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Development and numerical implementation of an anisotropic continuum damage model for concrete

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Outline

Introduction

- The non-linear behaviour of quasi-brittle materials under loading is mainly due to damage and micro-cracking rather than plastic deformation.
- Damage of such materials can be modelled using scalar, vector or higher order damage tensors.
- Failure of rock-like materials in tension is mainly due to the growth of the most critical micro-crack
- Failure of rock-like materials in compression can be seen as a cooperative action of a distributed microcrack array

http://mps-il.com

Ottosen's 4 parameter model

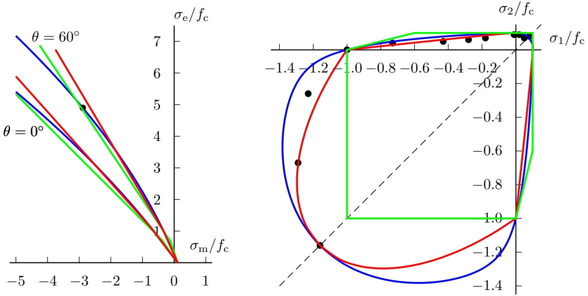
$$A\frac{J_2}{\sigma_c} + \Lambda\sqrt{J_2} + BI_1 - \sigma_{\rm c} = 0,$$

$$\Lambda = \begin{cases} k_1 \cos[\frac{1}{3}\arccos(k_2\cos 3\theta)] & \text{if} \quad \cos 3\theta \geqslant 0 \\ k_1 \cos[\frac{1}{3}\pi - \frac{1}{3}\arccos(-k_2\cos 3\theta)] & \text{if} \quad \cos 3\theta \leq 0 \end{cases}.$$

$$\cos 3\theta = \frac{3\sqrt{3}}{2}\frac{J_3}{J_2^{3/2}}, \text{: Lode angle}$$

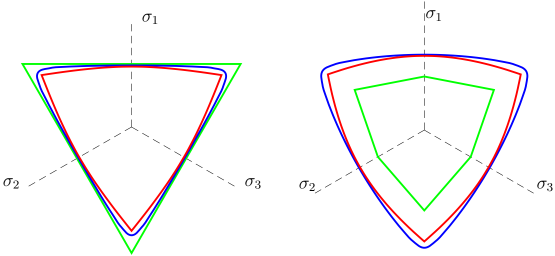
$\sigma_{\rm c}$: the uniaxial compressive strength
$I_1 = {\rm tr}\boldsymbol{\sigma}$: the first invariant of the stress tensor
$J_2 = \frac{1}{2}\boldsymbol{s}:\boldsymbol{s}$, $J_3 = \det \boldsymbol{s} = \frac{1}{3}{\rm tr}\boldsymbol{s}^3$: deviatoric invariants
A, B, k_1, k_2: material constants

Meridian plane & plane stress



Green line = Mohr-Coulomb with tension cut-off
Blue line = Ottosen's model
Red line = Barcelona model

Deviatoric plane



π – plane $\quad\quad$ $\sigma_{\mathrm{m}} = -f_{\mathrm{c}}$

Green line = Mohr-Coulomb with tension cut-off
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Thermodynamic formulation

Two potential functions

$\psi^{\mathrm{c}} = \psi^{\mathrm{c}}(S), \quad S = (\boldsymbol{\sigma}, \boldsymbol{D}, \kappa)$ Specific Gibbs free energy

$\gamma = \rho_0 \dot{\psi}^{\mathrm{c}} - \dot{\boldsymbol{\sigma}} : \boldsymbol{\epsilon}, \quad \gamma \geqslant 0$ Clausius-Duhem inequality

$\varphi(W; S), \quad W = (\boldsymbol{Y}, K)$ Dissipation potential

$$\gamma \equiv \boldsymbol{B}_Y : \boldsymbol{Y} + B_K K$$

$$\text{Define} \quad \boldsymbol{Y} = \rho_0 \frac{\partial \psi^c}{\partial \boldsymbol{D}} \quad K = -\rho_0 \frac{\partial \psi^c}{\partial \kappa}$$

$$\left(\rho_0 \frac{\partial \psi^c}{\partial \boldsymbol{\sigma}} - \boldsymbol{\epsilon} \right) : \dot{\boldsymbol{\sigma}} + \left(\dot{\boldsymbol{D}} - \boldsymbol{B}_Y \right) : \boldsymbol{Y} + (-\dot{\kappa} - B_K)\, K = 0$$

$$\boldsymbol{\epsilon} = \rho_0 \frac{\partial \psi^c}{\partial \boldsymbol{\sigma}}, \quad \dot{\boldsymbol{D}} = \boldsymbol{B}_Y, \quad \dot{\kappa} = -B_K$$

Specific model

Specific Gibbs free energy

$$\rho_0\psi^{\mathrm{c}}(\boldsymbol{\sigma},\boldsymbol{D},\kappa)$$
$$=\frac{1+\nu}{2E}\left[\mathrm{tr}\boldsymbol{\sigma}^2+\mathrm{tr}(\boldsymbol{\sigma}^2\boldsymbol{D})\right]-\frac{\nu}{2E}(1+\tfrac{1}{3}\mathrm{tr}\boldsymbol{D})(\mathrm{tr}\boldsymbol{\sigma})^2+\psi^{\mathrm{c},\kappa}(\kappa)$$

Elastic domain

$$\Sigma=\{(\boldsymbol{Y},K)|f(\boldsymbol{Y},K;\boldsymbol{\sigma})\leqslant 0\}$$

where the damage surface is defined as

$$f(\boldsymbol{Y},K;\boldsymbol{\sigma})=\frac{A\tilde{J}_2}{\sigma_{\mathrm{c}0}}+\Lambda\sqrt{\tilde{J}_2}+BI_1-(\sigma_{\mathrm{c}0}+K)=0$$

