A CONSTITUTIVE MODEL FOR STRAIN-RATE DEPENDENT DUCTILE-TO-BRITTLE TRANSITION

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OUTLINE

INTRODUCTION

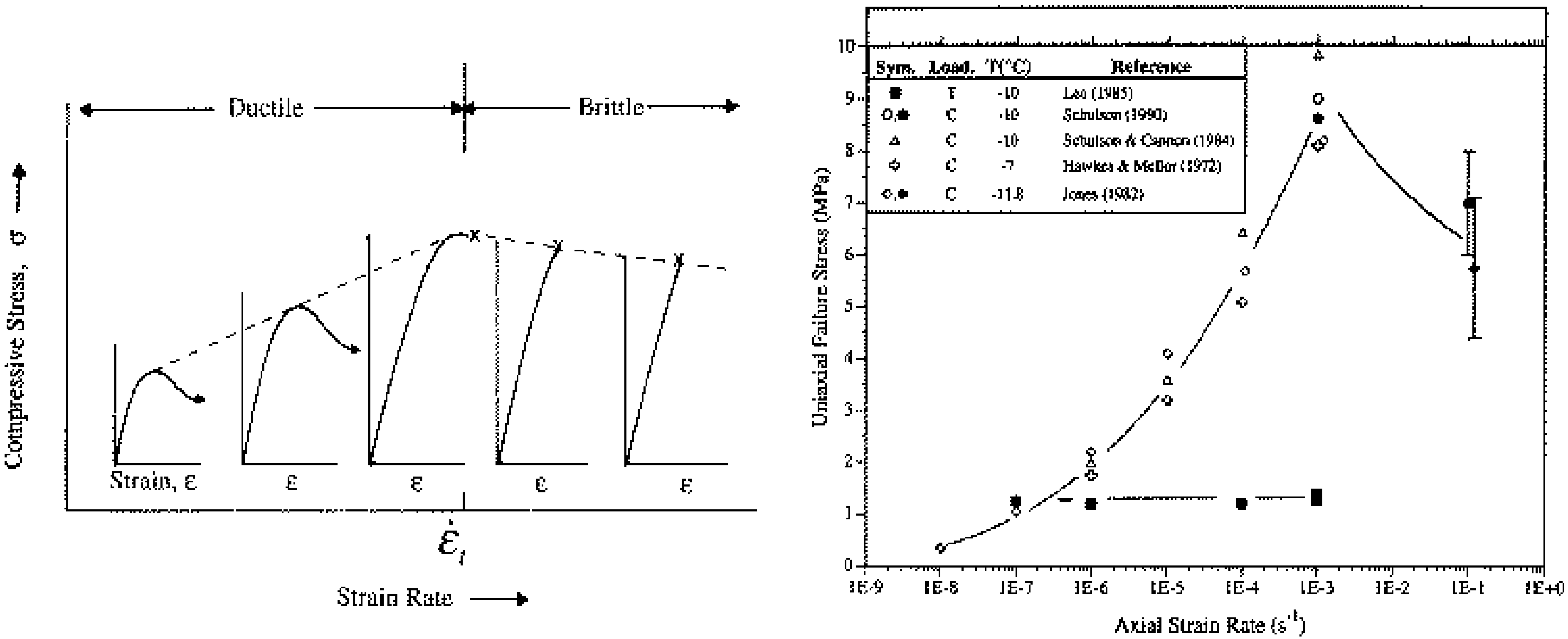


Fig. 2 in E. M. Schulson: Brittle failure of ice, *Engineering Fracture Mechanics* **68** (2001) 1839–1887.

