Ice failure analysis using strain-softening viscoplastic material model

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OUTLINE

- Characteristics of ice
- Viscoplastic material model
- Integration
- Example
- Concluding remarks

ICE

Failure (yield) surface rate dependent

Transition from ductile to brittle ($\sim 10^{-3}$ 1/s)

VISCOPLASTIC MODEL

Dynamic yield surface

$$f(\sigma_{ij}, K_\alpha, \dot{\epsilon}_{ij}^{\mathrm{vp}}, \dot{\kappa}_\alpha) = 0$$

Plastic potential

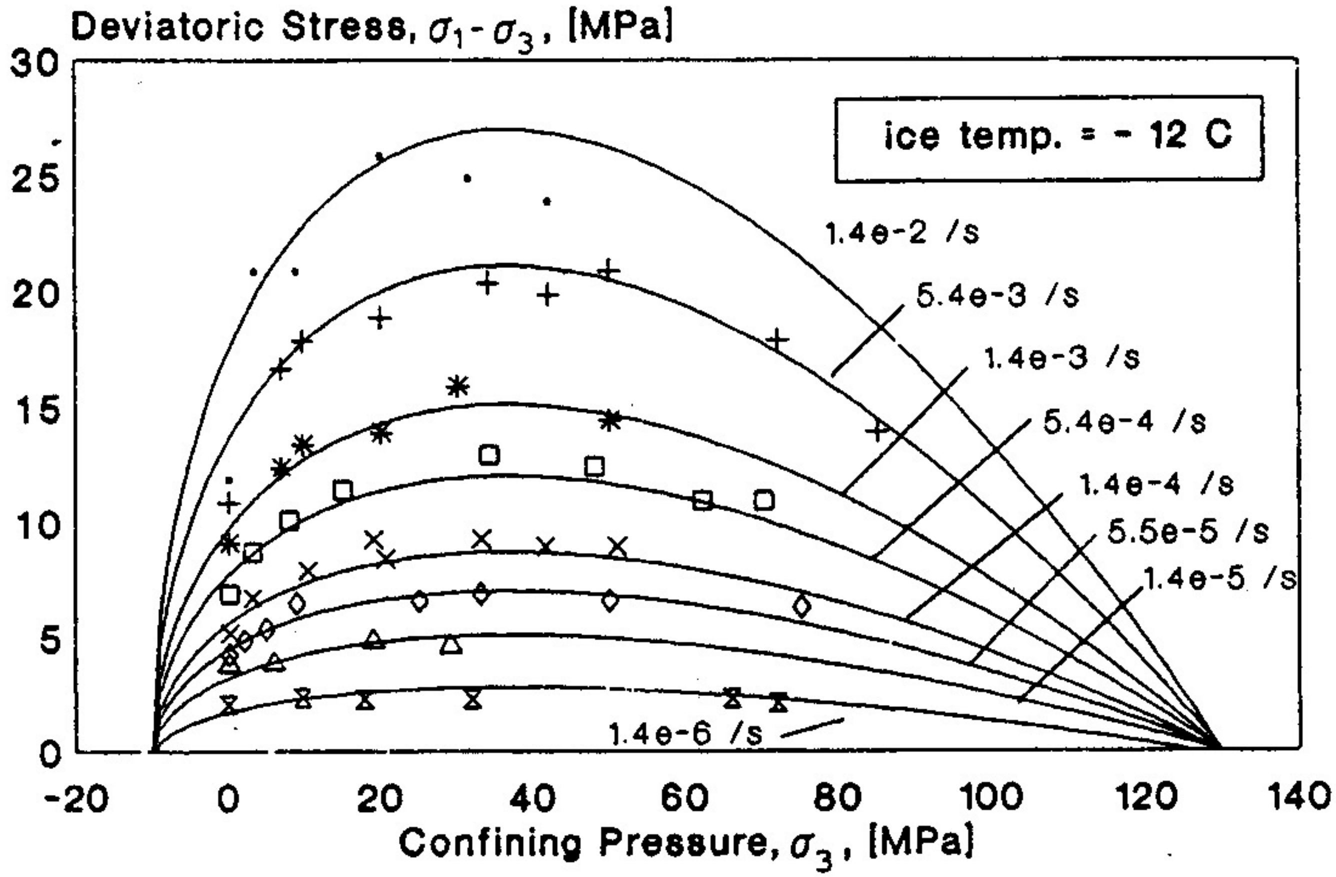
$$g = G(\sigma_{ij}, K_\alpha, \dot{\epsilon}_{ij}^{\mathrm{vp}}, \dot{\kappa}_\alpha)$$

