damage evolution,

$$h_{\mathrm{d}}(T) = \exp(-Q_{\mathrm{d}}/RT),$$

Q_{d} is the damage activation energy and R is the universal gas constant.
The reference value $Y_{\mathrm{r}} = {\sigma_{\mathrm{rd}}}^2/(2E)$, where σ_{rd} is a reference stress for the damage process.

Monkman-Grant parameter

Experimental relationship

$$C_{\mathrm{MG}} = (\dot{\varepsilon}^{\mathrm{c}}_{\mathrm{min}})^m t_{\mathrm{rup}} \approx \text{constant}.$$

For the two models the Monkman-Grant parameter have the values ($m = 1$)

$$\begin{aligned} C_{\mathrm{MG}} &= \dot{\varepsilon}^{\mathrm{c}}_{\mathrm{min}} t_{\mathrm{rup}} = \frac{1}{1+k+2r}\frac{t_{\mathrm{d}} h_{\mathrm{c}}}{t_{\mathrm{c}} h_{\mathrm{d}}}\left(\frac{\sigma}{\sigma_{\mathrm{r}}}\right)^{p-2r} && \text{model 1} \\ C_{\mathrm{MG}} &= \frac{t_{\mathrm{d}}}{t_{\mathrm{c}}} && \text{model 2.} \end{aligned}$$

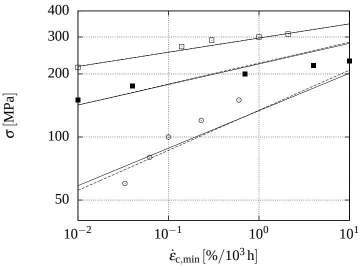
Model 2 can be obtained by imposing the following constrains to the model 1:

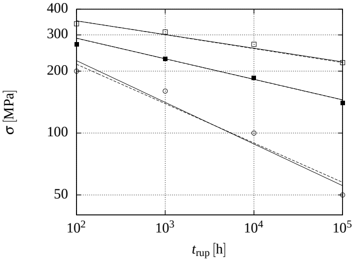
$$p = 2r, \qquad \frac{1}{1+k+2r}\frac{t_{\mathrm{d}} h_{\mathrm{c}}}{t_{\mathrm{c}} h_{\mathrm{c}}} = \text{constant}.$$

T24 material parameters

The calibrated model parameters for the 7CrMoVTiB10-10 steel (T24), $q_c = Q_c/R$ and $q_d = Q_d/R$, $p(T) = p_r(1 + a(T - T_r)/T_r)$ and $r(T) = r_r(1 + b(T - T_r)/T_r)$, $\sigma_{rc} = \sigma_{rd} = \sigma_r = \sigma_{y0}(T) = \sigma^* - cT$, with $\sigma^* = 1123$ MPa, $c = -1$ MPa/K.

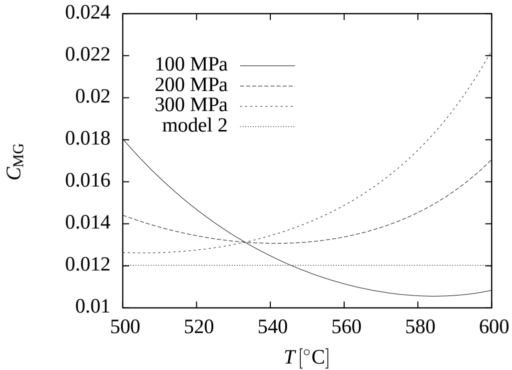
mod	t_c [s]	p_r	t_d [s]	a	q_c [K]	r_r	q_d [K]	b
1	3039.9	14.77	37.768	-4.804	7137.6	7.545	9350.1	-5.201
2	3414.1	14.59	41.26	-4.891	7137.6	-	-	-

