A geometric approach in computational stability analysis

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Dedicated to Niels

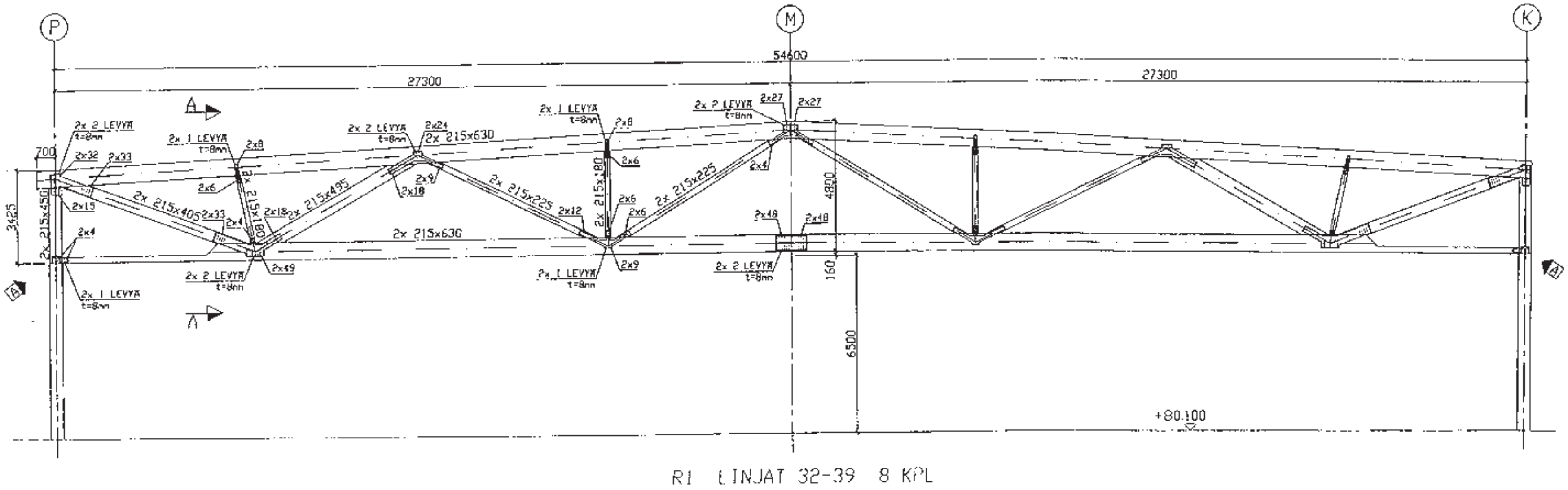
TAMPERE UNIVERSITY OF TECHNOLOGY

Outline

- Starting point
- The question of relevant buckling **mode**
- Stability eigenvalue problem
- Simple example
- Criticality manifold

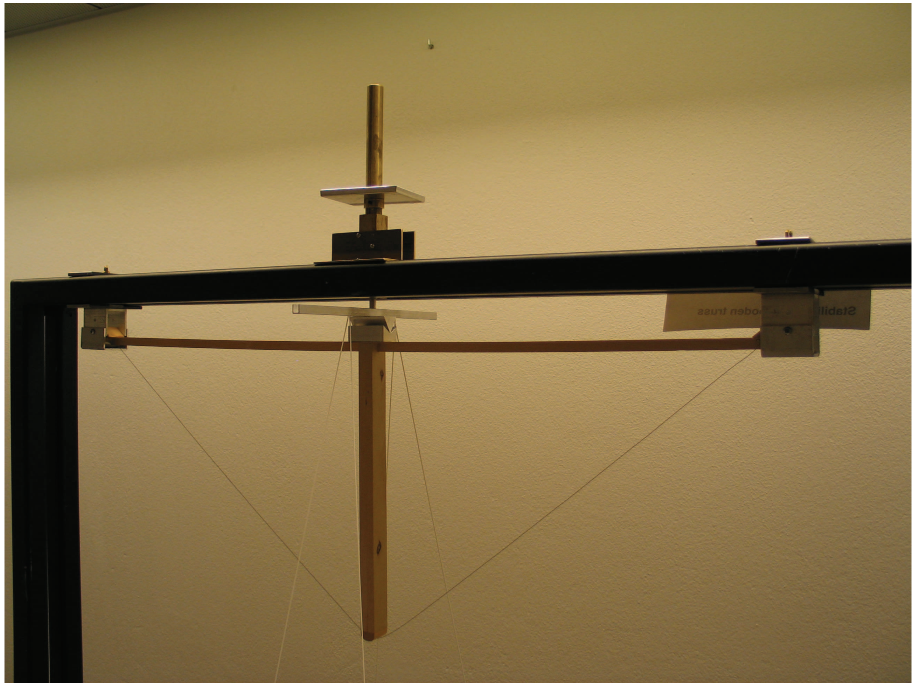
Photograph Accident Investigation Board Finland

