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A CONSTITUTIVE MODEL FOR STRAIN-RATE DEPENDENT DUCTILE-TO-BRITTLE TRANSITION

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Summary. In this paper a simple phenomenological model to describe ductile to brittle transition of rate-dependent solids is presented. The model is based on consistent thermodynamic formulation using proper expressions for the Helmholtz free energy and the dissipation potential. In the model the dissipation potential is additively split into damage and visco-plastic parts and the transition behaviour is obtained using a stress dependent damage potential. Damage is described by using a vectorial variable.

1 INTRODUCTION

Most materials exhibit rate-dependent inelastic behaviour. Increasing strain-rate usually increases the yield stress thus enlarging the elastic range. However, the ductility is gradually lost and for some materials there exist a rather sharp transition strain-rate zone after which the material behaviour is completely brittle.

In this paper a simple phenomenological approach to model ductile to brittle transition of rate-dependent solids is presented. It is an extension to the model presented in [1,2]using vectorial damage variable[3]. The model is based on consistent thermodynamic formulation using proper expressions for the Helmholtz free energy and dissipation potential. The dissipation potential is additively split into damage and visco-plastic parts and the transition behaviour is obtained using a stress dependent damage potential. The basic features of the model are discussed.

2 THERMODYNAMIC FORMULATION

The constitutive model is derived using a thermodynamic formulation, in which the material behaviour is described completely through the Helmholz free energy and the dissipation potential in terms of the variables of state and dissipation and considering that the Clausius-Duhem inequality is satisfied[4].

The Helmholtz free energy

$$\psi = \psi(\boldsymbol{\epsilon}_{\rm e}, \mathbf{D}) \tag{1}$$

is assumed to be a function of the elastic strains, $\boldsymbol{\epsilon}_{\rm e}$, and the damage vector $\mathbf{D}$. Assuming small strains, the total strain can be additively decomposed into elastic and inelastic strains $\boldsymbol{\epsilon}_{\rm i}$ as $\boldsymbol{\epsilon} = \boldsymbol{\epsilon}_{\rm e} + \boldsymbol{\epsilon}_{\rm i}$.

The Clausius-Duhem inequality, in the absence of thermal effects, is formulated as

$$\gamma \geq 0, \quad \gamma = -\rho\dot{\psi} + \boldsymbol{\sigma} : \dot{\boldsymbol{\epsilon}}, \tag{2}$$

where ρ is the material density. As usual in the solid mechanics, the dissipation potential

$$\varphi = \varphi(\boldsymbol{\sigma}, \mathbf{Y}) \tag{3}$$

is expressed in terms of the thermodynamic forces $\boldsymbol{\sigma}$ and $\mathbf{Y}$ dual to the fluxes $\dot{\boldsymbol{\epsilon}}_{\rm i}$ and $\dot{\mathbf{D}}$, respectively. The dissipation potential is associated with the power of dissipation, γ, such that

$$\gamma = \frac{\partial\varphi}{\partial\boldsymbol{\sigma}} : \boldsymbol{\sigma} + \frac{\partial\varphi}{\partial\mathbf{Y}} \cdot \mathbf{Y}. \tag{4}$$

Using definition (4) equation $(2)_2$ and defining that $\rho\partial\psi/\partial\mathbf{D} = -\mathbf{Y}$, result in equation

$$\left(\boldsymbol{\sigma} - \rho\frac{\partial\psi}{\partial\boldsymbol{\epsilon}_{\rm e}}\right) : \dot{\boldsymbol{\epsilon}}_{\rm e} + \left(\dot{\boldsymbol{\epsilon}}_{\rm i} - \frac{\partial\varphi}{\partial\boldsymbol{\sigma}}\right) : \boldsymbol{\sigma} + \left(\dot{\mathbf{D}} - \frac{\partial\varphi}{\partial\mathbf{Y}}\right) \cdot \mathbf{Y} = 0. \tag{5}$$

