Modelling Strain-Rate Dependent Ductile-to-Brittle Transition

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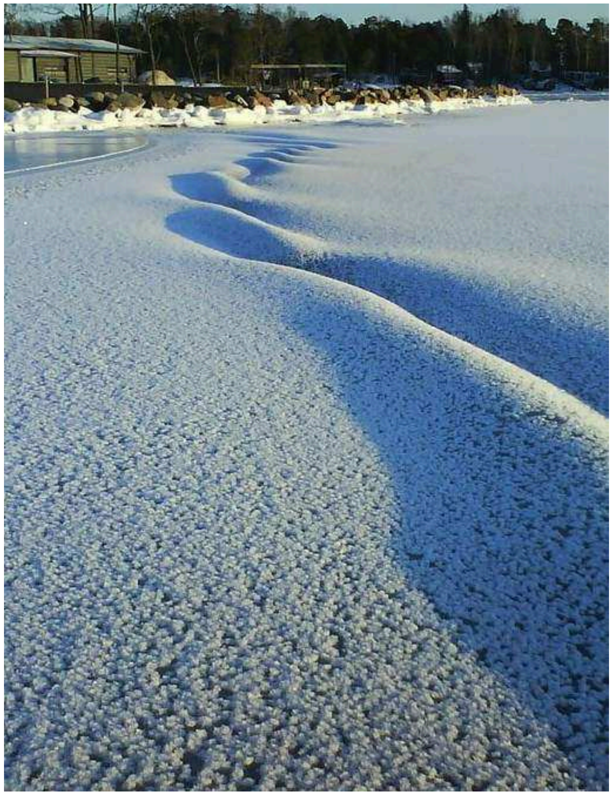
OUTLINE

- Motivation
- The model
  - Thermodyn. form.
  - Helmholtz free en.
  - Dissipation pot.
- Concluding remarks



Photographs Kari Kolari

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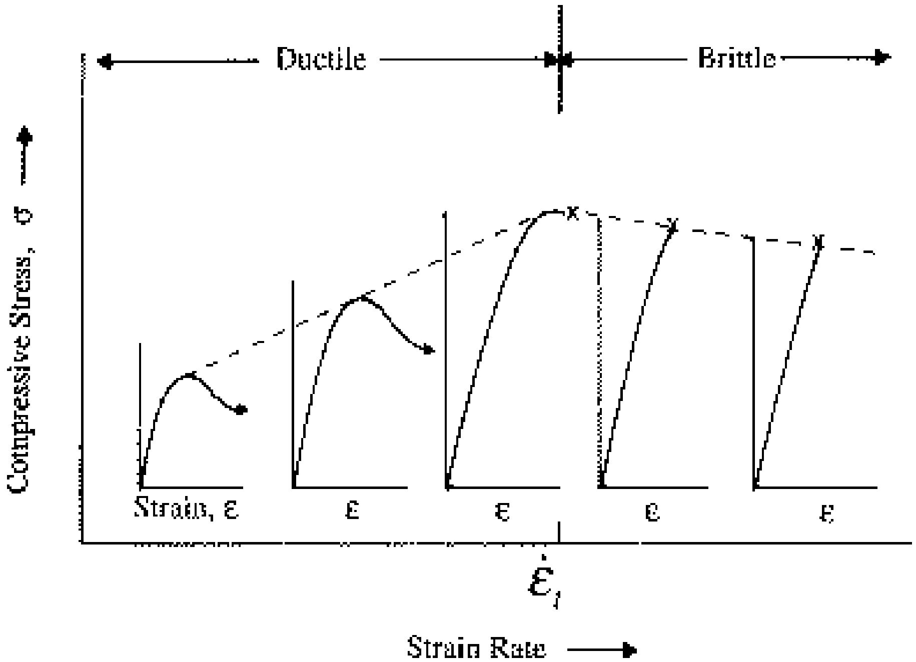
MOTIVATION

(a)



(b)