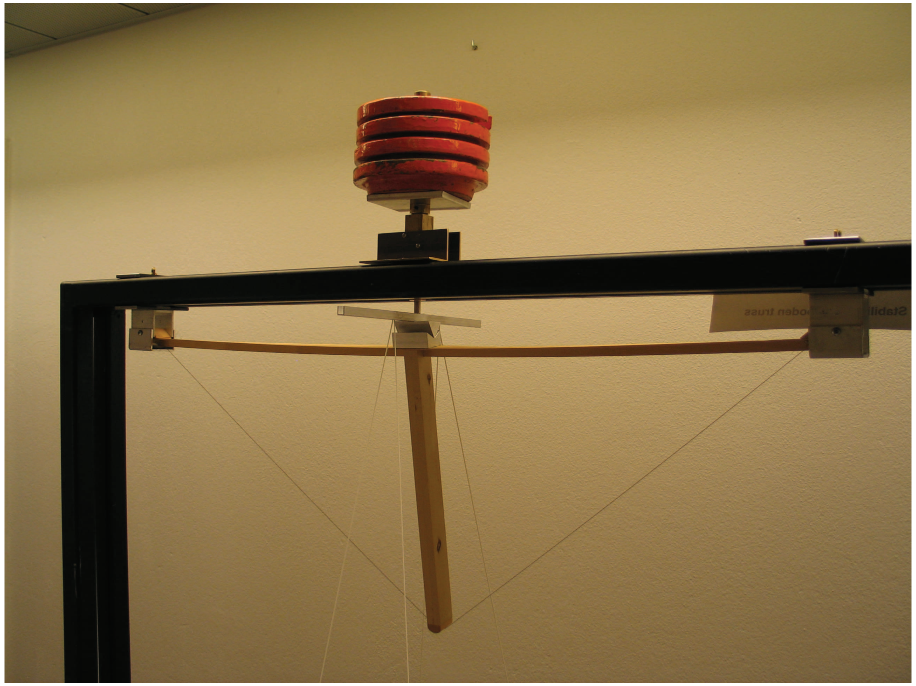
Collapse of Jyväskylä Fair Center roof
1st of Feb 2003



The questions

- What are the relevant buckling modes?
- Can they be adequately analysed by common computational tools?
- If not, how to analyse?





Stability eigenvalue problem

Definition for critical state: Find displacements $\boldsymbol{q}_{\mathrm{cr}}$, critical load λ_{cr} and the corresponding eigenmode $\boldsymbol{\phi}$ such, that

$$\boldsymbol{f}'(\boldsymbol{q}_{\mathrm{cr}}, \lambda_{\mathrm{cr}})\boldsymbol{\phi} = \boldsymbol{0} \quad \text{and} \quad \boldsymbol{f}(\boldsymbol{q}_{\mathrm{cr}}, \lambda_{\mathrm{cr}}) = \boldsymbol{0}, \tag{1}$$

where $\boldsymbol{f}' = \partial \boldsymbol{f}/\partial \boldsymbol{q}$. *The non-linear* mapping $\boldsymbol{f}$ defines the equilibrium path in the displacement $\boldsymbol{q}$ and load parameter λ space:

$$\boldsymbol{f}(\boldsymbol{q}, \lambda) \equiv \boldsymbol{r}(\boldsymbol{q}) - \lambda \boldsymbol{p}_r(\boldsymbol{q}) = \boldsymbol{0} \tag{2}$$

and constitutes the balance between internal- and external forces.

System (1) is a non-linear eigenvalue problem, which is HARD TO SOLVE!

