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ICE FAILURE ANALYSIS USING STRAIN-SOFTENING VISCOPLASTIC MATERIAL MODEL

Kari Kolari[1], **Reijo Kouhia**[2] **and Tuomo Kärnä**[1]

[1]VTT Building and Transport
PO Box 1806, FIN-02044 VTT e-mail: kari.kolari@vtt.fi, tuomo.karna@vtt.fi
web page: http://www.vtt.fi

[2]Laboratory of Structural Mechanics
P.O. Box 2100, FIN-02015 Helsinki University of Technology, Finland
e-mail: reijo.kouhia@hut.fi, web page: http://www.hut.fi/u/kouhia

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Abstract. *In this study the dynamic problem of ice-structure interaction is considered. Failure of ice is modeled by a strain softening viscoplastic material model where the dynamic yield function is of Nadreau-type. Either associated or non-assocated flow rules are considered and their implications are discussed both for physical and numerical points of view.*

1 INTRODUCTION

Failure of ice in the contact zone is a complex process. During a loading phase of ice-structure interaction the ice force increases. The loading phase in a contact point is followed by an unloading phase, where a major or secondary failure occurs at the contact point of ice edge. To model such phenomena, a proper description of the behaviour of ice is needed.

In this paper, a simple viscoplastic strain-softening material model is adopted. It is well known that a softening material behaviour results in a localisation of deformation and in a rate independent solid within the classical continuum description results in the loss of well-posedness of the governing initial-boundary value problem. The governing hyperbolic equations of motion ceases to be hyperbolic and became elliptic in the softening regime. Therefore, the domain is split into an elliptic part, in which the waves have imaginary wave speeds and are not able to propagate (standing waves) and into a hyperbolic part with propagating waves. As a consequence, a spurious sensitivity to discretizations is observed in numerical simulations of localisation problems. Several approaches to resolve this problem have been introduced in the literature. Perhaps the simplest one which preserves well-posedness of the governing equations is incorporation of the strain rate dependence to the constitutive model. The viscosity contribution introduces a material length scale even though the constitutive equations do not explicitly contain a parameter with the dimension of length. Other strategies like using micropolar continuum (or more generally microcontinuum) or higher-order gradient continuum models result in much more complicated equations and their numerical treatment is also more involved.

2 VISCOPLASTIC MATERIAL MODELS

Two major categories in formulating rate-dependent or viscoplastic material models exists: the overstress format and the consistency approach. Today, two mostly used overstress models are the Perzyna and the Duvaut-Lions models. In the Perzyna model [1], the direction of viscoplastic flow is determined by the gradient of a plastic potential function calculated at the current stress point. In the Duvault-Lions [2] model the viscoplastic flow is determined by the difference between the current stress point and the closest point projection onto a static yield surface, also called as the back-bone model. For Perzyna viscoplastic model, use of the postulate of maximum dissipation is rather involved. In the consistency models, a dynamic rate-dependent yield surface is defined which allows the use of the postulate of maximum dissipation in a straightforward manner, as shown by Ristinmaa and Ottosen [3].

Sluys has analysed the properties of several regularizing techniques for strain softening solids in his dissertation [4]. He has concluded that the viscoplastic models regularize the governing equations of motion at deformation states in fracture zones (mode-I localisation) and in shear bands (mode-II localisation). Wave propagation in viscoplastic solid is dispersive, which is necessary to capture localisation phenomena.

In the Perzyna model the viscoplastic strain rate is defined by

$$\dot{\epsilon}_{ij}^{\mathrm{vp}} = \frac{1}{\eta}\phi(F)\frac{\partial G}{\partial \sigma_{ij}}, \tag{1}$$

where η is the viscosity parameter and ϕ is some function of the yield function F and G is the plastic potential. Common choises for the overstress function ϕ are the power laws

$$\phi(F) = \left\langle \frac{F}{\sigma_0} \right\rangle^p \qquad \text{or} \qquad \phi(F) = \left\langle \frac{F}{\bar{\sigma}} \right\rangle^p, \tag{2}$$

in which p is a material parameter and $\bar{\sigma}, \sigma_0$ are the current yield stress and the initial value of it, respectively. The notation $\langle y \rangle$ refers to $yH(y)$ where H is the Heaviside unit step function.