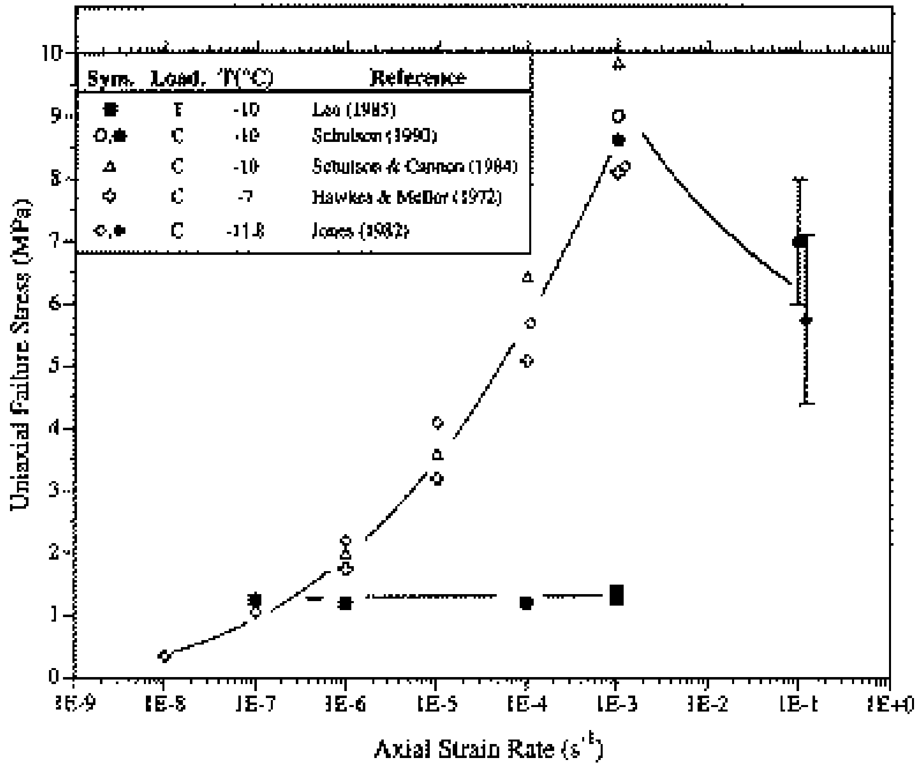


Fig. a in E. M. Schulson: Brittle failure of ice, *Engineering Fracture Mechanics* **68** (2001) 1839–1887.

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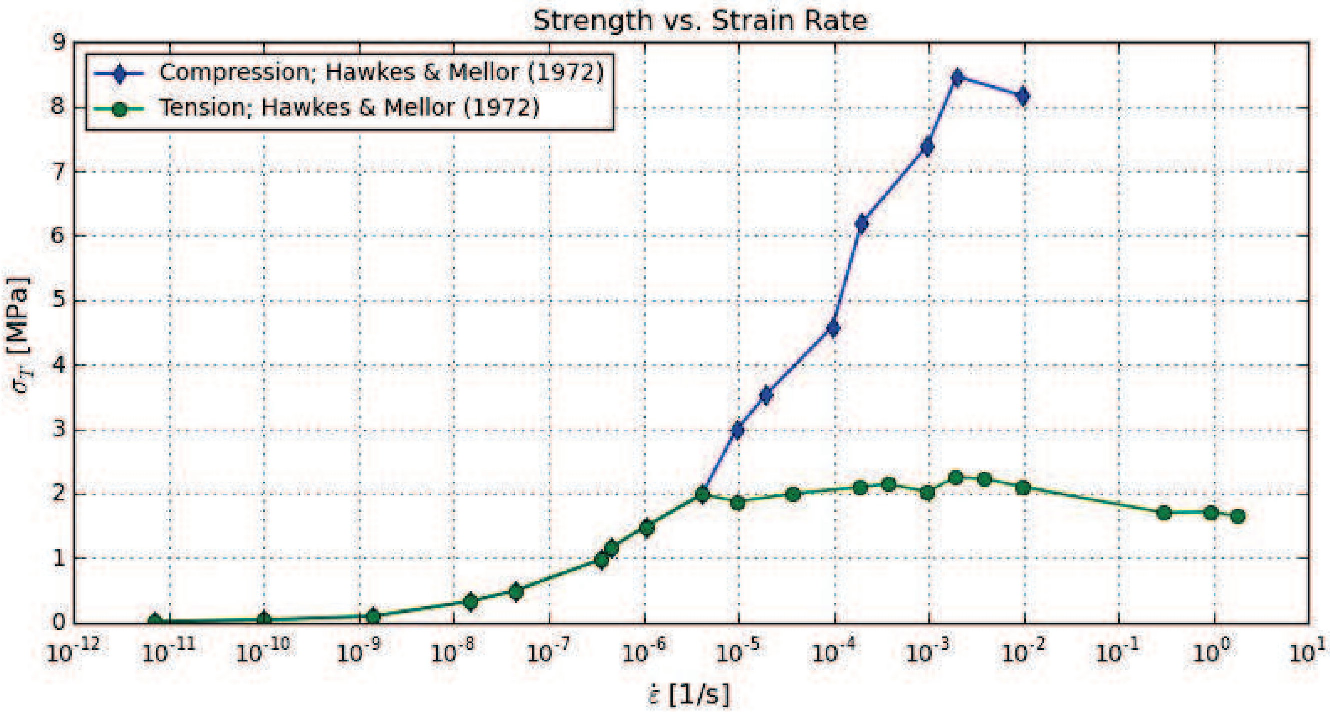
Strength vs. strain-rate

