AN INVESTIGATION INTO THE BEHAVIOUR OF THE KACHANOV-RABOTNOV TYPE CONTINUUM DAMAGE MODEL

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Summary In this paper the behaviour of the Kachanov-Rabotnov type continuum damage constitutive model are studied. Dispersion analysis as well as numerical finite element study of one-dimensional dynamic problem have been performed. In the case of non-gradient solid, it is found from numerical simulations that the width of the localisation zone is independent of the mesh size only if the loading rate is below a certain threshold value.

INTRODUCTION

In the late 1950's Kachanov and Rabotnov [4, 8] introduced a well known model to describe continuous degradation of a material. Originally they used the model to predict the creep rupture time. Since then, continuum damage mechanics has developed into an important and active field of continuum mechanics, see e.g. [5, 6, 7]. Progressing degradation of elastic properties will eventually lead to strain softening behaviour, which can result to ill-posedness of the underlying partial differential equation system. In numerical solution the ill-posedness caused by strain softening produces severe mesh size dependency. Several remedies have been proposed to alleviate or remove this deficiency: non-local integral constitutive models, adding spatial gradients, rate-dependency, etc. In the case of the Kachanov-Rabotnov continuum damage model, the evolution equation for damage has the following form

$$\dot{D} = \frac{1}{t_{\mathrm{d}}(1-D)^p}\left(\frac{Y}{Y_{\mathrm{r}}}\right)^r, \tag{1}$$

where $Y = -\rho\partial\psi/\partial D$ is the thermodynamic force dual to the damage rate $\dot{D}$, Y_{r} a freely chosen reference value for Y. Material parameters are p, r (dimensionless) and t_{d}, which has dimension of time. The most important parameters for stress-strain behaviour are r and t_{d}, while p effects to behaviour in the post-peak strain softening region.

Even though the Kachanov-Rabotnov damage model (1) involves rate-dependency, it does not completely resolve the pathological mesh sensitivity from the numerical solution. As it is shown in [2], to obtain a mesh independent numerical solution, the loading rate should be below a certain threshold. This is in contrast to numerical solutions in viscoplasticity.

GRADIENT ENHANCEMENT

One approach to obtain a well-posed model is to add gradient terms to the constitutive model. In damage mechanics the effect of higher-order strain gradients has been studied in [1]. Damage gradient is used in [3], however, their approach needs an additional balance equation. Here, the following gradient enhancement to the classical Kachanov-Rabotnov model is proposed:

$$\dot{D} = \frac{1}{t_{\mathrm{d}}(1-D)^p}\left(\frac{Y}{Y_{\mathrm{r}}}\right)^r \langle 1 + \ell^2 \Delta D\rangle, \tag{2}$$

where Δ is the Laplace operator, ℓ is an additional material parameter having dimension of length and $\langle \bullet \rangle$ denotes the Macaulay brackets, i.e. the positive part of its argument.

To investigate the regularizing behaviour of the Kachanov-Rabotnov continuum damage model, a one-dimensional finite element and finite difference analyses are carried out. A bar fixed at $x = L$ and subjected at $x = 0$ to a linearly increasing displacement $u(0) = \eta\epsilon_{\mathrm{r}} L/t_{\mathrm{d}}$ ($L = c_{\mathrm{e}} t_{\mathrm{d}}$, where c_{e} is the elastic wave speed) has been analysed with different uniform mesh sizes [2]. A standard central difference scheme is used to integrate the equations of motion and in the gradient model the simple Euler forward method is used to integrate the evolution equation (2).

For the non-gradient damage model (1), the width of the localisation zone is shown as a function of the loading rate η in figure 1 for three different mesh sizes $h = L/100, L/1000$ and $L/10000$ and for the cases $r = 2, p = 1$, and $r = 4, p = 1$. As it can be seen, the width of the damage localisation zone is mesh-size independent if the loading rate satisfies $\eta \lessapprox 0.75$. Linearly interpolated finite elements for the spatial discretisation are used.