THERMODYNAMIC FORMULATION

Clausius-Duhem Inequality $\quad \gamma = -\rho\dot{\psi} + \sigma : \dot{\epsilon} \geq 0$

Helmholtz free energy $\quad \psi(\epsilon_{\mathrm{e}}, \omega) \quad$ where $\quad \epsilon_{\mathrm{e}} = \epsilon - \epsilon_{\mathrm{i}}.$

Dissipation potential $\quad \varphi(\sigma, Y) \quad$ such that $\quad \gamma = \dfrac{\partial\varphi}{\partial\sigma} : \sigma + \dfrac{\partial\varphi}{\partial Y} Y \geq 0$

$$\Longrightarrow \qquad \left(\sigma - \rho\frac{\partial\psi}{\partial\epsilon_{\mathrm{e}}}\right) : \dot{\epsilon}_{\mathrm{e}} + \left(\dot{\epsilon}_{\mathrm{i}} - \frac{\partial\varphi}{\partial\sigma}\right) : \sigma + \left(-\dot{\omega} - \frac{\partial\varphi}{\partial Y}\right) Y = 0$$

$$\Longrightarrow \qquad \sigma = \rho\frac{\partial\psi}{\partial\epsilon_{\mathrm{e}}} \qquad \dot{\epsilon}_{\mathrm{i}} = \frac{\partial\varphi}{\partial\sigma} \qquad \dot{\omega} = -\frac{\partial\varphi}{\partial Y}$$

$\Longrightarrow \qquad \gamma \geq 0 \qquad$ satisfies CDI

Particular model

$$\rho\psi = \tfrac{1}{2}(1-D)\epsilon_\mathrm{e} : C_\mathrm{e} : \epsilon_\mathrm{e} = \tfrac{1}{2}\omega\epsilon_\mathrm{e} : C_\mathrm{e} : \epsilon_\mathrm{e}$$

$$\varphi(\sigma, Y) = \varphi_\mathrm{d}(Y)\varphi_\mathrm{tr}(\sigma) + \varphi_\mathrm{vp}(\sigma)$$

$\varphi_\mathrm{tr} \geq 0 \quad \varphi_\mathrm{tr} \approx 0$ when $\|\dot{\epsilon}_\mathrm{i}\| < \eta$ and $\varphi_\mathrm{tr} > 1$ when $\|\dot{\epsilon}_\mathrm{i}\| > \eta$

$$\varphi_\mathrm{d} = \frac{1}{r+1}\frac{Y_\mathrm{r}}{\tau_\mathrm{d}\omega}\left(\frac{Y}{Y_\mathrm{r}}\right)^{r+1}$$

$$\varphi_\mathrm{tr} = \frac{1}{pn}\left[\frac{1}{\tau_\mathrm{vp}\eta}\left(\frac{\bar{\sigma}}{\omega\sigma_\mathrm{r}}\right)^{p}\right]^{n} \qquad \sim \frac{\|\dot{\epsilon}_\mathrm{i}\|}{\eta}$$

$$\varphi_\mathrm{vp} = \frac{1}{p+1}\frac{\sigma_\mathrm{r}}{\tau_\mathrm{vp}}\left(\frac{\bar{\sigma}}{\omega\sigma_\mathrm{r}}\right)^{p+1}$$

