CONTINUUM DAMAGE MODEL FOR CONCRETE

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Introduction

- The non-linear behaviour of quasi-brittle materials, like concrete and rock under loading is mainly due to damage and micro-cracking rather than plastic deformation.
- Damage of quasi-brittle materials can be modelled using scalar, vector or higher order damage tensors.
- Failure in tension is mainly due to the growth of the most critical micro-crack.
- Failure in compression can be seen as a cooperative action of a distributed microcrack array.

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Ottosen's 4 parameter model

Failure surface

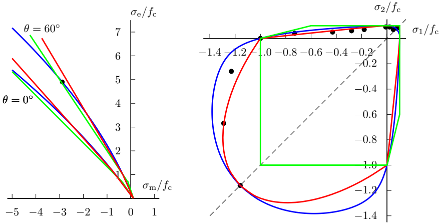
$$A\frac{J_2}{\sigma_c} + \Lambda\sqrt{J_2} + BI_1 - \sigma_\mathrm{c} = 0$$

$$\Lambda = \begin{cases} k_1 \cos[\frac{1}{3}\arccos(k_2\cos 3\theta)] & \text{if} \quad \cos 3\theta \geqslant 0 \\ k_1 \cos[\frac{1}{3}\pi - \frac{1}{3}\arccos(-k_2\cos 3\theta)] & \text{if} \quad \cos 3\theta \leq 0 \end{cases}$$

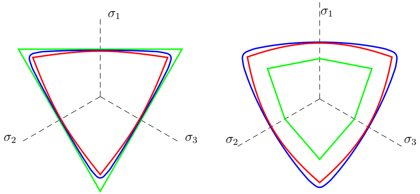
$$\cos 3\theta = \frac{3\sqrt{3}}{2}\frac{J_3}{J_2^{3/2}}$$

σ_c: the uniaxial compressive strength
$I_1 = \mathrm{tr}\boldsymbol{\sigma}$: the first invariant of the stress tensor
$J_2 = \frac{1}{2}\boldsymbol{s} : \boldsymbol{s}$, $J_3 = \det \boldsymbol{s} = \frac{1}{3}\mathrm{tr}\boldsymbol{s}^3$: deviatoric invariants
A, B, k_1, k_2: material constants

Meridian plane & plane stress



Deviatoric plane



π − plane $\qquad$ $\sigma_\mathrm{m} = -f_\mathrm{c}$

Green line = Mohr-Coulomb with tension cut-off
Blue line = Ottosen's model
Red line = Barcelona model

Thermodynamic formulation

Two potential functions

$$\psi^{\mathrm{c}} = \psi^{\mathrm{c}}(S), \quad S = (\boldsymbol{\sigma}, \boldsymbol{D}, \kappa) \qquad \text{Specific Gibbs free energy}$$

$$\gamma = \rho_0 \dot{\psi}^{\mathrm{c}} - \dot{\boldsymbol{\sigma}} : \boldsymbol{\varepsilon}, \quad \gamma \geqslant 0 \qquad \text{Clausius-Duhem inequality}$$

$$\varphi(W; S), \quad W = (\boldsymbol{Y}, K) \qquad \text{Dissipation potential}$$

$$\gamma \equiv \boldsymbol{B}_Y : \boldsymbol{Y} + B_K K$$

$$\text{Define} \quad \boldsymbol{Y} = \rho_0 \frac{\partial \psi^c}{\partial \boldsymbol{D}}, \quad K = -\rho_0 \frac{\partial \psi^c}{\partial \kappa}$$

$$\left(\rho_0 \frac{\partial \psi^c}{\partial \boldsymbol{\sigma}} - \boldsymbol{\varepsilon}\right) : \dot{\boldsymbol{\sigma}} + \left(\dot{\boldsymbol{D}} - \boldsymbol{B}_Y\right) : \boldsymbol{Y} + (-\dot{\kappa} - B_K) K = 0 \quad \forall \text{ admissible } \dot{\boldsymbol{\sigma}}, \boldsymbol{Y}, K$$