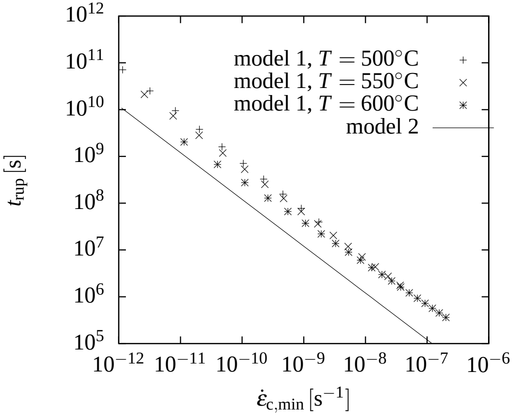




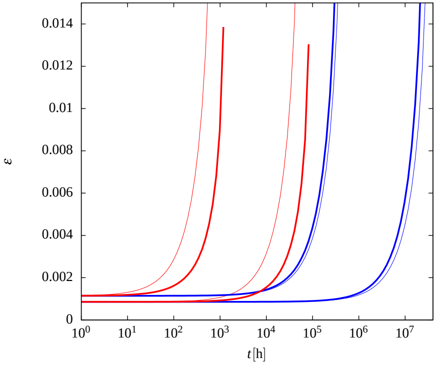
Minimum creep strain-rate (lhs) and the creep strengths (rhs).
Solid lines = model 1, dashed lines = model 2. Top $500°C$, middle $550°C$ bottom $600°C$.

Monkman-Grant parameter





Constant stress creep test



At temperatures $500°$C (blue curves) and $600°$C (red curves) with stress levels 150 MPa and 200 MPa. Model 2 results with thicker line.

Implementation to ANSYS as USERMAT subroutine

Evolution equations for stress and integrity

$$\begin{aligned}\dot{\boldsymbol{\sigma}} &= \boldsymbol{f}_\sigma(\boldsymbol{\sigma}, \omega), \\ \dot{\omega} &= f_\omega(\boldsymbol{\sigma}, \omega),\end{aligned}$$

where

$$\begin{aligned}\boldsymbol{f}_\sigma(\boldsymbol{\sigma}, \omega) &= \omega \boldsymbol{C}_\mathrm{e} \dot{\boldsymbol{\varepsilon}}_\mathrm{e} + \dot{\omega} \boldsymbol{C}_\mathrm{e} \boldsymbol{\varepsilon}_\mathrm{e} + \omega \dot{\boldsymbol{C}}_\mathrm{e} \boldsymbol{\varepsilon}_\mathrm{e} \\ &= \omega \boldsymbol{C}_\mathrm{e}(\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}_\mathrm{th} - \dot{\boldsymbol{\varepsilon}}_\mathrm{c}) + \left(\frac{\dot{\omega}}{\omega} \boldsymbol{C}_\mathrm{e} + \omega \dot{T} \frac{\partial \boldsymbol{C}_\mathrm{e}}{\partial T} \right) \boldsymbol{C}_\mathrm{e}^{-1} \boldsymbol{\sigma}, \\ f_\omega(\boldsymbol{\sigma}, \omega) &= -\frac{B_\mathrm{d}}{t_\mathrm{d}\, \omega^k} \left(\frac{Y}{Y_\mathrm{r}} \right)^r .\end{aligned}$$

Time discretisation using backward Euler scheme and linearisation

$$\left[\begin{array}{cc} \boldsymbol{H}_{11} & \boldsymbol{h}_{12} \\ \boldsymbol{h}_{21} & H_{22} \end{array}\right]\left\{\begin{array}{c} \delta\boldsymbol{\sigma} \\ \delta\omega \end{array}\right\} = \Delta t\left\{\begin{array}{c} \boldsymbol{f}_{\sigma} \\ f_{\omega} \end{array}\right\} - \left\{\begin{array}{c} \Delta\boldsymbol{\sigma} \\ \Delta\omega \end{array}\right\},$$

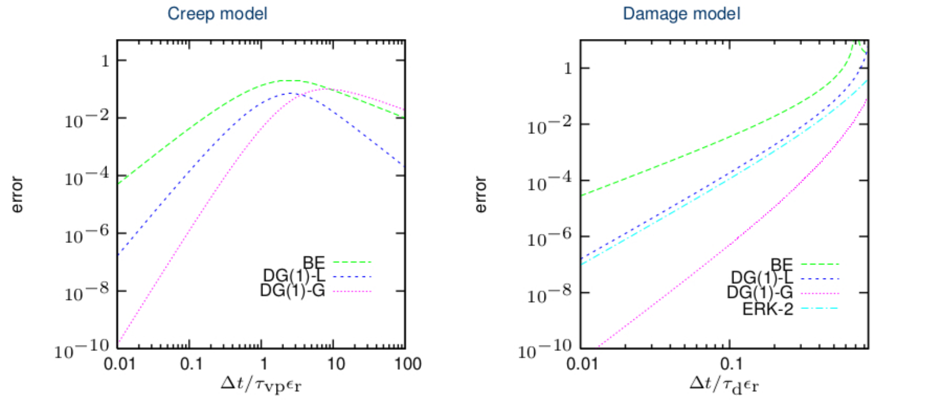
where

$$\begin{aligned}
\boldsymbol{H}_{11} &= \boldsymbol{I} - \Delta t\frac{\partial \boldsymbol{f}_{\sigma}}{\partial \boldsymbol{\sigma}}, \\
\boldsymbol{h}_{12} &= -\Delta t\frac{\partial \boldsymbol{f}_{\sigma}}{\partial \omega}, \quad \boldsymbol{h}_{21} = -\Delta t\frac{\partial f_{\omega}}{\partial \boldsymbol{\sigma}}, \\
H_{22} &= 1 - \Delta t\frac{\partial f_{\omega}}{\partial \omega}.
\end{aligned}$$

