A continuum based macroscopic unified low- and high cycle fatigue model

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ICMFF12 - 12th International Conference on Multiaxial Fatigue and Fracture
Bordeaux, France, June 24-26, 2019

Introduction - fatigue models

Problems in fatigue analyses:

- low-cycle- and high-cycle -fatigue regimes are treated separately,
- mostly based on well defined cycles,
- multiaxiality.

A more fundamental approach for HCF based on *evolution equations* proposed by Ottosen, Stenström and Ristinmaa in IJF 2008. https://doi.org/10.1016/j.ijfatigue.2007.08.009

In this study this idea is combined with a plasticity model to obtain a unified model.

Evolution equation based HCF model

Key ingredients are:

Endurance surface

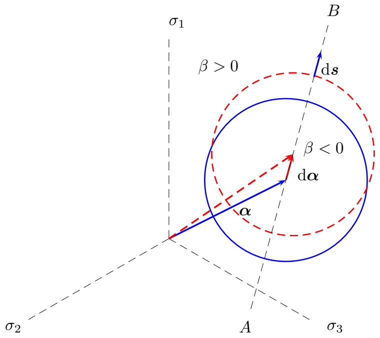
$$\beta(\boldsymbol{\sigma}, \{\boldsymbol{\alpha}\}; \text{parameters}) = 0$$

evolution equations for the fatigue damage D

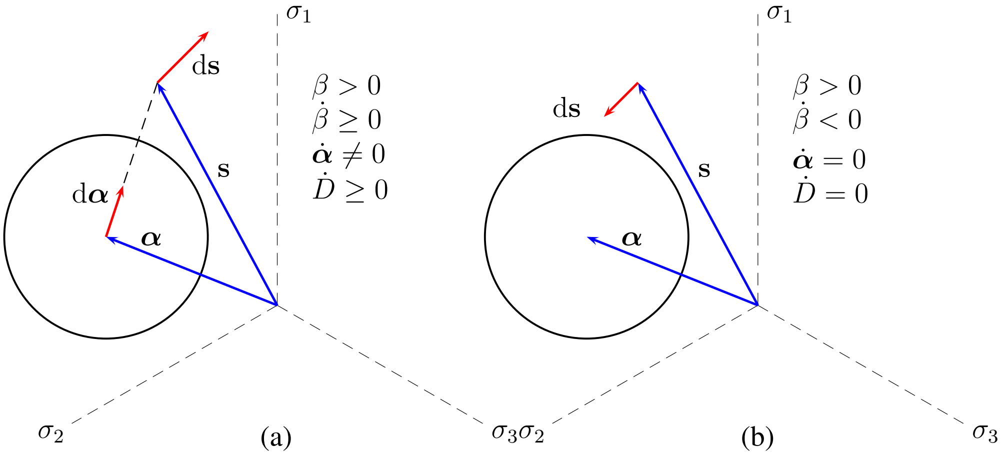
$$\dot{D} = g(\beta, D)\dot{\beta}$$

and the internal variables $\{\boldsymbol{\alpha}\}$

$$\{\dot{\alpha}\} = \{\boldsymbol{G}\}(\boldsymbol{\sigma}, \{\boldsymbol{\alpha}\})\dot{\beta}$$



Conditions for evolution



Original formulation for HCF

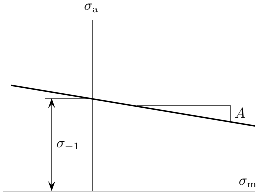
Endurance surface:

$$\beta = \frac{1}{\sigma_{-1}} \left[\sqrt{3\bar{J}_2} + AI_1 - \sigma_{-1} \right] = 0$$

where $\bar{J}_2 = \frac{1}{2}\mathrm{tr}\,(\boldsymbol{s} - \boldsymbol{\alpha})^2$, $\quad I_1 = \mathrm{tr}\,\boldsymbol{\sigma}$, $\quad A = \sigma_{-1}/\sigma_0 - 1$ and

$$\sigma_{-1} = \sigma_{\mathrm{af},R=-1}$$

$$\sigma_0 = \sigma_{\mathrm{af},R=0}$$



Evolution equations:

$$\dot{\boldsymbol{\alpha}} = C(\boldsymbol{s} - \boldsymbol{\alpha})\dot{\beta}, \qquad \dot{D} = K\exp(L\beta)\dot{\beta}$$

