Assessment of some integration methods for an evolution equation based high-cycle fatigue model

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Introduction - fatigue models

Problems in fatigue analyses:

- mostly based on well defined cycles,
- damage accumulation rules,
- multiaxiality,
- low-cycle- and high-cycle -fatigue regimes are treated separately.

A more fundamental approach for HCF based on *evolution equations* proposed by Ottosen, Stenström and Ristinmaa in IJF 2008. https://doi.org/10.1016/j.ijfatigue.2007.08.009

Evolution equation based HCF model

Key ingredients are:

Endurance surface

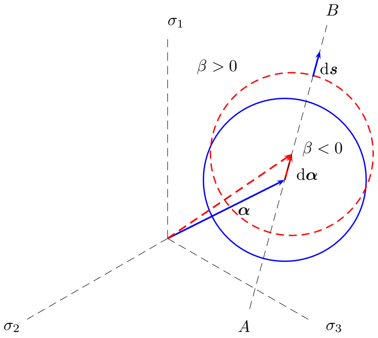
$$\beta(\boldsymbol{\sigma}, \{\boldsymbol{\alpha}\}; \text{parameters}) = 0$$

evolution equations for the fatigue damage D

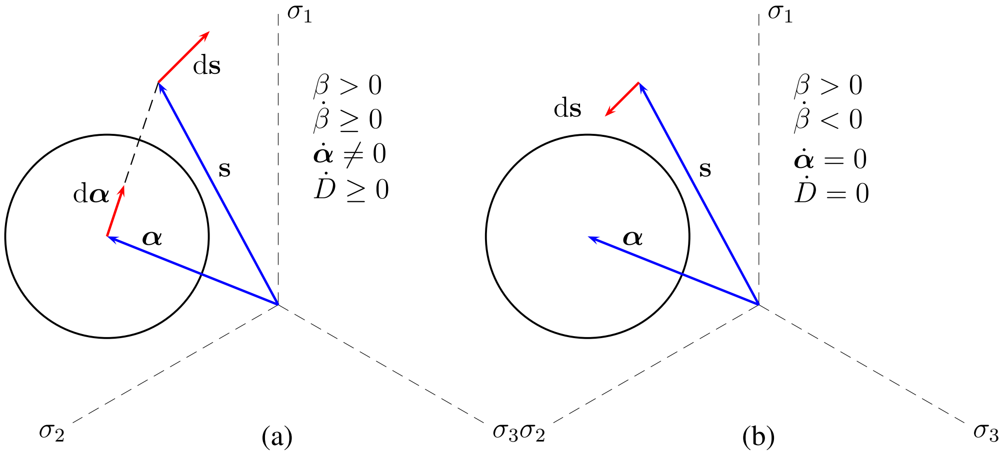
$$\dot{D} = g(\beta, D)\dot{\beta}$$

and the internal variables $\{\boldsymbol{\alpha}\}$

$$\{\dot{\alpha}\} = \{\boldsymbol{G}\}(\boldsymbol{\sigma}, \{\boldsymbol{\alpha}\})\dot{\beta}$$



Conditions for evolution



Original model by Ottosen et al.

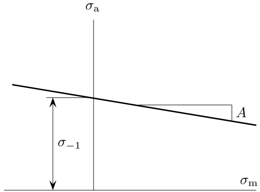
Endurance surface:

$$\beta = \frac{1}{\sigma_{-1}} \left[\sqrt{3\bar{J}_2} + AI_1 - \sigma_{-1} \right] = 0$$

where $\bar{J}_2 = \frac{1}{2}\mathrm{tr}(\boldsymbol{s} - \boldsymbol{\alpha})^2, \quad I_1 = \mathrm{tr}\boldsymbol{\sigma}, \quad A = \sigma_{-1}/\sigma_0 - 1$ and

$$\sigma_{-1} = \sigma_{\mathrm{af},R=-1}$$

$$\sigma_0 = \sigma_{\mathrm{af},R=0}$$



Evolution equations:

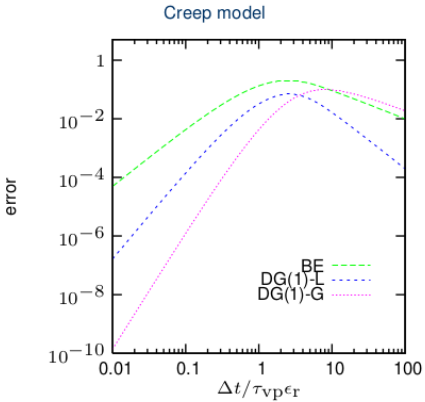
$$\dot{\boldsymbol{\alpha}} = C(\boldsymbol{s} - \boldsymbol{\alpha})\dot{\beta}, \qquad \dot{D} = K \exp(L\beta)\dot{\beta}$$

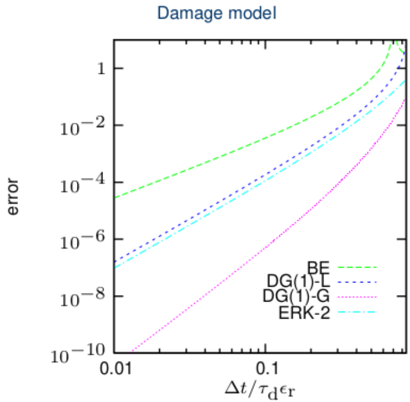
Integration of evolution equations

$$
\begin{aligned}
\dot{\boldsymbol{\alpha}} &= C(\boldsymbol{s} - \boldsymbol{\alpha})\dot{\beta}, \\
\dot{D} &= K \exp(L\beta)\dot{\beta}
\end{aligned}
$$

1. explicit Euler and Runge-Kutta methods RKNF, DOPRI
2. implicit Euler, discontinuous Galerkin

Amplification factor - one step solution

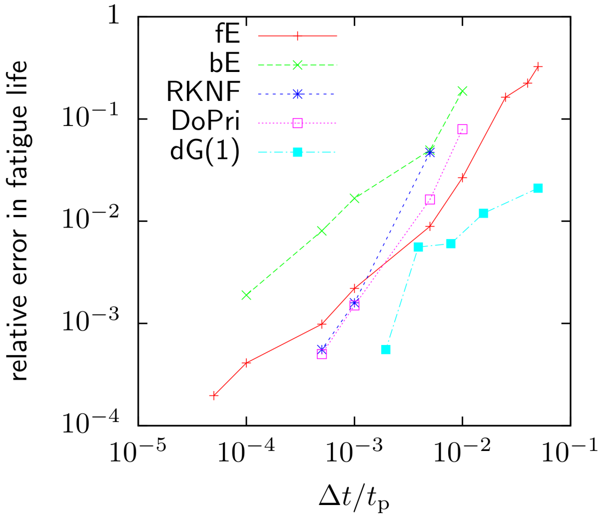




Convergence study - simple uniaxial stress history

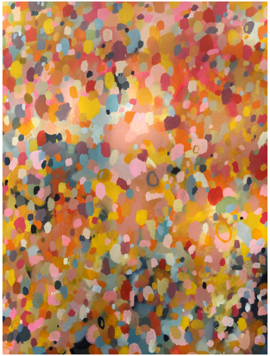
$$\sigma(t) = \sigma_\mathrm{m} + \sigma_\mathrm{a} \sin(2\pi t/t_\mathrm{p})$$
$$\sigma_\mathrm{m} = 0.8\sigma_{-1}, \quad \sigma_\mathrm{a} = \sigma_{-1}$$

Fatigue life 55880 cycles.



Concluding remarks

- Simple explicit Euler competetive integrator
- dG(1) performs also well
- RKNF, DoPri do not converge with optimal asymptotic rate



Eva Frengstad, Norwegian summer nights

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Thank you for your attention!