TAMPERE UNIVERSITY OF TECHNOLOGY

Computational modelling of HCF using a continuum based model

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Motivation

- **Classical fatigue models** can be considered as static criteria for alternating stress state and infinite life.
  For finite life predictions these criterion are augmented by damage accumulation rules based on cycles.
  - Problems:
    - Complex load histories - cycle counting methods based on well-defined cycles.
    - The effect of loading sequence is not taken into account.
- **Evolution equation based fatigue models** the endurance limit is described with a moving endurance surface.
  - The state variables in the endurance surface as well as damage are described using evolution equations.
  - Arbitrary loading histories can be treated in a unified manner.

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Endurance surface

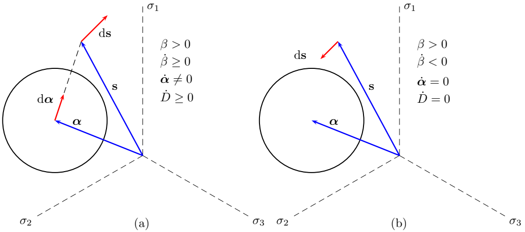
Basic idea

The endurance surface is defined in stress space as

$$\beta(\boldsymbol{\sigma}, \boldsymbol{\alpha}; \text{parameters}) = 0,$$

and the evolution of $\boldsymbol{\alpha}$ and damage D is defined as rate-equations

$$\dot{\boldsymbol{\alpha}} = \mathbf{A}(\boldsymbol{\sigma}, \boldsymbol{\alpha})\dot{\beta}, \qquad \dot{D} = g(\beta, D)\dot{\beta}.$$



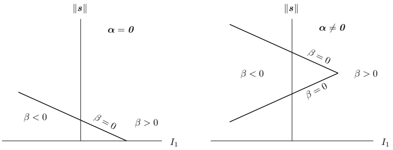
Isotropic HCF-model

Proposal by Ottosen, Stenström and Ristinmaa, 2008,

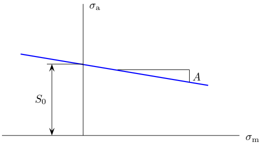
$$\beta = \frac{1}{S_0}\left(\bar{\sigma} + AI_1 - S_0\right) = 0, \qquad \beta \geq 0 \qquad \text{and} \qquad \dot{\beta} > 0,$$

$$\bar{\sigma} = \sqrt{3J_2(\boldsymbol{s} - \boldsymbol{\alpha})} = \sqrt{\tfrac{3}{2}(\boldsymbol{s} - \boldsymbol{\alpha}):(\boldsymbol{s} - \boldsymbol{\alpha})}, \qquad I_1 = \operatorname{tr}\boldsymbol{\sigma},$$

$$\dot{\boldsymbol{\alpha}} = C(\boldsymbol{s} - \boldsymbol{\alpha})\dot{\beta}, \qquad \dot{D} = K\exp(L\beta)\dot{\beta}.$$



Meridian plane



Reduces to Haigh-diag. in cyclic loading

Transversely isotropic HCF-model

Certain materials exhibit transversely isotropic symmetry as unidirectional composites or **forged metals**.

Shape of the endurance surface can depend of the invariants

$$I_1 = \operatorname{tr}\boldsymbol{\sigma},\ I_2 = \tfrac{1}{2}\operatorname{tr}\boldsymbol{\sigma}^2,\ I_3 = \tfrac{1}{3}\operatorname{tr}\boldsymbol{\sigma}^3,\ I_4 = \operatorname{tr}(\boldsymbol{\sigma}\boldsymbol{B}),\ I_5 = \operatorname{tr}(\boldsymbol{\sigma}^2\boldsymbol{B}),$$

where $\boldsymbol{B}$ is the structural tensor $\boldsymbol{B} = \boldsymbol{b} \otimes \boldsymbol{b}$ and $\boldsymbol{b}$ is the unit vector normal to the transverse isotropy plane.