The Jacobian matrix, i.e. "the algorithmic tangent" matrix has the form

$$\boldsymbol{C}^{\mathrm{alg}} = \omega\widetilde{\boldsymbol{H}}_{11}^{-1}\boldsymbol{C}_{\mathrm{e}}, \quad \text{where} \quad \widetilde{\boldsymbol{H}}_{11} = \boldsymbol{H}_{11} - \boldsymbol{h}_{12}H_{22}^{-1}\boldsymbol{h}_{21}^{\mathrm{T}}. \quad \text{Unsymmetric!}$$

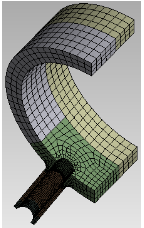
Error in one step solution

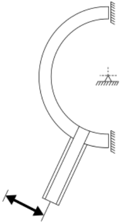


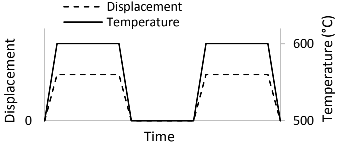
FE analysis and results

The mesh consists of mainly 20 node hexahedral ANSYS SOLID186 elements & some 10 node tetrahedal SOLID187 elements.
Prescribed displacement history at the end of the tube nozzle.

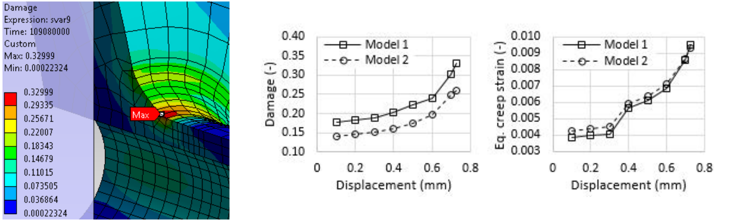
The computed lifetime is roughly 150 cycles. Ramp time 1 hour and hold time 200 hours. Internal pressure 14 MPa.





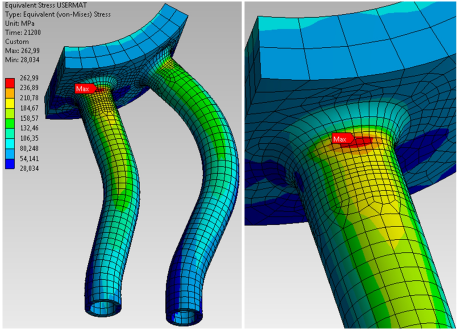


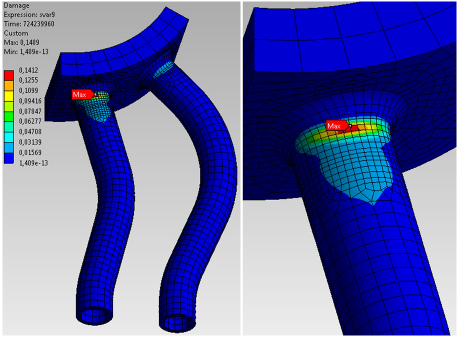
Results



Damage distribution near the most critical location of the header. The accumulated damage and the equivalent creep strain at the most critical location as functions of the prescribed displacement.

More close to the real structure





Concluding remarks

- Thermodynamically consistent model for high-temperature creep fatigue analyses has been developed.
- A specific model with Norton-Bailey type creep and Kachanov-Rabotnov type damage models are used.
- Two version of the damage evolution equations. One-version satisfies the Monkman-Grant hypothesis exactly. However, it restricts the model such that the behaviour is not realistic at lower temperatures.
- Materials parameters for the 7CrMoVTiB10-10 steel (T24) have been estimated in the temperature range 500-600 ^{0}C.
- Developed models have been implemented in the ANSYS FE-software by using the USERMAT subroutine.

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Thank You for Your Attention!

Watercolor by Pia Erlandsson