In the consistency model, the viscoplastic behaviour is governed by a dynamic yield surface

$$f(\sigma_{ij}, K_\alpha, \dot{\epsilon}_{ij}^{\mathrm{vp}}, \dot{\kappa}_{ij}) = 0 \tag{3}$$

and a function of plastic potential

$$g = G(\sigma_{ij}, K_\alpha, \dot{\epsilon}_{ij}^{\mathrm{vp}}, \dot{\kappa}_{ij}) \tag{4}$$

where κ_α denotes a set of internal variables and K_α are the hardening variables, i.e. the thermodynamic forces conjugated to the fluxes $\dot{\kappa}_\alpha$. Evolution laws for the viscoplastic flow and the internal variables are then postulated as follows:

$$\dot{\epsilon}_{ij}^{\mathrm{vp}} = \dot{\lambda}\frac{\partial G}{\partial \sigma_{ij}} = \dot{\lambda} m_{ij}, \qquad \dot{\kappa}_\alpha = \dot{\lambda}\frac{\partial G}{\partial K_\alpha}. \tag{5}$$

The Perzyna type overstress model can be formulated within the consistency approach, thus giving

$$f = F(\sigma_{ij}, K_\alpha) - \sigma_0 \left(\sqrt{\frac{3}{2}} \frac{\eta \dot{\bar{\epsilon}}^{\mathrm{vp}}}{\|m_{ij}\|} \right)^{1/p} = F(\sigma_{ij}, K_\alpha) - \sigma_0(\eta\dot{\lambda})^{1/p} \tag{6}$$

if the first form of equations (2) is used and $\dot{\bar{\epsilon}}^{\mathrm{vp}} = \sqrt{\frac{2}{3}\dot{\epsilon}_{ij}^{\mathrm{vp}}\dot{\epsilon}_{ij}^{\mathrm{vp}}}$ is the rate of the equivalent viscoplastic strain,

3 FAILURE SURFACE OF ICE

Several failure or yield surfaces have been proposed for ice. In this paper the following Nadreau-type yield surface has been used [5, 6]:

$$f = \sqrt{3J_2} - R(\dot{\bar{\epsilon}}^{\mathrm{vp}})P(p, K_\alpha) \tag{7}$$

where J_2 is the second invariant of the deviatoric stress and p is the hydrostatic pressure $p = -\frac{1}{3}I_1 = -\frac{1}{3}\sigma_{ii}$. The rate- and pressure functions have the forms

$$R(\dot{\bar{\epsilon}}^{\rm vp}) = r_0 + (\eta\dot{\bar{\epsilon}}^{\rm vp})^n \tag{8}$$

$$P(p, K_\alpha) = \frac{1}{(C+T)^2}(C + K_1 - p)(2C + T - p)(T + K_2 + p) \tag{9}$$

where C and T are the compressive and tensile strengths in hydrostatic stress state. Material parameter r_0 defines the static yield surface and two parameters η, n are needed to define the viscoplastic behaviour. Hardening parameters K_1, K_2 determine the hardening/softening of the material.

4 ALGORITHMIC TREATMENT

4.1 Perzyna model

For rate-dependent solids implicit time integrators are preferable. The critical time step of explicit methods for the Perzyna type viscoplastic model is of order $\Delta t_{\rm cr} \sim \eta\sigma_0/(pE)$, which results in a value of order 10^{-3} s or smaller for typical values of the material parameters for ice. Especially for quasi-static cases it is untolerably small. In this study the backward Euler scheme is used to integrate the viscoplastic constitutive models.