The key idea in the transversely isotropic model is to split the stress as

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}_L + \boldsymbol{\sigma}_T, \quad \text{where}$$
$$\boldsymbol{\sigma}_T = \boldsymbol{P}\boldsymbol{\sigma}\boldsymbol{P} = \boldsymbol{\sigma} - \boldsymbol{\sigma}\boldsymbol{B} - \boldsymbol{B}\boldsymbol{\sigma} + \sigma_b\boldsymbol{B},$$

and $\boldsymbol{P} = \boldsymbol{I} - \boldsymbol{B}$ is the projection tensor, $\sigma_b = I_4 = \boldsymbol{b}\cdot\boldsymbol{\sigma}\cdot\boldsymbol{b}$.

Transversely isotropic endurance surface

Endurance surface for transversely isotropic HC-fatigue model

$$\beta = \left\{ \bar{\sigma} + A_{_L} I_{_{L1}} + A_{_T} I_{_{T1}} - \left[(1-\zeta) S_{_T} + \zeta S_{_L} \right] \right\} / S_{_T} = 0,$$

where

$$\bar{\sigma} = \sqrt{3 J_2(\boldsymbol{s} - \boldsymbol{\alpha})}, \quad I_{L1} = \operatorname{tr} \boldsymbol{\sigma}_L = I_4, \quad I_{T1} = \operatorname{tr} \boldsymbol{\sigma}_T = I_1 - I_4,$$

and

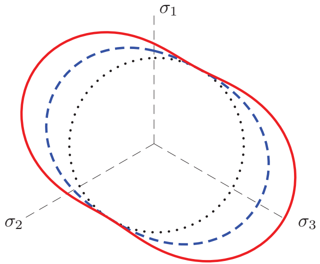
$$\zeta = \left(\frac{\boldsymbol{\sigma}_L : \boldsymbol{\sigma}_L}{\boldsymbol{\sigma} : \boldsymbol{\sigma}} \right)^n = \left(\frac{2 I_5 - I_4^2}{2 I_2} \right)^n .$$

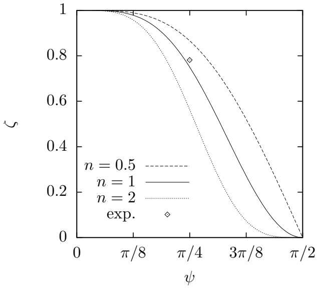
In uniaxial loading $\boldsymbol{\sigma} = \sigma \boldsymbol{n} \otimes \boldsymbol{n}$ the ζ-factor has the form

$$\zeta = (2 \cos^2 \psi - \cos^4 \psi)^n,$$

where ψ is the angle between $\boldsymbol{n}$ and $\boldsymbol{b}$.

Shape in the π-plane and ζ-factor





$S_L/S_T = 1$ dotted black line, 1.5 dashed blue line, 2 red line
$A_L = 0.225, A_T = 0.275, \boldsymbol{b} = (0, 0, 1)^T$

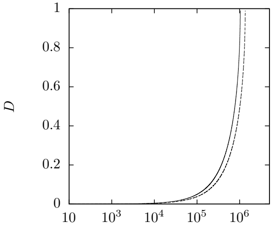
Damage evolution

Damage evolution equation modified to

$$\dot{D} = \frac{K}{(1-D)^k} \exp(L\beta)\dot{\beta},$$

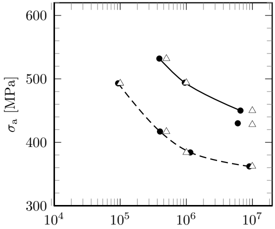
where the value $k = 1$ has been used.

Complicates sightly the parameter estimation.



Model calibration

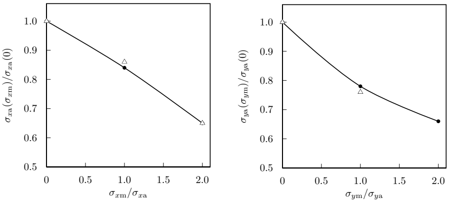
The model is calibrated for two steel grades: forged 34CrMo6 and isotropic AISI-SAE 4340 steel from $R = -1$ tests.