Asymmetry in the ductile to brittle transition rate in compression and tension: 10^{-3} 1/s in compression and 10^{-6} in tension



Fresh water ice -7 °C

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THERMODYNAMIC FORMULATION

Helmholtz free energy $\psi(\boldsymbol{\epsilon}_{\mathrm{e}}, \boldsymbol{d})$ where $\boldsymbol{\epsilon}_{\mathrm{e}} = \boldsymbol{\epsilon} - \boldsymbol{\epsilon}_{\mathrm{i}}$.

Clausius-Duhem Inequality $\gamma = -\rho\dot{\psi} + \boldsymbol{\sigma} : \dot{\boldsymbol{\epsilon}} \geq 0$

Dissipation potential $\varphi(\boldsymbol{\sigma}, \boldsymbol{y})$ such that $\gamma = \dfrac{\partial\varphi}{\partial\boldsymbol{\sigma}} : \boldsymbol{\sigma} + \dfrac{\partial\varphi}{\partial\boldsymbol{y}} \cdot \boldsymbol{y} \geq 0$

$$\Longrightarrow \quad \left(\boldsymbol{\sigma} - \rho\frac{\partial\psi}{\partial\boldsymbol{\epsilon}_{\mathrm{e}}}\right) : \dot{\boldsymbol{\epsilon}}_{\mathrm{e}} + \left(\dot{\boldsymbol{\epsilon}}_{\mathrm{i}} - \frac{\partial\varphi}{\partial\boldsymbol{\sigma}}\right) : \boldsymbol{\sigma} + \left(\dot{\boldsymbol{d}} - \frac{\partial\varphi}{\partial\boldsymbol{y}}\right) \cdot \boldsymbol{y} = 0$$

$$\Longrightarrow \quad \boldsymbol{\sigma} = \rho\frac{\partial\psi}{\partial\boldsymbol{\epsilon}_{\mathrm{e}}} \qquad \dot{\boldsymbol{\epsilon}}_{\mathrm{i}} = \frac{\partial\varphi}{\partial\boldsymbol{\sigma}} \qquad \dot{\boldsymbol{d}} = \frac{\partial\varphi}{\partial\boldsymbol{y}}$$

$\Longrightarrow \quad \gamma \geq 0$ satisfies CDI

Helmholtz free energy

Integrity basis

$$I_1 = \mathrm{tr}\,\boldsymbol{\epsilon}_{\mathrm{e}}, \quad I_2 = \tfrac{1}{2}\mathrm{tr}\,\boldsymbol{\epsilon}_{\mathrm{e}}^2, \quad I_3 = \tfrac{1}{3}\mathrm{tr}\,\boldsymbol{\epsilon}_{\mathrm{e}}^3, \quad I_4 = \|\boldsymbol{d}\|, \quad I_5 = \boldsymbol{d}\cdot\boldsymbol{\epsilon}_{\mathrm{e}}\cdot\boldsymbol{d}, \quad I_6 = \boldsymbol{d}\cdot\boldsymbol{\epsilon}_{\mathrm{e}}^2\cdot\boldsymbol{d}$$

