MODELLING OF ANISOTROPIC FATIGUE

Sami Holopainen[1], Reijo Kouhia [*1], Niels Saabye Ottosen[2], Matti Ristinmaa[2], and Timo Saksala[1]
[1]*Dep. of Mechanical Engineering and Industrial Design, Tampere University of Technology, Tampere, Finland*
[2]*Div. of Solid Mechanics, Lund University, Lund, Sweden*

Summary A continuum approach for anisotropic fatigue is described. The approach is based on the idea of a moving endurance surface in the stress space where the movement is described by a back-stress type tensor. The evolution associated with the movement is described by a rate type equation. In addition, damage accumulation is governed by a rate type evolution equation, thus facilitating its use under arbitrary complex loading conditions. The main emphasis of this paper is to discuss the possible forms of the endurance surface and pertinent evolution equations to model high-cycle anisotropic fatigue. Suggestions towards a unified model capturing the low-cycle regime are also given.

INTRODUCTION

Fatigue of materials under variable loads is a complicated physical process which can even result in catastrophic failure of engineering components. It is characterized by nucleation, coalescence and stable growth of cracks. Nucleation of cracks starts from stress concentrations near persistent slip bands, grain interfaces and inclusions [1, 2, 3].

In high-cycle fatigue, the macroscopic behavior of the material is primarily elastic, while in the low-cycle fatigue regime considerable macroscopic plastic deformations take place. Transition between low- and high-cycle fatigue occurs between $10^3 - 10^4$ cycles. In recent years, it has been observed that fatigue failures may occur for very large fatigue cycles in the order of $10^9 - 10^{10}$ cycles, i.e. below the previously assumed fatigue limits.

Main emphasis is here given to high-cycle fatigue modelling. Many different approaches have been proposed which can roughly be classified into stress invariant, or average stress based and critical plane approaches. In these approaches, damage accumulation is usually based on cycle-counting, which makes their use questionable under complex load histories [4, 5].

A different strategy for high-cycle fatigue modelling was proposed by Ottosen et al. [4]. In this approach, which could be classified as evolutionary, the concept of a moving endurance surface in the stress space is postulated together with a damage evolution equation. The endurance surface is expressed in terms of the first stress invariant and the second invariant of the reduced deviatoric stress tensor where the center of the surface in the deviatoric plane is defined by a deviatoric back stress tensor, as is done similarly in kinematic plasticity models. Therefore, the load history is memorized by the back-stress tensor. In this model, arbitrary stress states are treated in a unified manner for different loading histories, thus avoiding cycle-counting techniques.

EVOLUTION EQUATION BASED CONTINUUM FATIGUE MODELS

The key ingedients of the continuum based fatigue model are the assumption of the existence of an endurance surface defined in the stress space and the evolution equations describing the development of damage and internal variables needed in the model. A function β that depends on the stress tensor and some internal variables is established and the endurance surface is then defined as

$$\beta(\boldsymbol{\sigma}, \{\boldsymbol{\alpha}\}; \text{parameters}) = 0, \tag{1}$$

where $\boldsymbol{\sigma}$ is the stress tensor and $\{\boldsymbol{\alpha}\}$ denotes the set of internal variables. Evolution of the internal variables and the damage are described by the evolution equations

$$\{\dot{\alpha}\} = \{\boldsymbol{G}\}(\boldsymbol{\sigma}, \{\boldsymbol{\alpha}\})\dot{\beta}, \qquad \text{and} \qquad \dot{D} = g(\beta, D)\dot{\beta}. \tag{2}$$

The form of the functions $\boldsymbol{G}$ and g are important for modelling the finite life durability, while the endurance surface dictates the infinite life resistance. In contrast to plasticity the stress state can lie outside the endurance surface and the evolution of the internal variables and the damage take place only when $\beta \geq 0$ and $\dot{\beta} > 0$.

Isotropic fatigue

The original version of the model was developed for isotropic high-cycle fatigue [4] where the endurance surface was given in terms of two stress invariants: $I_1 = \operatorname{tr}\boldsymbol{\sigma}$ and $\bar{\sigma} = \sqrt{3J_2(\boldsymbol{s}-\boldsymbol{\alpha})} = \sqrt{(3/2)(\boldsymbol{s}-\boldsymbol{\alpha}):(\boldsymbol{s}-\boldsymbol{\alpha})}$. For the back stress evolution, the Ziegler type model was chosen and for the damage evolution equation the function g proportional to $\exp(\beta)$ has proven to be successfull. The endurance surface contains only two material parameters, which can be determined from two fatigue tests for infinite life. In the evolution laws three additional material parameters are needed.

An alternative formulation utilizing the idea of [4] for the endurance surface was presented by Brighenti et al. [6] containing all the three stress invariants of the isotropy group.