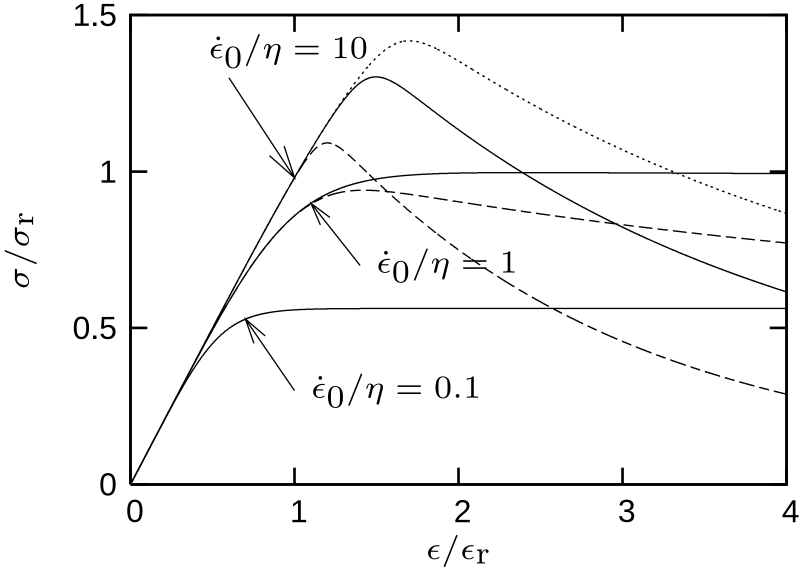
MODEL CHARACTERISTICS

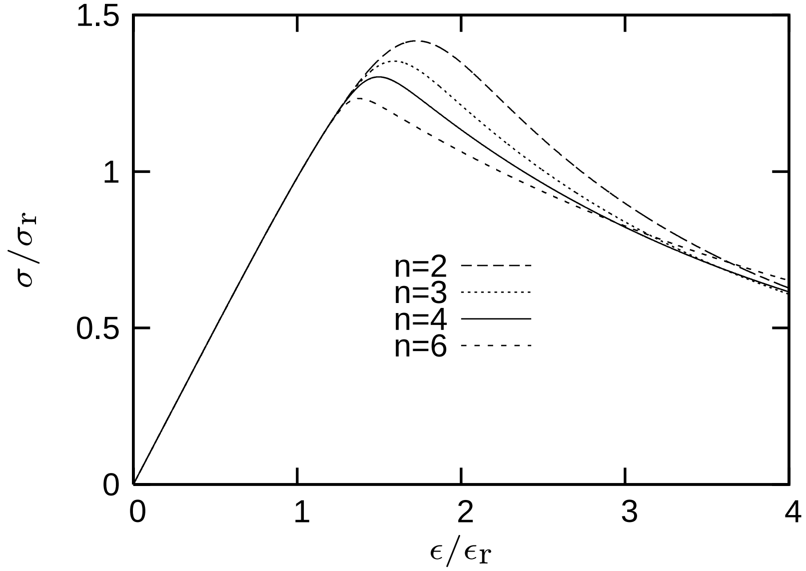
- Elastic stiffness is reduced monotonously due to damage
- The model does not include any specific yield stress
- In the absence of damage, the inelastic model behaves in a constant uniaxial strain-rate loading as

$$\sigma \longrightarrow (\tau_{\mathrm{vp}}\dot{\epsilon}_0)^{1/p}\sigma_{\mathrm{r}} \quad \text{when} \quad t \longrightarrow \infty$$

- The constraint for the integrity $\omega \in [0, 1]$ is satisfied automatically
- CDI is satisfied *a priori* for any admissible isothermal process
- The dissipation potential is a non-convex function with respect to the thermodynamic forces σ and Y

Stress-strain behaviour





ALGORITMIC TREATMENT

$$\dot{\boldsymbol{\sigma}} = \boldsymbol{f}_\sigma(\boldsymbol{\sigma}, \omega) = \omega \boldsymbol{C}_\mathrm{e}(\dot{\boldsymbol{\epsilon}} - \dot{\boldsymbol{\epsilon}}_\mathrm{i}) + \frac{f_\omega}{\omega}\boldsymbol{\sigma}$$

$$\dot{\omega} = f_\omega(\boldsymbol{\sigma}, \omega) = -\frac{\varphi_\mathrm{tr}}{\tau_\mathrm{d}\omega}\left(\frac{Y}{Y_\mathrm{r}}\right)^r$$

