On the direct solution of critical equilibrium states

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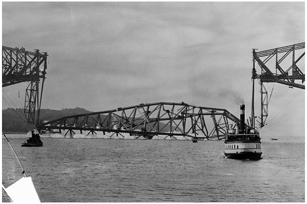
Introduction

- Definition of criticality
- Some Algorithms
- A proposal
- Example



Stability eigenvalue problem

Definition for a critical state: Find displacements $\boldsymbol{q}_{\rm cr}$, critical load $\lambda_{\rm cr}$ and the corresponding eigenmode $\boldsymbol{\phi}$ such, that

$$\begin{cases} \boldsymbol{f}'(\boldsymbol{q}_{\rm cr}, \lambda_{\rm cr})\boldsymbol{\phi} & = \boldsymbol{0} \\ \boldsymbol{f}(\boldsymbol{q}_{\rm cr}, \lambda_{\rm cr}) & = \boldsymbol{0} \end{cases} \tag{1}$$

where $\boldsymbol{f}' = \partial \boldsymbol{f} / \partial \boldsymbol{q}$. *The non-linear* mapping $\boldsymbol{f}$ defines the equilibrium path in the displacement $\boldsymbol{q}$ and load parameter λ space:

$$\boldsymbol{f}(\boldsymbol{q}, \lambda) \equiv \boldsymbol{r}(\boldsymbol{q}) - \lambda \boldsymbol{p}_r(\boldsymbol{q}) = \boldsymbol{0}$$

and constitutes the balance between internal- and external forces.

System (1) is a non-linear eigenvalue problem, which is
HARD TO SOLVE!

Solution algorithm for non-linear eigenproblem

Extended system:

$$\boldsymbol{g}(\boldsymbol{x}) = \boldsymbol{g}(\boldsymbol{q}, \boldsymbol{\phi}, \boldsymbol{\lambda}) = \begin{cases} \hat{\boldsymbol{f}}(\boldsymbol{q}, \boldsymbol{\lambda}) \equiv \boldsymbol{f}(\boldsymbol{q}, \lambda) + \boldsymbol{f}_0(\boldsymbol{q}, \boldsymbol{\lambda}) = \boldsymbol{0} \\ \boldsymbol{h}(\boldsymbol{q}, \boldsymbol{\phi}, \boldsymbol{\lambda}) \equiv \boldsymbol{f}'(\boldsymbol{q}, \boldsymbol{\lambda})\boldsymbol{\phi} + \boldsymbol{h}_0(\boldsymbol{\phi}, \boldsymbol{\lambda}) = \boldsymbol{0} \\ \boldsymbol{c}(\boldsymbol{q}, \boldsymbol{\phi}, \boldsymbol{\lambda}) = \boldsymbol{0}. \end{cases}$$

Newton's linearisation step results in

$$\boldsymbol{A}\delta\boldsymbol{x} = -\boldsymbol{g},$$

where

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{K}_f & \boldsymbol{0} & \boldsymbol{P} \\ \boldsymbol{Z} & \boldsymbol{K}_h & \boldsymbol{N} \\ \boldsymbol{C}_q & \boldsymbol{C}_\phi & \boldsymbol{C}_\lambda \end{bmatrix}, \quad \delta\boldsymbol{x} = \begin{Bmatrix} \delta\boldsymbol{q} \\ \delta\boldsymbol{\phi} \\ \delta\boldsymbol{\lambda} \end{Bmatrix}, \quad \boldsymbol{g} = \begin{Bmatrix} \hat{\boldsymbol{f}} \\ \boldsymbol{h} \\ \boldsymbol{c} \end{Bmatrix}.$$

Block elimination

The solution vector is partitioned as

$$\delta \boldsymbol{q} = \boldsymbol{q}_f + \boldsymbol{Q}_p \delta \boldsymbol{\lambda},$$
$$\delta \boldsymbol{\phi} = \boldsymbol{\phi}_h + \boldsymbol{\Phi}_n \delta \boldsymbol{\lambda}.$$

Vectors $\boldsymbol{q}_f, \boldsymbol{\phi}_h$ and the $n \times p$ matrices $\boldsymbol{Q}_p, \boldsymbol{\Phi}_n$ solved from

$$\boldsymbol{K}_f \boldsymbol{q}_f = -\hat{\boldsymbol{f}}, \qquad \boldsymbol{K}_f \boldsymbol{Q}_p = -\boldsymbol{P},$$
$$\boldsymbol{K}_h \boldsymbol{\phi}_h = -\boldsymbol{h} - \boldsymbol{Z} \boldsymbol{q}_{\mathrm{f}}, \qquad \boldsymbol{K}_h \boldsymbol{\Phi}_n = -\boldsymbol{N} - \boldsymbol{Z} \boldsymbol{Q}_p,$$

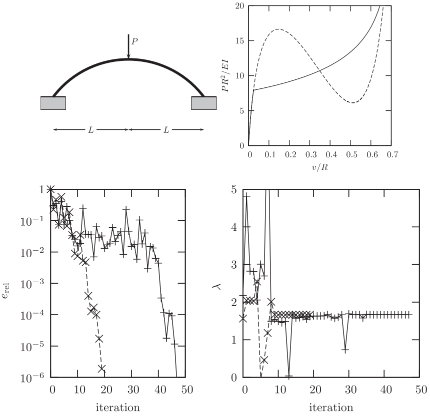
and for the control parameters

$$\delta \boldsymbol{\lambda} = -(\boldsymbol{C}_\lambda + \boldsymbol{C}_q \boldsymbol{Q}_p + \boldsymbol{C}_\phi \boldsymbol{\Phi}_n)^{-1} (\boldsymbol{c} + \boldsymbol{C}_q \boldsymbol{q}_f + \boldsymbol{C}_\phi \boldsymbol{\phi}_h).$$

Suitable strategy if direct linear solver is used.

Problems with the direct method

Since Newton's method is only *locally convergent* the method might fail if started from the unloaded, undeformed state.



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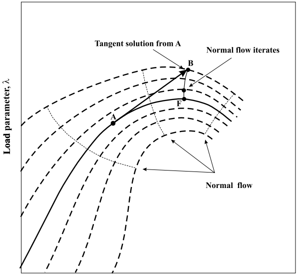
Mixed strategy

1. Compute crude approximation to the lowest critical load.
2. Use orthogonal trajectory method (normal flow) to get a nearby point on the equilibrium path.
3. Use the extended system started from the computed equilibrium point.



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Concluding remarks

An algorithm for direct critical point search which is belived to increase the robustness of the basic direct solution algorithm has been proposed.

Thank you for your attention!