LCF-HCF approach

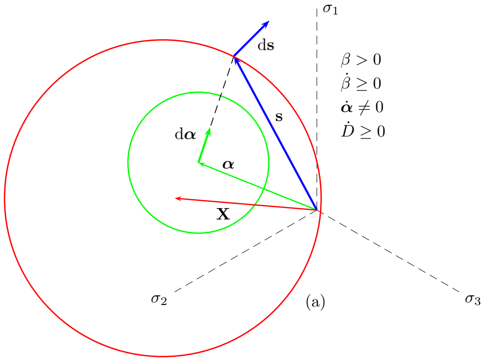
Couples with the plasticity model, Chaboche type model adopted:

$$f(\boldsymbol{\sigma}, \boldsymbol{X}, R) = \sqrt{\tfrac{3}{2}(\boldsymbol{s} - \boldsymbol{X}) : (\boldsymbol{s} - \boldsymbol{X})} - (\sigma_\mathrm{y} + R) = 0$$

$$\dot{\boldsymbol{\varepsilon}}^\mathrm{p} = \dot{\lambda}\frac{\partial f}{\partial \boldsymbol{\sigma}}, \qquad \dot{\varepsilon}^\mathrm{p}_\mathrm{eff} = \sqrt{\tfrac{2}{3}\dot{\boldsymbol{\varepsilon}}^\mathrm{p} : \dot{\boldsymbol{\varepsilon}}^\mathrm{p}}$$

$$R = \sum R_i, \qquad \dot{R}_i = \gamma R_{\infty,i}\left(1 - R_i/R_{\infty,i}\right)\dot{\varepsilon}^\mathrm{p}_\mathrm{eff}$$

$$\boldsymbol{X} = \sum \boldsymbol{X}_i, \qquad \dot{\boldsymbol{X}}_i = \tfrac{2}{3}X_{\infty,i}\dot{\boldsymbol{\varepsilon}}^\mathrm{p} - \gamma_i\dot{\varepsilon}^\mathrm{p}_\mathrm{eff}\boldsymbol{X}_i$$



LCF-HCF approach - damage evolution

$$\frac{\mathrm{d}D}{\mathrm{d}t} = g(\beta)\frac{\mathrm{d}\beta}{\mathrm{d}t} + M\frac{\mathrm{d}}{\mathrm{d}t}(\exp(Q\beta)\varepsilon^{\mathrm{p}}_{\mathrm{eff}})$$

where the high cycle part is modified to

$$g(\beta) = K\left(1 + \frac{1-\exp(-\tilde{L}(\beta-b))}{a+\exp(-\tilde{L}(\beta-b))}\right) \approx K\exp L\beta \quad \text{when} \quad \beta \lesssim 1$$

Parameters M and Q from two standard cyclic tests with different amplitudes:

$$\frac{\mathrm{d}D}{\mathrm{d}N} \approx M\exp(Q\beta)\frac{\mathrm{d}\varepsilon^{\mathrm{p}}_{\mathrm{eff}}}{\mathrm{d}N} = 4M\exp(Q\beta)\varepsilon^{\mathrm{p}}_{\mathrm{a}}$$

$$Q = \frac{1}{\beta(N_1)-\beta(N_2)}\ln\frac{N_2\varepsilon^{\mathrm{p}}_{\mathrm{a2}}}{N_1\varepsilon^{\mathrm{p}}_{\mathrm{a1}}}, \qquad M = \frac{1}{4N_i\exp(Q\beta(N_i))\varepsilon^{\mathrm{p}}_{\mathrm{a}i}}$$