Invariants in terms of Y

$$\tilde{J}_2 = \frac{1}{1+\nu}\left[E\,\mathrm{tr}\boldsymbol{Y} - \tfrac{1}{6}(1-2\nu)(\mathrm{tr}\boldsymbol{\sigma})^2\right]$$

$$\tilde{J}_3 = \frac{2}{3(1+\nu)}\left\{E\left[\mathrm{tr}(\boldsymbol{\sigma}\boldsymbol{Y}) - \mathrm{tr}\boldsymbol{\sigma}\,\mathrm{tr}\boldsymbol{Y}\right] + \tfrac{1}{9}(1-2\nu)(\mathrm{tr}\boldsymbol{\sigma})^3\right\}$$

$$\varphi(\boldsymbol{Y}, K; \boldsymbol{\sigma}) = \mathrm{I}_\Sigma(\boldsymbol{Y}, K; \boldsymbol{\sigma})$$

where I_Σ is the indicator function

$$\mathrm{I}_\Sigma(\boldsymbol{Y}, K; \boldsymbol{\sigma}) = \begin{cases} 0 & \text{if} \quad (\boldsymbol{Y}, K) \in \Sigma \\ +\infty & \text{if} \quad (\boldsymbol{Y}, K) \notin \Sigma \end{cases}$$

$$(\boldsymbol{B}_Y, B_K) = \begin{cases} (\boldsymbol{0}, 0), & \text{if } f(\boldsymbol{Y}, K; \boldsymbol{\sigma}) < 0 \\ \left(\dot{\lambda}\dfrac{\partial f}{\partial \boldsymbol{Y}}, \dot{\lambda}\dfrac{\partial f}{\partial K}\right), \ \dot{\lambda} \geq 0, & \text{if } f(\boldsymbol{Y}, K; \boldsymbol{\sigma}) = 0 \end{cases}$$

$$\dot{\boldsymbol{D}} = \dot{\lambda}\frac{\partial f}{\partial \boldsymbol{Y}}, \quad \dot{\kappa} = -\dot{\lambda}\frac{\partial f}{\partial K}$$

1 Introduction

2 Ottosen's 4 parameter model

3 Thermodynamic formulation

4 Specific model

5 Some results

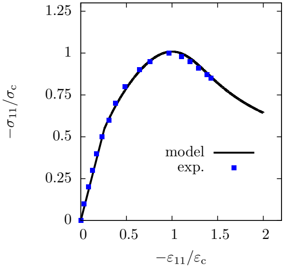
6 Conclusions and *future work*

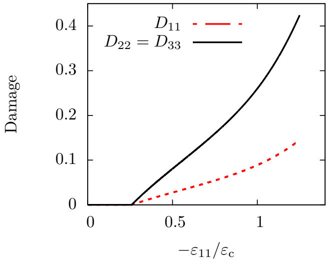
Some results

Uniaxial compression - ultimate compressive stength $\sigma_c = 32.8\,\text{MPa}$
$\sigma_{c0} = 18\,\text{MPa}, \quad \sigma_{t0} = 1\,\text{MPa}, \quad (I_1, \sqrt{J_2}) = (-5\sqrt{3}\sigma_{c0}, 4\sigma_{c0}/\sqrt{2})$
$A = 2.694, B = 5.597, k_1 = 19.083, k_2 = 0.998$

$$K = [a_1(\kappa/\kappa_{\max}) + a_2(\kappa/\kappa_{\max})^2]/[1 + b(\kappa/\kappa_{\max})^2]$$

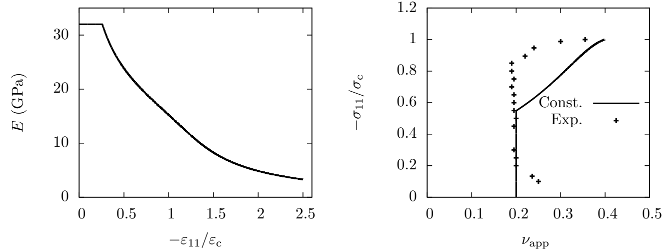
$a_1 = 85.3\,\text{MPa}, \quad a_2 = -12.65\,\text{MPa}, \quad b = 0.7032$



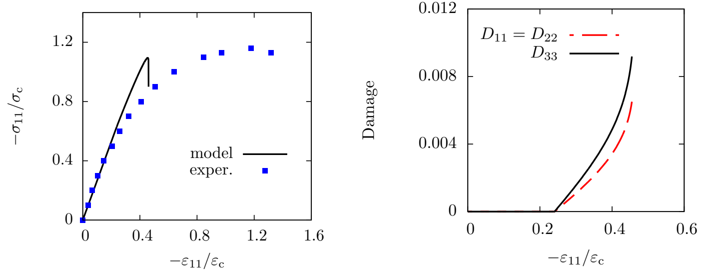


Experimental results from Kupfer et al. 1969.

Young's modulus and apparent Poisson's ratio

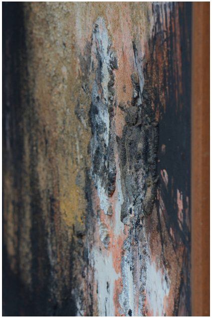


Biaxial compression



Conclusions and future work

- Continuum damage formulation of the Ottosen's 4 parameter model
- Can model axial splitting
- *Implementation into FE software (own codes, ABAQUS)*
- *Development of directional hardening model*
- *Regularization by higher order gradients*



Juana Francés (1926-1990) ∼ 1960

Thank you for your attention!