Polynomial eigenvalue problem

Expanding the nl-ev-problem into Taylor's series wrt the state $(\boldsymbol{q}_*, \lambda_*)$

$$\boldsymbol{q} = \boldsymbol{q}_* + \Delta\lambda \boldsymbol{q}_1 + \frac{1}{2}(\Delta\lambda)^2 \boldsymbol{q}_2 + \cdots$$

results in a polynomial ev-problem:

$$\left(\boldsymbol{K}_{0|*} + \Delta\lambda \boldsymbol{K}_{1|*} + \frac{1}{2}(\Delta\lambda)^2 \boldsymbol{K}_{2|*} + \cdots\right) \boldsymbol{\phi} = \boldsymbol{0}$$

$$\boldsymbol{K}_{0|*} = \boldsymbol{f}'_*,$$

$$\boldsymbol{K}_{1|*} = \left.\frac{\mathrm{d}\boldsymbol{f}}{\mathrm{d}\lambda}\right|_* = \boldsymbol{f}''_* \boldsymbol{q}_1 + \dot{\boldsymbol{f}}'_*, \qquad \dot{\boldsymbol{f}} = \frac{\partial \boldsymbol{f}}{\partial \lambda},$$

$$\boldsymbol{K}_{2|*} = \left.\frac{\mathrm{d}^2\boldsymbol{f}}{\mathrm{d}\lambda^2}\right|_* = \boldsymbol{f}''_* \boldsymbol{q}_2 + \boldsymbol{f}'''_* \boldsymbol{q}_1 \boldsymbol{q}_1 + 2\dot{\boldsymbol{f}}''_* \boldsymbol{q}_1 + \ddot{\boldsymbol{f}}'_*$$

Linear stability eigenvalue problem

In the linear stability eigenvalue analysis the reference state is usually the initial state: $(\boldsymbol{q}_*, \lambda_*) = (\mathbf{0}, 0)$.[1] The eigenvalue problem to be solved is

$$\left(\boldsymbol{K}_{0|0} + \lambda \boldsymbol{K}_{1|0}\right) \boldsymbol{\phi} = \mathbf{0}$$

where the matrices are (assuming dead weight loading, i.e. $\dot{\boldsymbol{f}}' \equiv \mathbf{0}$)

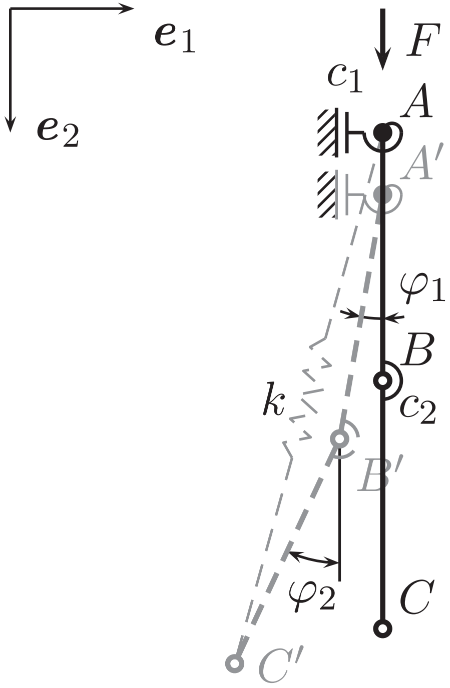
$$\boldsymbol{K}_{0|0} = \boldsymbol{f}'(\mathbf{0}, 0), \qquad \boldsymbol{K}_{1|0} = \boldsymbol{f}''(\mathbf{0}, 0)\boldsymbol{q}_1,$$

and the pre-buckling displacement field, $\boldsymbol{q}_1$, is solved from $\boldsymbol{K}_{0|0}\boldsymbol{q}_1 = \boldsymbol{p}_r$.

Simple to solve, but CAN THE SOLUTION BE COMPLETELY USELESS!

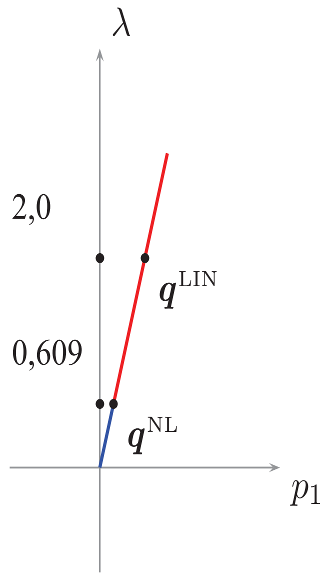
[1]This is usually the case in commercial FE software.