Evolution laws

$$\dot{\epsilon}_{ij}^{\mathrm{vp}} = \dot{\lambda}\frac{\partial G}{\partial \sigma_{ij}} = \dot{\lambda} m_{ij}, \qquad \dot{\kappa}_\alpha = \dot{\lambda}\frac{\partial G}{\partial K_\alpha}.$$

FAILURE SURFACE OF ISOTROPIC ICE

Nadreau & Michel 1986, Kormann & Brown 1990, exp. data Jones 1982



MODIFIED NADREAU YIELD SURFACE

$$f = \sqrt{3J_2} - R(\dot{\bar{\epsilon}}^{\mathrm{vp}})P(p, K_\alpha)$$

Nadreau & Michel 1986, Kormann & Brown 1990

$$R(\dot{\bar{\epsilon}}^{\mathrm{vp}}) = (\eta\dot{\bar{\epsilon}}^{\mathrm{vp}})^n$$

$$P(p, K_\alpha) = \frac{1}{\sqrt{C - T}}(C - p)\sqrt{\sqrt{3}(T + p)}$$

where $p = -\frac{1}{3}\sigma_{ii}$,

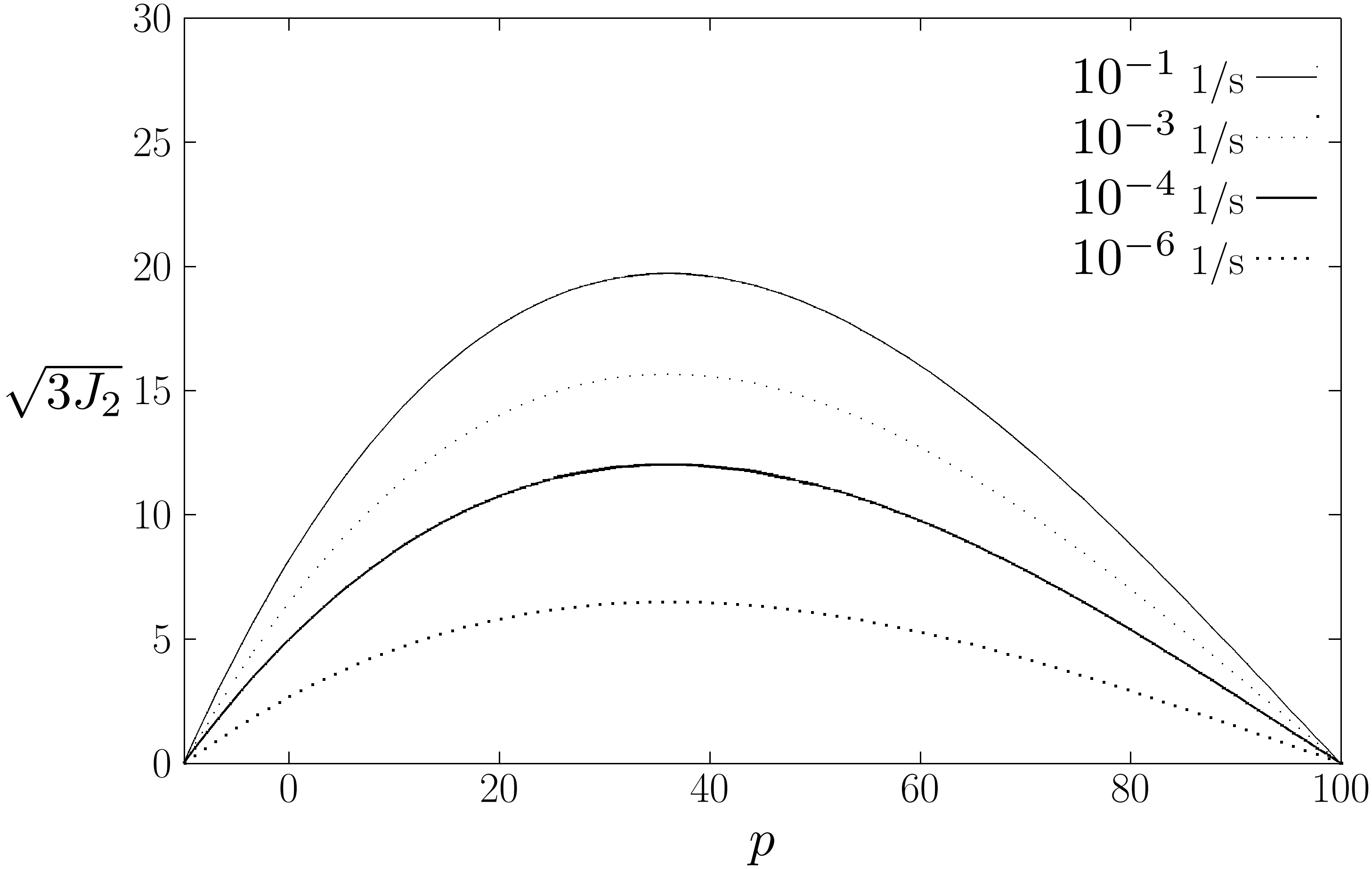
OUR CHOICE

$$R(\dot{\bar{\epsilon}}^{\mathrm{vp}}) = r_0 + (r_\infty - r_0)\tanh((\eta(\dot{\epsilon}_0 + \dot{\bar{\epsilon}}^{\mathrm{vp}}))^n)$$

$$P(p, K) = \frac{1}{(C+T)^2}(C + K - p)(2C + T - p)(T + p)$$

$$K = k(\kappa, \dot{\kappa})$$

$$\eta = 1000\ \mathrm{s}, r_0 = 0.05, r_\infty = 0.5, p = 0.2, \dot{\epsilon}_0 = 10^{-7}\ 1/\mathrm{s}$$

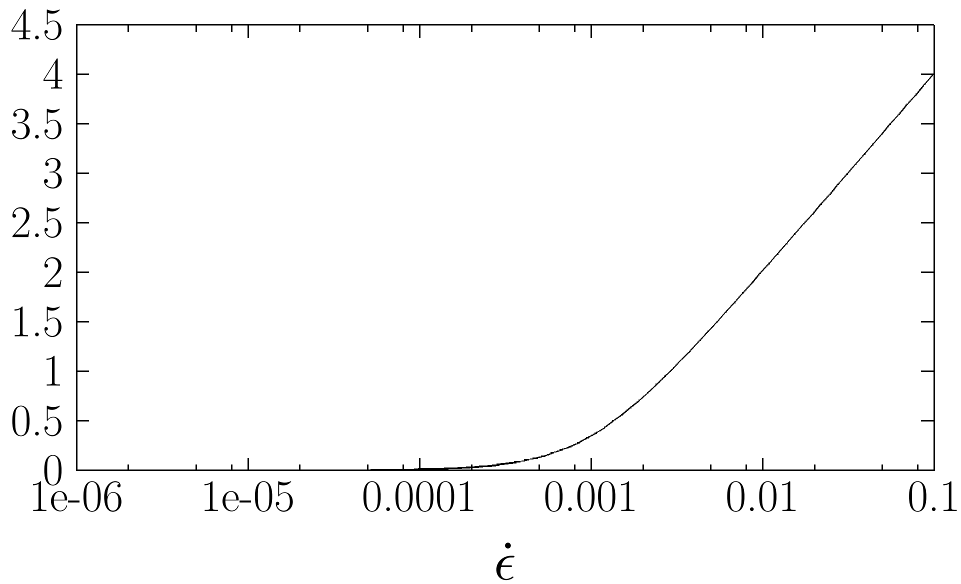


Units MPa