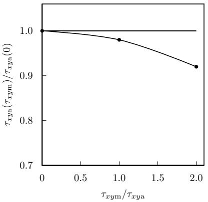
material	S_L [MPa]	S_T [MPa]	A_L	A_T	C	K	L
34CrMo6	447	360	0.225	0.300	33.6	$12.8 \cdot 10^{-5}$	4.0
AISI-SAE 4340	490	490	0.225	0.225	0.11	$1.46 \cdot 10^{-5}$	8.7

Results

The model describes well the mean stress effect in cyclic tension as well as the non-linear effect on mean shear stress on the fatigue strength.



cyclic normal stress in
longitudinal and transverse directions

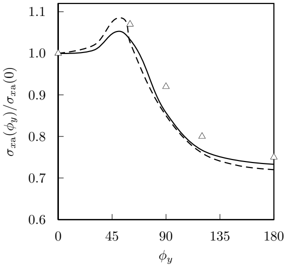


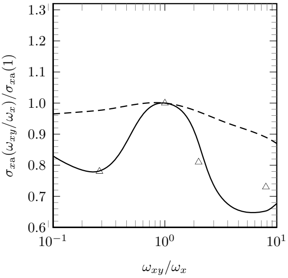
mean shear stress effect
on fatigue stress

Effect of phase- and frequency difference

$$\sigma_x = \sigma_{x\mathrm{m}} + \sigma_{x\mathrm{a}} \sin(\omega t)$$
$$\sigma_y = \sigma_{x\mathrm{m}} + \sigma_{x\mathrm{a}} \sin(\omega t - \phi_y)$$
$$\sigma_{x\mathrm{m}} = 1.105\sigma_{x\mathrm{a}}, \quad R = 0.05$$

$$\sigma_x = \sigma_{x\mathrm{a}} \sin(\omega_x t)$$
$$\tau_{xy} = \tfrac{1}{2}\sigma_{x\mathrm{a}} \sin(\omega_{xy} t)$$





data for isotropic AISI SAE 4340 (dashed line), 34CrMo6 (solid line)

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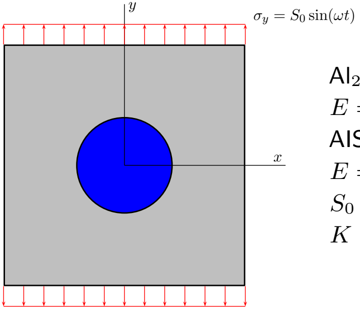
Test case - Inclusion problem

The model is implemented in Abaqus FE program using the UMAT subroutine.

Al_2O_3 inclusion in a steel plate in plane strain.



Al_2O_3 inclusion:

$E = 375$ GPa, $\nu = 0.22$,

AISI-SAE 4340 steel:

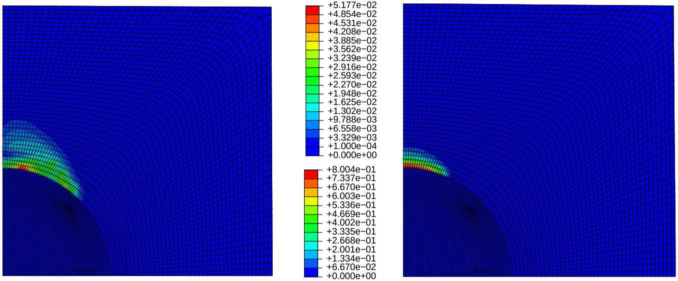
$E = 210$ GPa, $\nu = 0.3$

$S_0 = 490$ MPa, $A = 0.225, C = 0.11$

$K = 1.46 \cdot 10^{-5}, L = 8.7$

Influence of damage to the behaviour

Damage fields after the cycle 5500 and 8300. Effect of damage taken into account in the constitutive model



Fatigue life ≈ 8300 cycles.

6 Conclusions and *future developments*

Conclusions and future developments

- An evolution equation based multiaxial transversely isotropic HCF model is developed.
- It can be used for arbitrary loading histories.
- The model is implemented in the Abaqus FE-software using the UMAT subroutine.

- *evolution equation for ansiotropic damage*,
- *verification of evolution equations from micromechanics*, and
- *extension to LCF and high-temperature creep fatigue are under development*.

Thank you for your attention!

Amy Winehouse
Acrylic painting by Kelli Gedvil 2013.