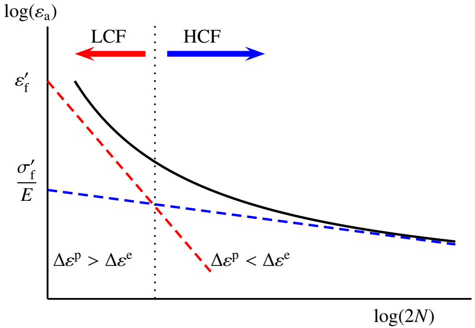
LCF-parameters

Using the Coffin-Manson and Ramberg-Osgood relations: $\varepsilon_a^p = {\varepsilon'}_f (2N)^{-c}, \quad \sigma_a = \sigma'_c (\varepsilon_a^p)^{n_c}$

$$Q = \frac{1-c}{\beta(N_1) - \beta(N_2)} \ln\left(\frac{N_2}{N_1}\right)$$

$$M = \frac{1}{4N_i \exp(Q\beta(N_i)){\varepsilon'}_f (2N_i)^{-c}}$$

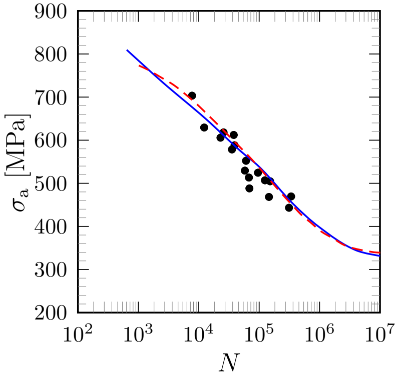
$$\beta(N_i) = \frac{1}{\sigma_{-1}} \left[(1+A)\sigma'_c(\varepsilon'_f)^{-c}(2N_i)^{-cn_c} - \sigma_{-1}\right]$$



Preliminary results, S-N curve for AISI 4340

$\sigma_{-1} = 315\,\mathrm{MPa}, \quad A = 0.225, \quad C = 1.2,$
$K = 2.5 \cdot 10^{-6}, \quad \tilde{L} = 14.5, \quad a = 0.005, \quad b = 0.5,$
$M = 10^{-11}, \quad Q = 16.$
$\sigma_{\mathrm{y}} = 331\,\mathrm{MPa}, \quad X_{\infty,1} = 35921\,\mathrm{MPa},$
$X_{\infty,2} = 6972\,\mathrm{MPa}, \quad X_{\infty,3} = 4222\,\mathrm{MPa},$
$\gamma_1 = 651, \quad \gamma_2 = 53.3, \quad \gamma_3 = 5.7$, no isotr. hardening,

Chaboche model data from
Y. Gorash, D. MacKenzie, *Open Engineering*, **7**, 126 (2017)
https://doi.org/10.1515/eng-2017-0019



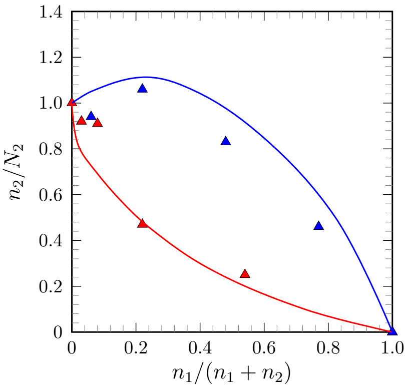
Present model fit with blue solid line. Dashed red line fit by Gorash, MacKenzie.
Experimental results (black dots) from N.E. Dowling: Mean stress effects in stress-life and strain-life fatigue. SAE Technical Paper 1 (2004), 1-14.

Two-level test

Two-level loading 735 → 810 MPa (blue), 810 → 735 MPa (red).

Experimental data shown by triangles from
W.H. Erickson, C.E. Work, A study of the accumulation of fatigue damage in steel, *64th Annual Meeting of ASTM*, 704-718 (1961).

Present model predictions by solid lines.



Concluding remarks and *future work*

- Continuum based Unified LCF-HCF model.
- Multiaxial, applicable to arbitrary loading history.
- Applicable for post-processing.
- Can be easily extended to include anisotropic, gradient and stochastic effects.
- *Parameter estimation.*
- *Micromechanical motivation of the evolution equations.*

Human fatigue illustrated by Akseli Gallen-Kallela 1894

Acknowledgements: The work was partially funded by TEKES - The National Technology Foundation of Finland (Business Finland from January 1, 2018), project MaNuMiES.

Thank you for your attention!