or

$$\hat{I}_5 = \hat{\boldsymbol{d}}\cdot\boldsymbol{\epsilon}_{\mathrm{e}}\cdot\hat{\boldsymbol{d}}, \quad \hat{I}_6 = \hat{\boldsymbol{d}}\cdot\boldsymbol{\epsilon}_{\mathrm{e}}^2\cdot\hat{\boldsymbol{d}} \quad \text{where} \quad \hat{\boldsymbol{d}} = \boldsymbol{d}/\|\boldsymbol{d}\| = \boldsymbol{d}/I_4$$

$$\begin{aligned}\rho\psi = {} & (1 - I_4)\left(\tfrac{1}{2}\lambda I_1^2 + 2\mu I_2\right) \\ & + H(\sigma^{\perp})\frac{\lambda\mu}{\lambda + 2\mu} I_4 (I_1^2 - 2 I_1 \hat{I}_5 + \hat{I}_5^2) + (1 - H(\sigma^{\perp})) I_4 (\tfrac{1}{2}\lambda I_1^2 + \mu \hat{I}_5^2) \\ & + \mu I_4 \left(2 I_2 + \hat{I}_5^2 - 2 I_6\right) + \alpha I_5\end{aligned}$$

Heaviside function $H(\sigma^{\perp})$ takes into account the “crack” opening/closure
Stresses are continuous when the “crack” closes

Dissipation potential

Decomposed as

$$\varphi(\boldsymbol{\sigma}, \boldsymbol{y}) = \varphi_{\mathrm{d}}(\boldsymbol{y})\varphi_{\mathrm{tr}}(\boldsymbol{\sigma}) + \varphi_{\mathrm{vp}}(\boldsymbol{\sigma})$$

where

$$\varphi_{\mathrm{d}} = \frac{1}{2(r+1)}\frac{Y_{\mathrm{r}}}{\tau_{\mathrm{d}}(1-I_4)}H(\epsilon_1 - \epsilon_{\mathrm{tresh}})\left(\frac{(\boldsymbol{y}+\boldsymbol{y}_0)\cdot\boldsymbol{M}\cdot(\boldsymbol{y}+\boldsymbol{y}_0)}{Y_{\mathrm{r}}^2}\right)^{r+1}$$

$$\varphi_{\mathrm{tr}} = \frac{1}{pn}\left[\frac{1}{\tau_{\mathrm{vp}}\eta}\left(\frac{\bar{\sigma}}{(1-I_4)\sigma_{\mathrm{r}}}\right)^{p}\right]^{n}$$

$$\varphi_{\mathrm{vp}} = \frac{1}{p+1}\frac{\sigma_{\mathrm{r}}}{\tau_{\mathrm{vp}}}\left(\frac{\bar{\sigma}}{(1-I_4)\sigma_{\mathrm{r}}}\right)^{p+1}$$

and

$$\boldsymbol{M} = \boldsymbol{n}\otimes\boldsymbol{n}, \quad \boldsymbol{y}_0 = \beta Y_{\mathrm{r}}\boldsymbol{n}$$

MODEL CHARACTERISTICS

- Elastic stiffness is reduced monotonously due to damage
- Qualitatively predicts correct brittle failure mode in compression/tension
  Animation
- The constraint for the damage $\|\boldsymbol{d}\| \in [0,1]$ is satisfied automatically
- The transition function φ_{tr} deals with the change in the mode of deformation through the damage evolution such that

$$\varphi_{\mathrm{tr}} \geq 0 \quad \text{and} \quad \varphi_{\mathrm{tr}} \approx 0 \text{ when } \|\dot{\boldsymbol{\epsilon}}_{\mathrm{i}}\| < \eta \quad \text{and} \quad \varphi_{\mathrm{tr}} > 1 \text{ when } \|\dot{\boldsymbol{\epsilon}}_{\mathrm{i}}\| > \eta;$$

- CDI is satisfied *a priori* for any admissible isothermal process
- The dissipation potential is a non-convex function with respect to the thermodynamic forces $\boldsymbol{\sigma}$ and $\boldsymbol{y}$