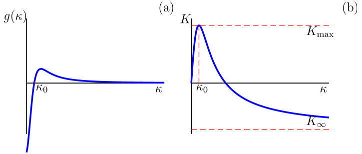
$$\Rightarrow \quad \boldsymbol{\varepsilon} = \rho_0 \frac{\partial \psi^c}{\partial \boldsymbol{\sigma}}, \quad \dot{\boldsymbol{D}} = \boldsymbol{B}_Y, \quad \dot{\kappa} = -B_K$$

Specific model

Specific Gibbs free energy

$$\rho_0\psi^{\mathrm{c}}(\boldsymbol{\sigma},\boldsymbol{D},\kappa)=\frac{1}{4G}\left[\mathrm{tr}\boldsymbol{s}^2+\mathrm{tr}(\boldsymbol{s}^2\boldsymbol{D})\right]+\frac{1}{18K_{\mathrm{b}}}(1+\chi\mathrm{tr}\boldsymbol{D})(\mathrm{tr}\boldsymbol{\sigma})^2+\int_0^{\kappa}\int_0^{\kappa'}g(\kappa'')\mathrm{d}\kappa''\mathrm{d}\kappa'$$

$$g(\kappa)=\frac{H_0}{\kappa_0}\frac{h_2\left(\kappa/\kappa_0\right)^2-2h_1\left(\kappa/\kappa_0\right)-1}{\left[h_2\left(\kappa/\kappa_0\right)^2+1\right]^2}$$



Dissipation potential:

$$\varphi(\boldsymbol{Y},K;\boldsymbol{\sigma},\boldsymbol{\varepsilon})=\mathrm{I}(\boldsymbol{Y},K;\boldsymbol{\sigma},\boldsymbol{\varepsilon}),\qquad \mathrm{I}(\boldsymbol{Y},K;\boldsymbol{\sigma},\boldsymbol{\varepsilon})=\begin{cases}0, & \text{if } (\boldsymbol{Y},K)\in\Sigma\\ +\infty, & \text{if } (\boldsymbol{Y},K)\notin\Sigma\end{cases}$$

Elastic domain

$$\Sigma=\{(\boldsymbol{Y},K)|f(\boldsymbol{Y},K;\boldsymbol{\sigma})\leqslant 0\}$$

where the damage surface is defined as

$$f(\boldsymbol{Y},K;\boldsymbol{\sigma},\boldsymbol{\varepsilon})=\frac{AJ_2}{\sigma_{\mathrm{c}}}+\Lambda\sqrt{J_2}+BI_1-(\sigma_{\mathrm{c0}}+K)+\mathrm{tr}\left[\boldsymbol{A}^{\mathrm{T}}\left(\boldsymbol{Y}-\boldsymbol{Y}(\boldsymbol{\sigma})\right)\right]$$

Specific model (cont'd)

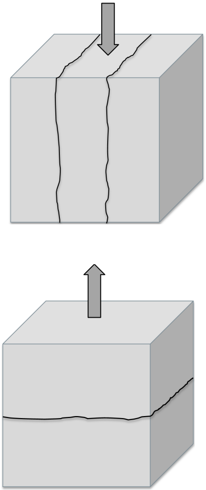
The symmetric positive definite second-order tensor $\boldsymbol{A}$ is defined as

$$\boldsymbol{A}(\boldsymbol{\sigma}, \boldsymbol{\varepsilon}) = \frac{1}{1 + \beta_1 \langle \mathrm{tr}\ \boldsymbol{\sigma} \rangle / \sigma_{\mathrm{t}}} \left(\frac{\boldsymbol{\varepsilon}_+}{\|\boldsymbol{\varepsilon}_+\|} + \beta_2 \boldsymbol{I} \right)$$

and $\boldsymbol{\varepsilon}_+$ is the positive part of the elastic strain tensor, i.e.

$$\boldsymbol{\varepsilon}_+ = \sum_{i=1}^{3} \langle \varepsilon_i \rangle \boldsymbol{\phi}_i \otimes \boldsymbol{\phi}_i$$

where $\varepsilon_i,\ i = 1, \ldots, 3$ are the eigenvalues of the elastic strain tensor, $\boldsymbol{\phi}_i$ stands for the corresponding eigenvectors. β_1, β_2 are positive non-dimensional parameters and σ_{t} is the uniaxial tensile strength. $\|\boldsymbol{\varepsilon}_+\| = \sqrt{\mathrm{tr}\,(\boldsymbol{\varepsilon}_+^2)}$, and $\langle \bullet \rangle$ denotes the Macaulay brackets.