Then, if eq. (5) holds for any evolution of $\dot{\epsilon}_{\rm e}$, σ and Y, inequality (2) is satisfied and the following relevant constitutive relations are obtained:

$$\boldsymbol{\sigma} = \rho\frac{\partial\psi}{\partial\boldsymbol{\epsilon}_{\rm e}}, \qquad \dot{\boldsymbol{\epsilon}}_{\rm i} = \frac{\partial\varphi}{\partial\boldsymbol{\sigma}}, \qquad \dot{\mathbf{D}} = \frac{\partial\varphi}{\partial\mathbf{Y}}. \tag{6}$$

3 PARTICULAR MODEL

In the present formulation the Helmholtz free energy, ψ, is a function depending on the symmetric second order strain tensor $\boldsymbol{\epsilon}_{\rm e}$ and the damage vector $\mathbf{D}$, the integrity basis thus consists of the following six invariants

$$I_1 = \operatorname{tr}\boldsymbol{\epsilon}_{\rm e}, \quad I_2 = \tfrac{1}{2}\operatorname{tr}\boldsymbol{\epsilon}_{\rm e}^2, \quad I_3 = \tfrac{1}{3}\operatorname{tr}\boldsymbol{\epsilon}_{\rm e}^3, \quad I_4 = \|\mathbf{D}\|, \quad I_5 = \mathbf{D}\cdot\boldsymbol{\epsilon}_{\rm e}\cdot\mathbf{D}, \quad I_6 = \mathbf{D}\cdot\boldsymbol{\epsilon}_{\rm e}^2\cdot\mathbf{D}. \tag{7}$$

A particular expression for the free energy, describing the elastic material behaviour with the directional reduction effect due to damage, is given by[3]

$$\begin{aligned}\rho\psi = {} & (1 - I_4)\left(\tfrac{1}{2}\lambda I_1^2 + 2\mu I_2\right) \\ & + H(\sigma^\perp)\frac{\lambda\mu}{\lambda + 2\mu}(I_4 I_1^2 - 2I_1 I_5 I_4^{-1} + I_5^2 I_4^{-3}) + (1 - H(\sigma^\perp))(\tfrac{1}{2}\lambda I_4 I_1^2 + \mu I_5^2 I_4^{-3}) \\ & + \mu\left(2I_4 I_2 + I_5^2 I_4^{-3} - 2I_6 I_4^{-1}\right),\end{aligned} \tag{8}$$

where λ and μ are the Lamé parameters, H is the Heaviside step-function and

$$\sigma^\perp = \lambda I_1 + 2\mu\hat{\mathbf{D}}\cdot\boldsymbol{\epsilon}_{\rm e}\cdot\hat{\mathbf{D}}, \quad \text{and} \quad \hat{\mathbf{D}} = \mathbf{D}/I_4. \tag{9}$$

To model the ductile-to-brittle transition due to increasing strain-rate, the dissipation potential is decomposed into the brittle damage part, φ_{d}, and the ductile viscoplastic part, φ_{vp}, as

$$\varphi(\boldsymbol{\sigma}, \mathbf{Y}) = \varphi_{\mathrm{d}}(\mathbf{Y})\varphi_{\mathrm{tr}}(\boldsymbol{\sigma}) + \varphi_{\mathrm{vp}}(\boldsymbol{\sigma}), \tag{10}$$

where the transition function, φ_{tr}, deals with the change in the mode of deformation when the strain-rate $\dot{\boldsymbol{\epsilon}}_{\mathrm{i}}$ increases. Applying an overstress type of viscoplasticity[5,6,7] and the principle of strain equivalence[8,9], the following choices are made to characterize the inelastic material behaviour:

$$\varphi_{\mathrm{d}} = \frac{1}{2r+2}\frac{Y_{\mathrm{r}}}{\tau_{\mathrm{d}}(1-I_4)}H(\epsilon_1 - \epsilon_{\mathrm{tresh}})\left(\frac{\mathbf{Y}\cdot\mathbf{M}\cdot\mathbf{Y}}{Y_{\mathrm{r}}^2}\right)^{r+1}, \tag{11}$$