[*]Corresponding author. Email: reijo.kouhia@tut.fi

Transversally isotropic fatigue

For transverse isotropy, the unit vector $\boldsymbol{b}$ designates the privileged longitudinal direction. The integrity basis of a scalar function depending on a symmetric second order tensor and a vector contains six invariants. Since the vector $\boldsymbol{b}$ is a unit vector, the integity basis contains the following invariants

$$I_1 = \operatorname{tr}\boldsymbol{\sigma}, \quad I_2 = \tfrac{1}{2}\operatorname{tr}\boldsymbol{\sigma}^2, \quad I_3 = \tfrac{1}{3}\operatorname{tr}\boldsymbol{\sigma}^3, \quad I_4 = \operatorname{tr}(\boldsymbol{\sigma}\boldsymbol{B}), \quad I_5 = \operatorname{tr}(\boldsymbol{\sigma}^2\boldsymbol{B}), \tag{3}$$

where $\boldsymbol{B} = \boldsymbol{b}\otimes\boldsymbol{b}$ is the structural tensor for transverse isotropy. In terms of the stress deviator $\boldsymbol{s}$ the invariants 2,4 and 5 are defined as

$$J_2 = \tfrac{1}{2}\operatorname{tr}(\boldsymbol{s}^2), \quad J_4 = \operatorname{tr}(\boldsymbol{s}\boldsymbol{B}), \quad J_5 = \operatorname{tr}(\boldsymbol{s}^2\boldsymbol{B}). \tag{4}$$

In the model proposed in [7], the key idea is to split the stress tensor into the longitudinal and transverse parts $\boldsymbol{\sigma} = \boldsymbol{\sigma}_{\rm L} + \boldsymbol{\sigma}_{\rm T}$, where the transverse component is obtained from

$$\boldsymbol{\sigma}_{\rm T} = \boldsymbol{P}\boldsymbol{\sigma}\boldsymbol{P} = \boldsymbol{\sigma} - \boldsymbol{\sigma}\boldsymbol{B} - \boldsymbol{B}\boldsymbol{\sigma} + I_4\boldsymbol{B}, \tag{5}$$

where $\boldsymbol{P}$ is the projection tensor, $\boldsymbol{P} = \boldsymbol{I} - \boldsymbol{B}$.

It is known that fatigue failure is also dependent on the maximum shear stresses occurring in the longitudinal planes and in the transverse isotropy plane. These quantities can be expressed in terms of the deviatoric invariants J_2, J_4, and J_5 as

$$\tau_{\rm L}^2 = J_5 - J_4^2, \qquad \tau_{\rm T}^2 = J_2 + \tfrac{1}{4}J_4^2 - J_5. \tag{6}$$

The endurance surface for transversely isotropic fatigue given in [7] can be considered as the simplest possible extension of the evolution equation based fatigue model to transversally isotropic case. Only two additional parameters are needed in comparison to the isotropic model in [4].

In this paper different forms of the endurance surface in terms of invariants $I_1, \ldots I_5$ or I_1, I_3, J_2, J_4 and J_5 or $I_1, I_3, \tau_{\rm L}, \tau_{\rm T}$ are discussed. Special emphasis is given to the ease of determining the material parameters.

Orthotropic fatigue

In the case of orthotropic symmetry group, the integrity basis contains seven invariants, for example $\operatorname{tr}(\boldsymbol{\sigma}\boldsymbol{B}_1)$, $\operatorname{tr}(\boldsymbol{\sigma}\boldsymbol{B}_2)$, $\operatorname{tr}(\boldsymbol{\sigma}\boldsymbol{B}_3)$, $\operatorname{tr}(\boldsymbol{\sigma}^2\boldsymbol{B}_1)$, $\operatorname{tr}(\boldsymbol{\sigma}^2\boldsymbol{B}_2)$, $\operatorname{tr}(\boldsymbol{\sigma}^2\boldsymbol{B}_3)$ and $\operatorname{tr}(\boldsymbol{\sigma}^3)$, where $\boldsymbol{B}_i = \boldsymbol{b}_i\otimes\boldsymbol{b}_i$ are the structural tensors or orthotropic symmetry group. Also the necessity to have an anisotropic description for the damage variable is discussed.

Low-cycle fatigue

Extension of the evolution equation based continuum fatigue model to the low-cycle regime inevitably couples the constititive model into the fatigue model [8]. Here, the modifications in the endurance surface and the evolution equations are briefly discussed.

CONCLUSIONS

Different forms of endurance surfaces and pertinent evolution equations for various symmetries when modelling high-cycle fatigue are discussed. Behaviour of these models have been investigated. Special emphasis is given to simplicity and minimality in the number of material parameters, which is of paramount importance in fatigue analysis due to the expense of fatigue testing.

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