STRAIN RATE DEPENDENCY FOR HARDENING

Transition function

$$\frac{1}{2}\left[c(\log(\eta\dot{\epsilon}) - \log(\eta\dot{\epsilon}_{\mathrm{tr}})) + \ln\cosh c(\log(\eta\dot{\epsilon}) - \log(\eta\dot{\epsilon}_{\mathrm{tr}})) - \ln\frac{1}{2}\right]$$



$$\dot{\epsilon}_{\mathrm{tr}} = 10^3 \qquad c = 2$$

INTEGRATION

Backward Euler integration

$$f(\boldsymbol{\sigma}, \lambda, \dot{\lambda}) = 0$$

$$f^i + \frac{\partial f}{\partial \boldsymbol{\sigma}} \delta \boldsymbol{\sigma} + \frac{\partial f}{\partial \lambda} \delta \lambda + \frac{\partial f}{\partial \dot{\lambda}} \delta \dot{\lambda} = 0$$

$$\delta \lambda = \frac{f^i}{a},$$

where

$$\boxed{a = \mathbf{n}^T \mathbf{H} \tilde{\mathbf{m}} - \frac{\partial f}{\partial \lambda} - \frac{1}{\Delta t} \frac{\partial f}{\partial \dot{\lambda}}}$$

$$\mathbf{H} = \left((\mathbf{C}^{\mathrm{el}})^{-1} + \Delta \lambda \frac{\partial \mathbf{m}}{\partial \boldsymbol{\sigma}} \right)^{-1}$$

$$\tilde{\mathbf{m}} = \mathbf{m} + \Delta \lambda \frac{\partial \mathbf{m}}{\partial \lambda} + \frac{\Delta \lambda}{\Delta t} \frac{\partial \mathbf{m}}{\partial \dot{\lambda}}.$$