The following algorithmic treatment follows closely the formulation given in refs. [7, 8] Incremental relation for stress and strain can be written as

$$\Delta\boldsymbol{\sigma} = \mathbf{C}^{\rm el}(\Delta\boldsymbol{\epsilon} - \Delta\boldsymbol{\epsilon}^{\rm vp}) \tag{10}$$

where the viscoplastic strain increment is

$$\Delta\boldsymbol{\epsilon}^{\rm vp} = \Delta\lambda\mathbf{m}, \qquad \text{where} \qquad \mathbf{m} = \frac{\partial G}{\partial\boldsymbol{\sigma}} \qquad \text{and} \qquad \Delta\lambda = \frac{\Delta t}{\eta}\phi(F). \tag{11}$$

The algorithmically consistent tangent matrix, which is needed in the global equilibrium equations, can be derived from the iterative counterpart of (11) and the iterative change of the viscoplastic strain is

$$\delta\boldsymbol{\epsilon}^{\rm vp} = \Delta\lambda\frac{\partial\mathbf{m}}{\partial\boldsymbol{\sigma}}\delta\boldsymbol{\sigma} + (\Delta\lambda\frac{\partial\mathbf{m}}{\partial\lambda} + \mathbf{m})\delta\lambda \tag{12}$$

Substituting expression (12) into the iterative counterpart of equation (10) results in

$$\delta\boldsymbol{\sigma} = \mathbf{H}\delta\boldsymbol{\epsilon} - \mathbf{H}\left(\mathbf{m} + \Delta\lambda\frac{\partial\mathbf{m}}{\partial\lambda}\right)\delta\lambda, \qquad \text{where} \qquad \mathbf{H} = \left((\mathbf{C}^{\rm el})^{-1} + \Delta\lambda\frac{\partial\mathbf{m}}{\partial\boldsymbol{\sigma}}\right)^{-1}. \tag{13}$$

The change in the viscoplastic multiplier can be solved from a scalar non-linear equation

$$r(\boldsymbol{\sigma}, \lambda) = \Delta\lambda - \frac{\Delta t}{\eta}\phi(\boldsymbol{\sigma}, \lambda) = 0. \tag{14}$$

By using the Newton's method the iterative change $\delta\lambda$ can be solved, giving

$$\delta\lambda = -\frac{r}{A} + \frac{\Delta t}{\eta A}\mathbf{n}^T\mathbf{H}\delta\boldsymbol{\epsilon}, \tag{15}$$

where

$$A = 1 + \frac{\Delta t}{\eta}\left[\mathbf{n}^T\mathbf{H}\left(\mathbf{m} + \Delta\lambda\frac{\partial\mathbf{m}}{\partial\lambda}\right) - \frac{\partial\phi}{\partial\lambda}\right] \qquad \text{and} \qquad \mathbf{n} = \frac{\partial F}{\partial\boldsymbol{\sigma}}. \tag{16}$$

Substituting the iterative change of the viscoplastic multiplier (15) back to the equation (13) gives the desired Jacobian matrix

$$\mathbf{C} = \mathbf{H} - \frac{\Delta t}{\eta A}\mathbf{H}\left(\mathbf{m} + \Delta\lambda\frac{\partial\mathbf{m}}{\partial\lambda}\right)\mathbf{n}^T\mathbf{H}. \tag{17}$$

This algorithmic tangent matrix is necessary for the Newton's method to obtain asymptotically quadratic convergence of the global equilibrium equations.

The local nonlinear problem (14) is also solved by the Newton's method. Increasing the power p in the constitutive model, makes the local problem more difficult and more iterations are needed in the Newton process to reach the asymptotic convergence domain.