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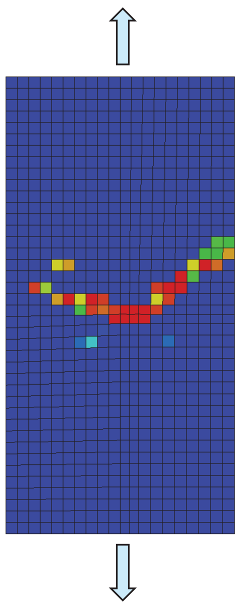
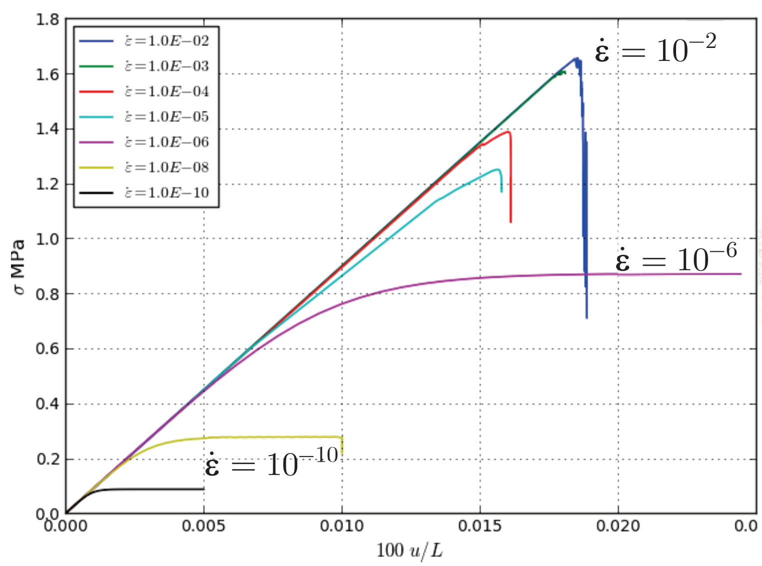
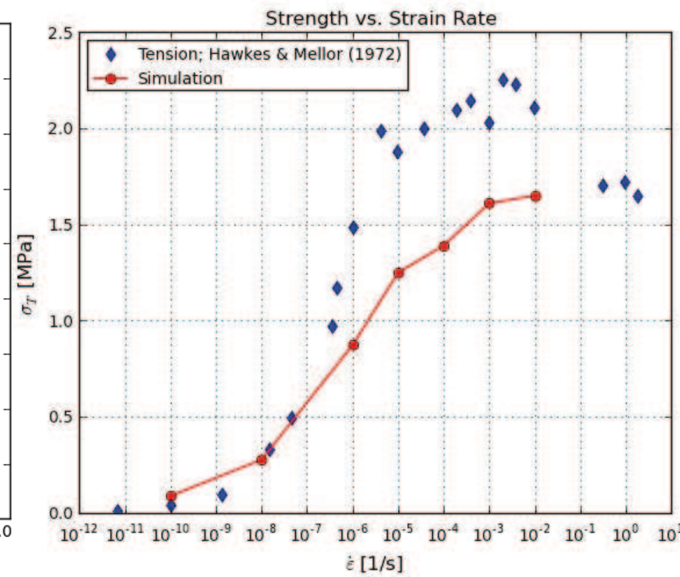
EXAMPLE - tension test

Implemented in ABAQUS as a user subroutine
$E = 9$ MPa, $\sigma_{\mathrm{r}} = 0.9$ MPa, $\nu = 0.3$, $\tau_{\mathrm{d}} = 100$ s, $\eta = 10^{-6}$ 1/s,
$\tau_{\mathrm{vp}} = 400 \cdot 10^{3}$ s, $p = n = 4$, $r = 1.5$ $\beta = 0.025$







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CONCLUSIONS AND FURTHER DEVELOPMENTS

- Thermodynamically consistent formulation
- Easily extensible to more realistic damage and plasticity models
- Length scale ?
- Brittle compressive failure is a challenge
- Alternative formulation using $\psi^*(\sigma, \alpha)$ and $\varphi(\dot{\epsilon}_\mathrm{i}, Z)$!

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