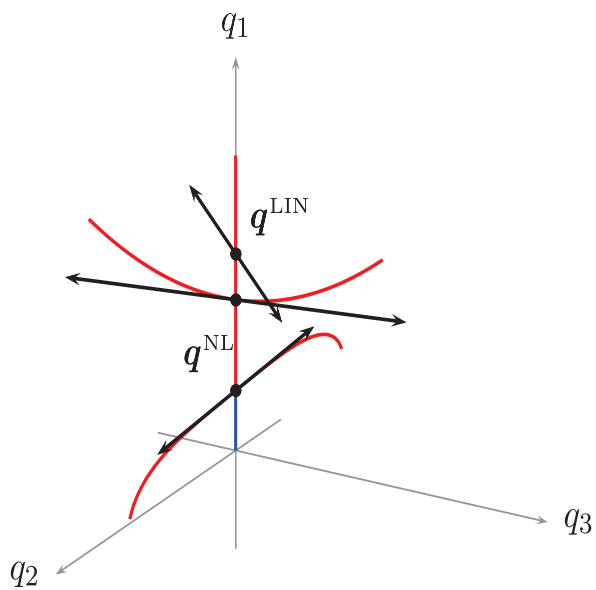
Simple example - state space



$\lambda^{\mathrm{LIN}} = 2{,}0$

$\lambda^{\mathrm{NL}} = 0{,}609$



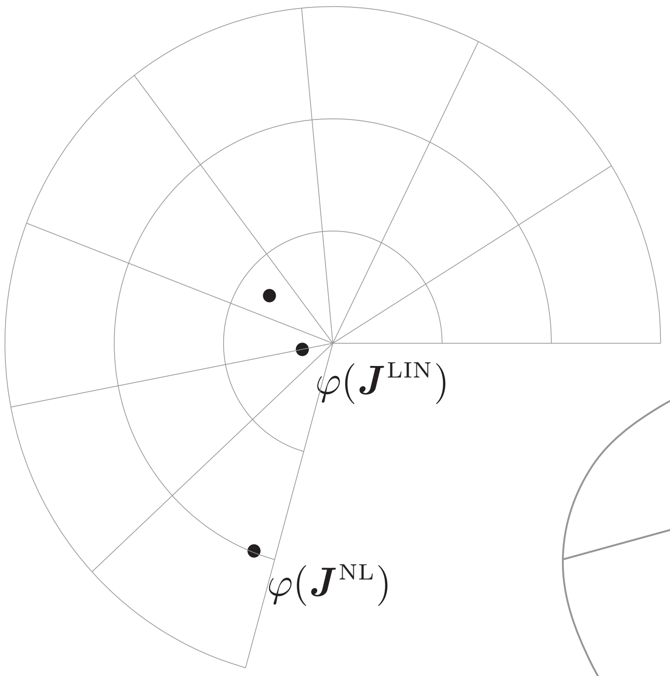


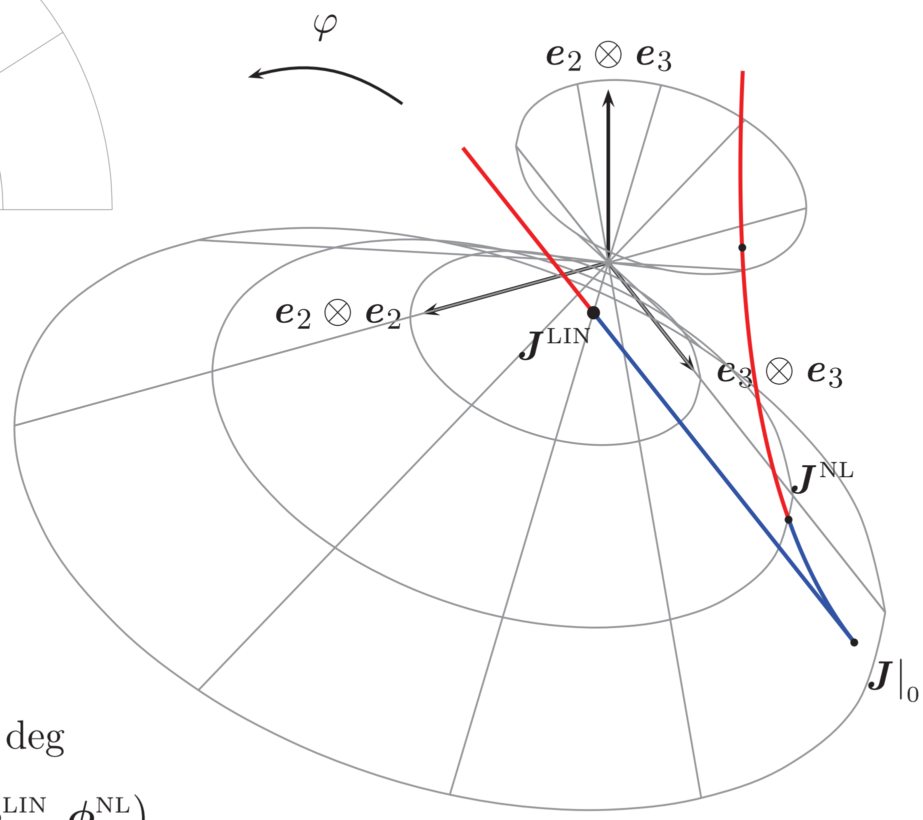
$$\frac{\lambda^{\mathrm{NL}} - \lambda^{\mathrm{LIN}}}{\lambda^{\mathrm{NL}}} = \text{-}2{,}281$$