4.2 Consistency model

Numerical integration of the consistency model (3), (5) is similar to the rate-intependent plasticity. The dynamic yield function (3) can be written as

$$f(\boldsymbol{\sigma}, \lambda, \dot{\lambda}) = 0. \tag{18}$$

During the viscoplastic flow, the stress stays on the yield surface and the consistency condition at some iterate i is

$$f^i + \frac{\partial f}{\partial\boldsymbol{\sigma}}\delta\boldsymbol{\sigma} + \frac{\partial f}{\partial\lambda}\delta\lambda + \frac{\partial f}{\partial\dot{\lambda}}\delta\dot{\lambda} = 0. \tag{19}$$

Since the strain rate is constant durin a step, the rate of the plastic multiplier can be approximated by $\dot{\lambda} = \Delta\lambda/\Delta t$ and the consistency condition can be written as

$$f^i + \mathbf{n}^T\delta\sigma + \left(\frac{\partial f}{\partial\lambda} + \frac{1}{\Delta t}\frac{\partial f}{\partial\dot{\lambda}}\right)\delta\lambda = 0. \tag{20}$$

Analogous to (12) the iterative change in the viscoplastic strain is

$$\delta\boldsymbol{\epsilon}^{\mathrm{vp}} = \Delta\lambda\frac{\partial\mathbf{m}}{\partial\boldsymbol{\sigma}}\delta\boldsymbol{\sigma} + \tilde{\mathbf{m}}\delta\lambda, \qquad \text{where} \qquad \tilde{\mathbf{m}} = \mathbf{m} + \Delta\lambda\frac{\partial\mathbf{m}}{\partial\lambda} + \frac{\Delta\lambda}{\Delta t}\frac{\partial\mathbf{m}}{\partial\dot{\lambda}}. \tag{21}$$

Substituting expression (12) into the consistency condition (20) results in

$$\delta\boldsymbol{\sigma} = \mathbf{H}\delta\boldsymbol{\epsilon} - \mathbf{H}\tilde{\mathbf{m}}\delta\lambda, \tag{22}$$

where the matrix $\mathbf{H}$ is given as in eq. (13). The iterative change of the viscoplastic multiplier can now be solved from the consistency condition, giving

$$\delta\lambda = \frac{f^i}{a}, \qquad \text{where} \qquad a = \mathbf{n}^T\mathbf{H}\tilde{\mathbf{m}} - \frac{\partial f}{\partial\lambda} - \frac{1}{\Delta t}\frac{\partial f}{\partial\dot{\lambda}}. \tag{23}$$

Newton's method is usually applied to solve the non-linear equations (14) or (20). In certain cases the convergence can be extremely slow. Recently, de Souza Neto [9] proposed a robust algorithm to solve the Perzyna type viscoplastic problem using a logarithmic transformation of equation (14). If the strain rate exponent n in the consistency model is below one, the derivative $\partial f/\partial\dot{\lambda}$ tends to infinity when $\dot{\epsilon}^{\text{vp}}$ approaches zero, and thus $a \to \infty$. Logarithmic scaling of the equation does not solve this problem. To overcome the difficulty a scaling for the rate of plastic multiplier is used in a form

$$x = \ln(\dot{\lambda}), \tag{24}$$

and the consistency condition (20) is expressed in terms of stress $\boldsymbol{\sigma}$ and the scaled quantity x. The iterative change of x is solved from the consistency condition, resulting in

$$\delta x = \frac{f^i}{a_{\text{sc}}}, \qquad \text{where} \qquad a_{\text{sc}} = \exp(x)\Delta t\mathbf{n}^T\mathbf{H}\tilde{\mathbf{m}} - \frac{\partial f}{\partial\lambda}\frac{\partial\lambda}{\partial x} - \frac{\partial f}{\partial\dot{\lambda}}\frac{\partial\dot{\lambda}}{\partial x}, \tag{25}$$

and the parameter a_{sc} in the scaled formulation is finite when $\dot{\epsilon}^{\text{vp}} \to 0$.

5 CONCLUDING REMARKS

A consistency viscoplasticity formulation is developed for ice-structure interaction problems. A pressure dependent dynamic failure surface of Nadreau-type is adopted and implicit scheme to solve the non-linear local problem is used. To avoid numerical difficulties in the solution process due to the rate-dependent term, a logarithmic scaling of the rate of the plastic multiplier is used. Since the failure surface is of teardrop-shaped, it forms a cone about the pressure axis. Therefore non-associated formulation is preferred where the plastic potential is a smooth surface. Further studies will be focused on developing transversally isotropic failure surface better suitable for columnar ice.

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