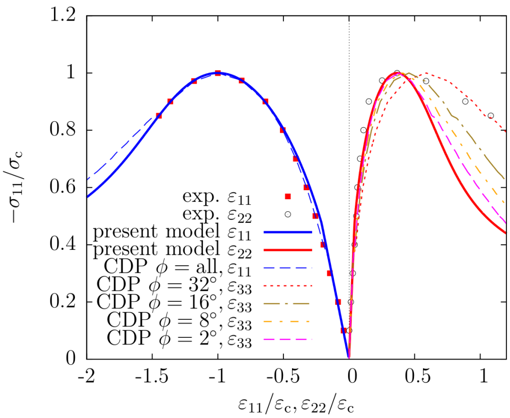
Calibration of material parameters

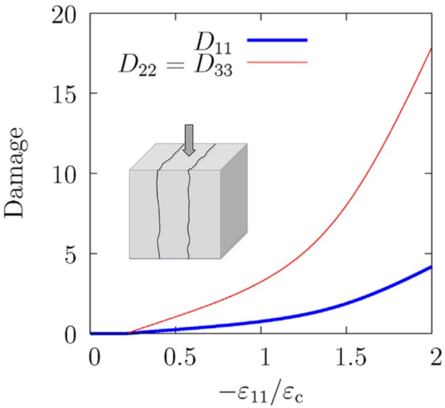
- The parameter set A, B, k_1 and k_2 are adjusted to represent the damage surface with four failure states.
- Damage starts to develop at fairly low stress levels $\Rightarrow$ the damage initiation stress in compression σ_{c0}.
- For parameters χ, κ_0, β_1 and β_2, the uniaxial compression and tension + the biaxial test results are needed.
- **All parameters can be calculated from analytical expressions.**

quantity	value	unit	notes	model parameter	value	unit
E	31.9	GPa		A	1.512	-
ν	0.2	-		B	3.597	-
σ_c	30.9	MPa		k_1	12.963	-
σ_t	2.78	MPa		k_2	0.9864	-
σ_{bc}	35.8	MPa		K_∞	-6.72	MPa
$I_{1,4}$	-267.6	MPa	Ottosen-Ristinmaa-05	H_0	40.2	MPa
$\sqrt{J_{2,4}}$	87.4	MPa	Ottosen-Ristinmaa-05	h_1	-0.1253	-
σ_{c0}	10.8	MPa	$0.35\sigma_c$	h_2	0.7494	-
$\varepsilon_{11,c}$	0.0022	-		κ_0	3.5151	-
$\varepsilon_{22,c}$	0.000806	-		χ	0.00162	-
$\varepsilon_{11,t}$	0.00009	-		β_1	75.843	-
$\varepsilon_{11,bc}$	0.0026	-		β_2	0.21551	-
$\varepsilon_{33,bc}$	0.0033	-				
σ_{pp}	26.282	MPa	post peak value			
$\varepsilon_{11,pp}$	0.0031887	-	post peak value			

Experimental data from H. Kupfer, H.K. Hilsdorf, and H. Rüsch. Behaviour of concrete under biaxial stresses. *Journal of the American Concrete Institute*, 66(8):656666, August 1969.