Backward Euler + Newton linearisation

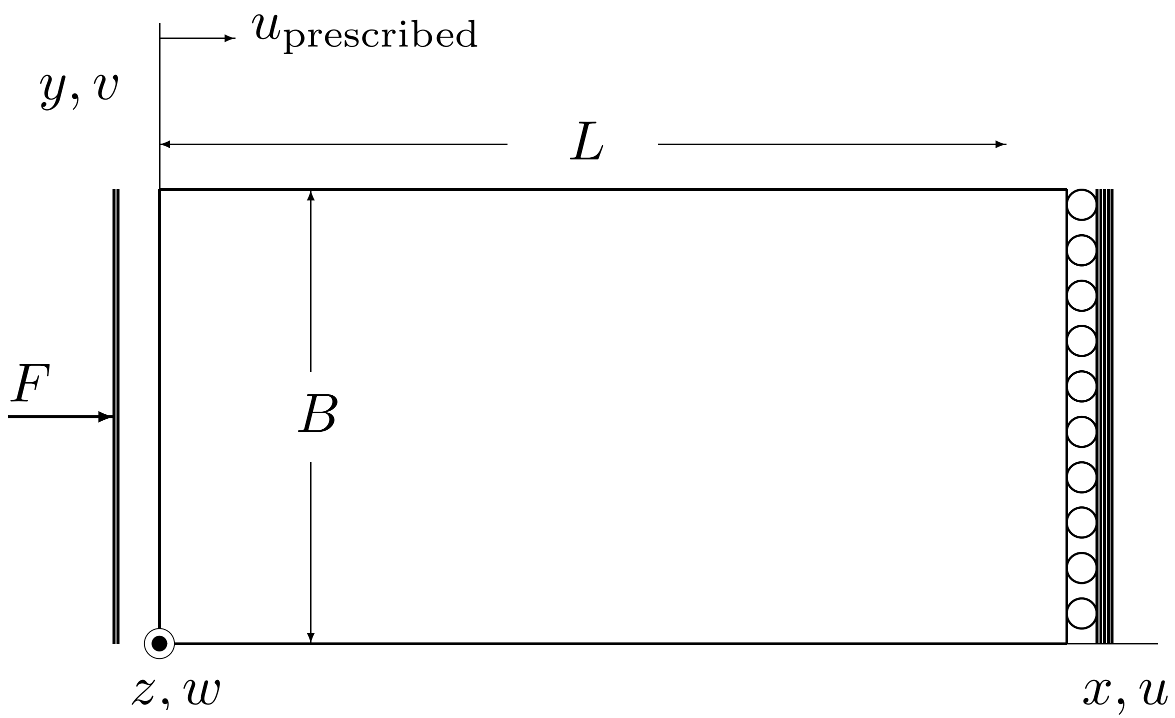
$$\begin{bmatrix} \boldsymbol{H}_{11} & \boldsymbol{h}_{12} \\ \boldsymbol{h}_{21}^T & H_{22} \end{bmatrix} \left\{ \begin{array}{c} \delta\boldsymbol{\sigma} \\ \delta\omega \end{array} \right\} = \Delta t \left\{ \begin{array}{c} \boldsymbol{f}_\sigma \\ f_\omega \end{array} \right\} - \left\{ \begin{array}{c} \Delta\boldsymbol{\sigma} \\ \Delta\omega \end{array} \right\}$$

$$\boldsymbol{H}_{11} = \boldsymbol{I} - \Delta t \frac{\partial \boldsymbol{f}_\sigma}{\partial \boldsymbol{\sigma}} \quad \boldsymbol{h}_{12} = -\Delta t \frac{\partial \boldsymbol{f}_\sigma}{\partial \omega} \quad \boldsymbol{h}_{21}^T = -\Delta t \frac{\partial f_\omega}{\partial \boldsymbol{\sigma}} \quad H_{22} = 1 - \Delta t \frac{\partial f_\omega}{\partial \omega}$$