$$1 - |\langle \boldsymbol{\phi}^{\mathrm{LIN}}, \boldsymbol{\phi}^{\mathrm{NL}} \rangle| = 0{,}2442$$

$$\angle(\boldsymbol{\phi}^{\mathrm{LIN}}, \boldsymbol{\phi}^{\mathrm{NL}}) = 40{,}9 \quad \mathrm{deg}$$

Simple example - criticality manifold

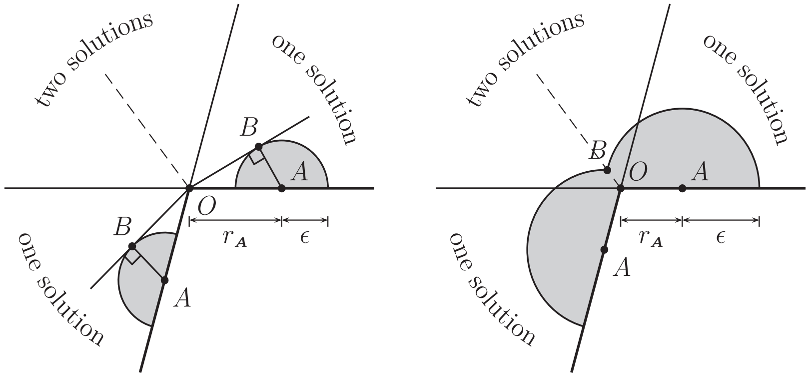


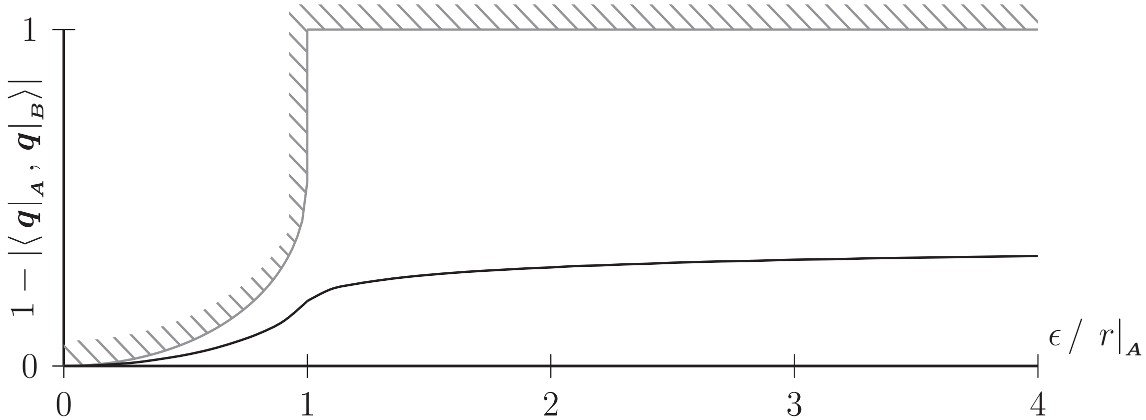


$$\angle\big(\varphi(\boldsymbol{J}^{\mathrm{LIN}}), \varphi(\boldsymbol{J}^{\mathrm{NL}})\big) = \; 57{,}85 \;\; \mathrm{deg}$$

$$\ldots = \sqrt{2}\,\angle\big(\boldsymbol{\phi}^{\mathrm{LIN}}, \boldsymbol{\phi}^{\mathrm{NL}}\big)$$

Error in the eigenmode





Concluding remarks

- Estimation of the error in the buckling mode
- Generalization of the error estimate to a general n-dimensional case
- Practical implications