Unconfined uniaxial compression test

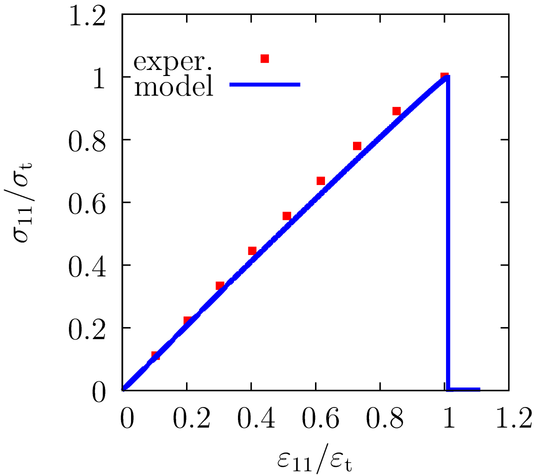


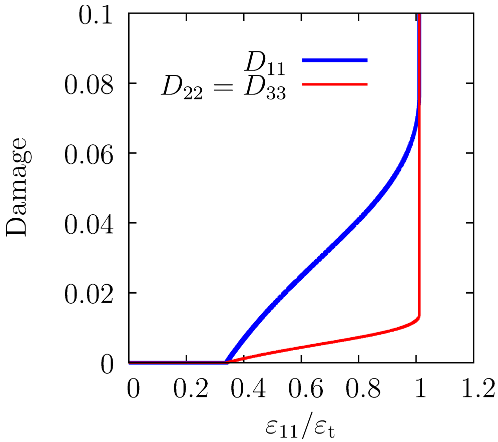


(lhs) Stress-strain relation in unconfined uniaxial compression. The Abaqus CDP model responses are shown for four values of the dilatation angle. Experimental data is from Kupfer et al. 1969.

(rhs) Damage-strain relation, axial splitting along the direction of uniaxial compression.

Unconfined uniaxial tensile test

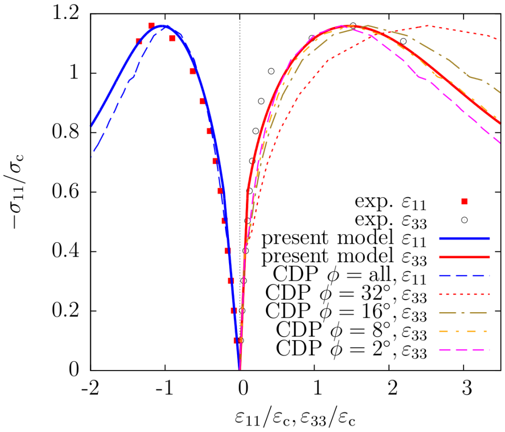


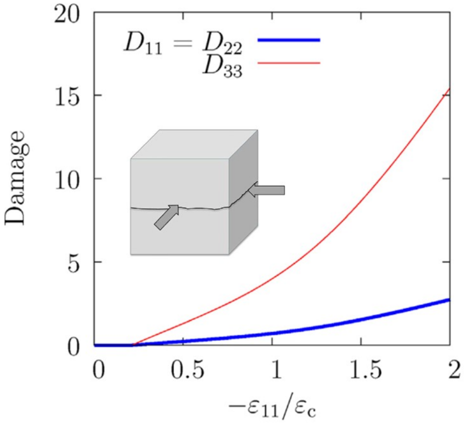


(lhs) Stress-strain behaviour in uniaxial tension with experimental results from Kupfer et al. 1969.

(rhs) Damage-strain behaviour, damage is the largest in the direction of stress.

Equibiaxial compression test





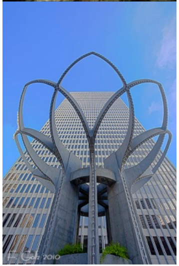
(lhs) Stress-strain behaviour in equibiaxial compression ($\sigma_{11} = \sigma_{22}$) with experimental results from Kupfer et al. 1969. The Abaqus CDP model responses are shown for four values of the dilatation angle. Notice that different dilatation angle gives the best fit to experimental data in comparison to unconfined uniaxial compression for the CDP model.

(rhs) Damage-strain behaviour, damage is the largest in the 33-direction, i.e. the fracture mode corresponds splitting along the compressive plane illustrated.

Conclusions and *future work*

- Continuum damage formulation mimicking the behaviour of the Ottosen's 4 parameter concrete model
- Predicts correctly both failure loads and modes, e.g. axial splitting
- Simple determination of the model parameters
- *Implementation into FE software (Elmer, Abaqus)*
- *Cyclic compression tests*
- *Inclusion of permanent plastic deformations*
- *Regularization by higher order gradients*

Thank you for your attention!



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