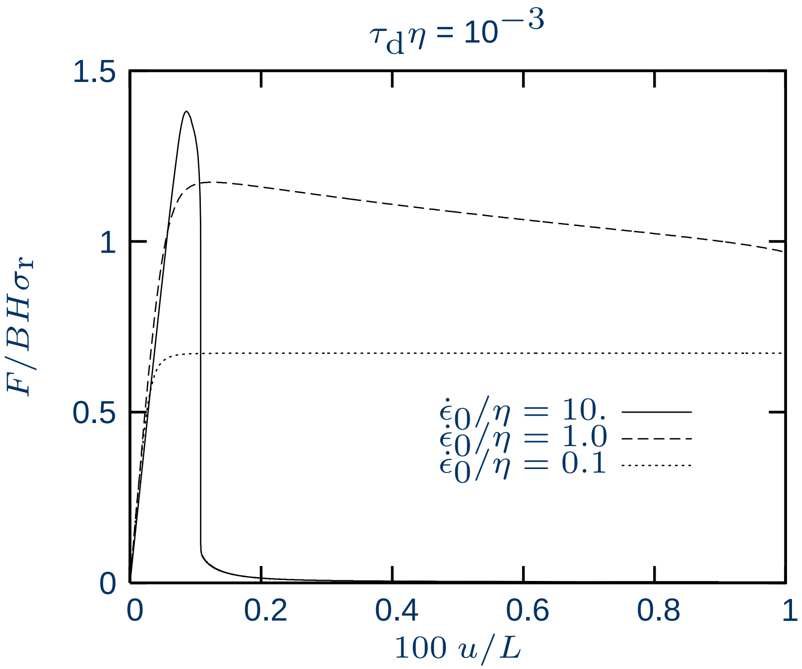
ATS $\quad \boldsymbol{C} = \omega(\boldsymbol{H}_{11} - \boldsymbol{h}_{12} H_{22}^{-1} \boldsymbol{h}_{21}^T)^{-1} \boldsymbol{C}_\mathrm{e}$

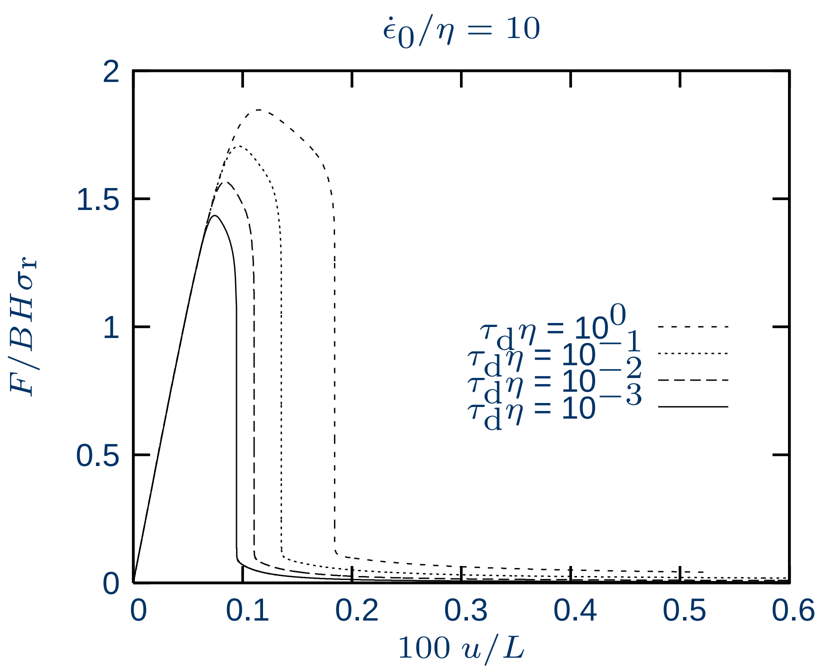
NUMERICAL EXAMPLE

von-Mises solid $\bar{\sigma} = \sigma_{\text{eff}}$, E = 40 GPa, $\nu = 0.3$, σ_{r} = 20 MPa, $\tau_{\text{vp}} = 1000$ s
transition strain rate $\eta = 10^{-3}$ s^{-1}, $p = r = n = 4$



Load-displacement curves, 12×6 mesh





CONCLUSIONS AND FURTHER DEVELOPMENTS

- Thermodynamically consistent formulation
- Easily extensible to more realistic damage and plasticity models
- Length scale ??
- More robust local itereration than bE — dG(1) or dG(1)-Lobatto ??
- Dynamics !!
- Alternative formulation using $\psi^*(\sigma, \alpha)$ and $\varphi(\dot{\epsilon}_{\mathrm{i}}, Z)$!!