Modelling Strain-Rate Dependent Ductile-to-Brittle Transition

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ABSTRACT

Most materials exhibit rate-dependent inelastic behaviour. Increasing strain-rate usually increases the yield stress thus enlarging the elastic range. However, the ductility is gradually lost and for some materials there exist a rather sharp transition strain-rate zone after which the material behaviour is completely brittle.

A phenomenological approach to model ductile to brittle transition of rate-dependent solids is presented. It is an extension to the model presented in [1] using vectorial damage variable [3]. The constitutive model is derived using a thermodynamic formulation, in which the material behaviour is described completely through the Helmholz free energy and the dissipation potential in terms of the variables of state and dissipation and considering that the Clausius-Duhem inequality is satisfied [2].

The Helmholtz free energy, $\psi = \psi(\boldsymbol{\epsilon}_\mathrm{e}, \mathbf{d})$, is assumed to be a function of the elastic strains, $\boldsymbol{\epsilon}_\mathrm{e}$, and the damage vector $\mathbf{d}$. Assuming small strains, the total strain can be additively decomposed into elastic and inelastic strains $\boldsymbol{\epsilon}_\mathrm{i}$ as $\boldsymbol{\epsilon} = \boldsymbol{\epsilon}_\mathrm{e} + \boldsymbol{\epsilon}_\mathrm{i}$. The dissipation potential is additively split into brittle damage, φ_d, and visco-plastic, φ_vp, parts as

$$\varphi(\boldsymbol{\sigma}, \mathbf{y})\varphi = \varphi(\boldsymbol{\sigma}, \mathbf{y}) = \varphi_\mathrm{d}(\mathbf{y})\varphi_\mathrm{tr}(\boldsymbol{\sigma}) + \varphi_\mathrm{vp}(\boldsymbol{\sigma}), \tag{1}$$

and is expressed in terms of the thermodynamic forces $\boldsymbol{\sigma}$ and $\mathbf{y}$ dual to the fluxes $\dot{\boldsymbol{\epsilon}}_\mathrm{i}$ and $\dot{\mathbf{d}}$, respectively. The transition function, φ_tr, deals with the change in the mode of deformation when the strain-rate $\dot{\boldsymbol{\epsilon}}_\mathrm{i}$ increases.

In the present formulation the Helmholtz free energy, ψ, is a function depending on the symmetric second order strain tensor $\boldsymbol{\epsilon}_\mathrm{e}$ and the damage vector $\mathbf{d}$, the integrity basis thus consists of the following six invariants

$$I_1 = \mathrm{tr}\,\boldsymbol{\epsilon}_\mathrm{e}, \quad I_2 = \tfrac{1}{2}\mathrm{tr}\,\boldsymbol{\epsilon}_\mathrm{e}^2, \quad I_3 = \tfrac{1}{3}\mathrm{tr}\,\boldsymbol{\epsilon}_\mathrm{e}^3, \quad I_4 = \|\mathbf{d}\|, \quad I_5 = \mathbf{d}\cdot\boldsymbol{\epsilon}_\mathrm{e}\cdot\mathbf{d}, \quad I_6 = \mathbf{d}\cdot\boldsymbol{\epsilon}_\mathrm{e}^2\cdot\mathbf{d}. \tag{2}$$

A particular expression for the free energy, describing the elastic material behaviour with the directional reduction effect due to damage, is given by [3]

$$\begin{aligned}\rho\psi = (1-I_4)\left(\tfrac{1}{2}\lambda I_1^2 + 2\mu I_2\right) + H(\sigma^\perp)\frac{\lambda\mu}{\lambda+2\mu}(I_4 I_1^2 - 2I_1 I_5 I_4^{-1} + I_5^2 I_4^{-3}) \\ + (1-H(\sigma^\perp))(\tfrac{1}{2}\lambda I_4 I_1^2 + \mu I_5^2 I_4^{-3}) + \mu\left(2I_4 I_2 + I_5^2 I_4^{-3} - 2I_6 I_4^{-1}\right), \quad (3)\end{aligned}$$

where λ and μ are the Lamé parameters, H is the Heaviside step-function and $\sigma^\perp = \lambda I_1 + 2\mu\hat{\mathbf{d}}\cdot\boldsymbol{\epsilon}_\mathrm{e}\cdot\hat{\mathbf{d}}$, and $\hat{\mathbf{d}} = \mathbf{d}/I_4$. Applying an overstress type of viscoplasticity [5] and the principle of strain equivalence [4], the following choices are made to characterize the inelastic material behaviour:

$$\varphi_\mathrm{d} = \frac{1}{2r+2}\frac{Y_\mathrm{r}}{\tau_\mathrm{d}(1-I_4)}H(\epsilon_1 - \epsilon_\mathrm{tresh})\left(\frac{(\mathbf{y}-\mathbf{y}_0)\cdot\mathbf{M}\cdot(\mathbf{y}-\mathbf{y}_0)}{Y_\mathrm{r}^2}\right)^{r+1}, \quad (4)$$

$$\varphi_\mathrm{tr} = \frac{1}{pn}\left[\frac{1}{\tau_\mathrm{vp}\eta}\left(\frac{\bar{\sigma}}{(1-I_4)\sigma_\mathrm{r}}\right)^p\right]^n, \quad (5)$$

$$\varphi_\mathrm{vp} = \frac{1}{p+1}\frac{\sigma_\mathrm{r}}{\tau_\mathrm{vp}}\left(\frac{\bar{\sigma}}{(1-I_4)\sigma_\mathrm{r}}\right)^{p+1}, \quad (6)$$

where parameters τ_d, r and n are associated with the damage evolution, and parameters τ_vp and p with the visco-plastic flow. In addition, η denotes the inelastic transition strain-rate. The damage treshold strain is ϵ_tresh and the largest principal strain is denoted as ϵ_1. Initiation of damage requires a small seed $\mathbf{y}_0$. Direction of the damage vector is defined through the tensor $\mathbf{M} = \mathbf{n}\otimes\mathbf{n}$, where $\mathbf{n}$ is the eigenvector of the elastic strain tensor corresponding to the largest principal strain ϵ_1 and $\otimes$ denotes the tensor product. The relaxation times τ_d and τ_vp have the dimension of time and the exponents $r, p \geq 0$ and $n \geq 1$ are dimensionless. $\bar{\sigma}$ is a scalar function of stress, e.g. the effective stress $\sigma_\mathrm{eff} = \sqrt{3J_2}$, where J_2 is the second invariant of the deviatoric stress. The reference values Y_r and σ_r can be chosen arbitrarily, and they are used to make the expressions dimensionally reasonable. Since only isotropic elasticity is considered, the reference value Y_r has been chosen as $Y_\mathrm{r} = \sigma_\mathrm{r}^2/E$, where E is the Young's modulus.

References

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