$$\varphi_{\mathrm{tr}} = \frac{1}{pn}\left[\frac{1}{\tau_{\mathrm{vp}}\eta}\left(\frac{\bar{\sigma}}{(1-I_4)\sigma_{\mathrm{r}}}\right)^{p}\right]^{n}, \tag{12}$$

$$\varphi_{\mathrm{vp}} = \frac{1}{p+1}\frac{\sigma_{\mathrm{r}}}{\tau_{\mathrm{vp}}}\left(\frac{\bar{\sigma}}{(1-I_4)\sigma_{\mathrm{r}}}\right)^{p+1}, \tag{13}$$

where parameters τ_{d}, r and n are associated with the damage evolution, and parameters τ_{vp} and p with the visco-plastic flow. In addition, η denotes the inelastic transition strain-rate. The damage treshold strain is $\epsilon_{\mathrm{tresh}}$ and the largest principal strain is denoted as ϵ_1. Direction of the damage vector is defined through the tensor

$$\mathbf{M} = \mathbf{n}\otimes\mathbf{n} \tag{14}$$

where $\mathbf{n}$ is the eigenvector of the elastic strain tensor corresponding to the largest principal strain ϵ_1 and $\otimes$ denotes the tensor product. The relaxation times τ_{d} and τ_{vp} have the dimension of time and the exponents $r, p \geq 0$ and $n \geq 1$ are dimensionless. $\bar{\sigma}$ is a scalar function of stress, e.g. the effective stress $\sigma_{\mathrm{eff}} = \sqrt{3J_2}$, where J_2 is the second invariant of the deviatoric stress. The reference values Y_{r} and σ_{r} can be chosen arbitrarily, and they are used to make the expressions dimensionally reasonable. Since only isotropic elasticity is considered, the reference value Y_{r} has been chosen as $Y_{\mathrm{r}} = \sigma_{\mathrm{r}}^2/E$, where E is the Young's modulus.

Making use of eqs. (6), choices (8)-(13) yield the desired constitutive equations.

This particular model has the following general properties:

- Elastic stiffness is reduced monotonously due to damage.
- The model does not include any specific yield stress.
- In the absence of damage evolution, the inelastic model behaves under a constant uniaxial strain-rate loading as

  $$\sigma \to (\tau_{\mathrm{vp}}\dot{\epsilon}_0)^{1/p}\sigma_{\mathrm{r}} \quad \text{when} \quad t\to\infty,$$

  where $\dot{\epsilon}_0$ is a prescribed strain-rate;
- In the evolution of damage, the constraint for the damage $D = I_4 = \|\mathbf{D}\|$ that $D \in [0,1]$ is satisfied automatically, since initially $D = 0$, $\dot{\mathbf{D}} \geq 0$ and $\dot{\mathbf{D}} \to 0$ as $D \to 1$;
- The transition function φ_{tr} deals with the change in the mode of deformation through the damage evolution such that

  $$\varphi_{\mathrm{tr}} \geq 0 \quad \text{and} \quad \varphi_{\mathrm{tr}} \approx 0 \text{ when } \|\dot{\boldsymbol{\epsilon}}_{\mathrm{i}}\| < \eta \quad \text{and} \quad \varphi_{\mathrm{tr}} > 1 \text{ when } \|\dot{\boldsymbol{\epsilon}}_{\mathrm{i}}\| > \eta;$$

- Inequality (2) is satisfied *a priori* for any admissible isothermal process. Moreover, the dissipation potential (10) is a non-convex function with respect to the thermodynamic forces $\boldsymbol{\sigma}$ and $\mathbf{Y}$.
- The evolution of damage $(6)_3$ with the potential (11) will result in splitting damage in compression, while for tensile loading damage occurs on the plane perpendicular to the tensile stress[10].
- The form (8) of the Helmholtz free energy takes into account the directionality of damage. The crack deactivation criteria is based on the elastic normal stress acting on the damage plane.

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