PROBLEM WITH THE POWER LAW

$$\dot{\epsilon}^{\mathrm{vp}} \longrightarrow 0 \qquad \text{then} \qquad \frac{\partial f}{\partial \dot{\lambda}} \longrightarrow \infty \qquad a \longrightarrow -\infty$$

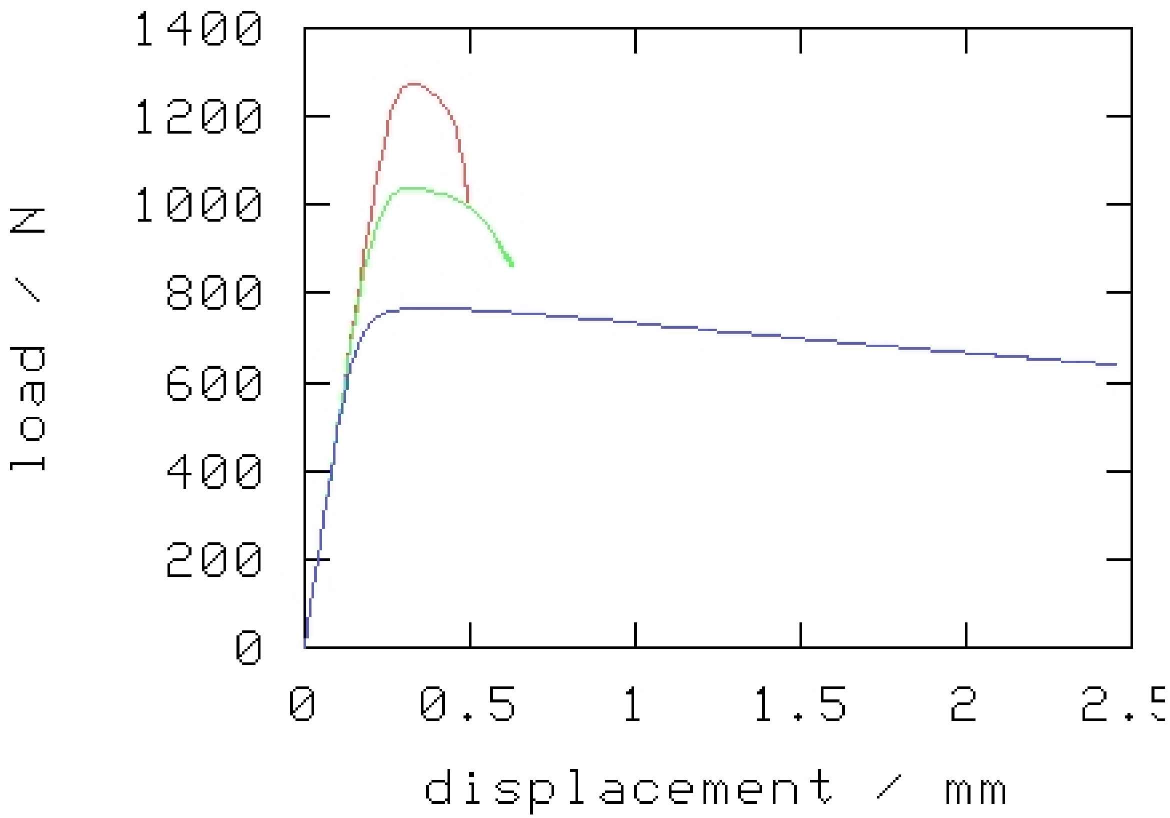
SCALING

$$x = \ln(\eta \dot{\lambda})$$

$$\delta x = \frac{f^i}{a_{\mathrm{sc}}}, \qquad a_{\mathrm{sc}} = \exp(x) \Delta t \mathbf{n}^T \mathbf{H} \tilde{\mathbf{m}} - \frac{\partial f}{\partial \lambda}\frac{\partial \lambda}{\partial x} - \frac{\partial f}{\partial \dot{\lambda}}\frac{\partial \dot{\lambda}}{\partial x}$$

EXAMPLE

Loading average strain rate $10^{-4}, 10^{-3}$ and 10^{-2} 1/s
6x12 trilinear $\bar{\mathbf{B}}$-element mesh (sample size 100 x 200 x 1 mm)



CONCLUDING REMARKS

- Consistency viscoplasticity formulation
- Both strain rate hardening and softening effects
- Further studies on developing yield surface for culumnar ice
- Comparisons to tests