In figure 2 the damage profiles at time intervals $t_{\mathrm{d}}/100$ have been shown for the gradient model (2) when finite element and finite difference approach for the spatial discretisation is used. The additional material parameter has been $\ell = L/10$. Homogeneous natural boundary conditions have been used for the damage.

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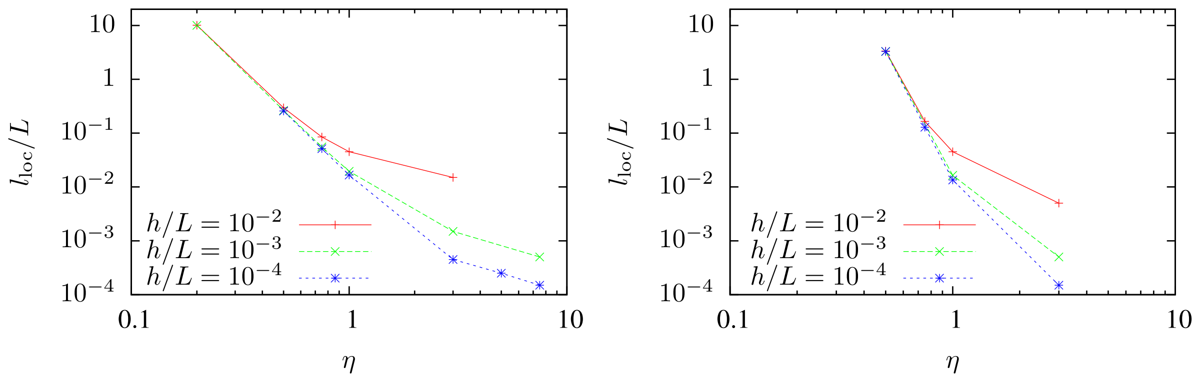

Figure 1: Damage localisation width as a function of the prescribed loading rate for the non-gradient model, $r = 2$ (lhs), $r = 4$ (rhs). In both cases $p = 1$.

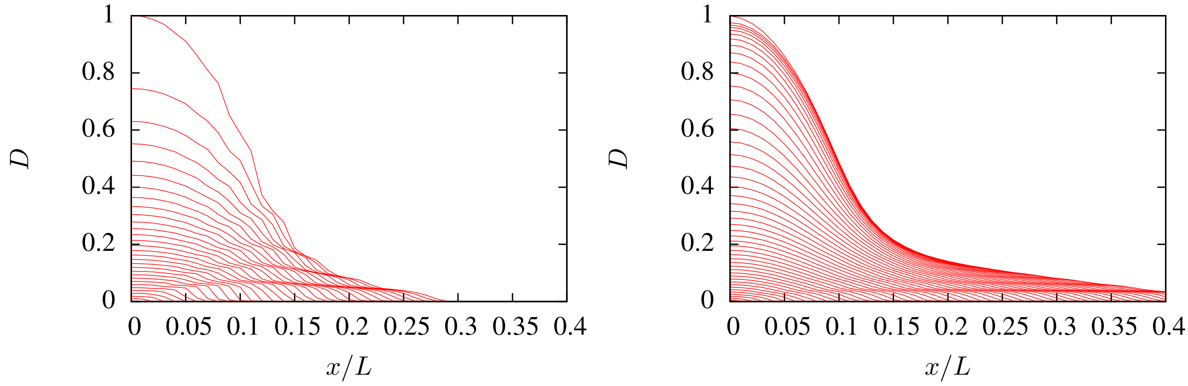

Figure 2: Damage profiles at different time instances for the gradient model, $h/L = 1/100$, $r = 4, p = 1, \ell = L/10$ and $\eta = 1$. Finite element (lhs) and finite difference solutions (rhs) shown.

CONCLUDING REMARKS

The classical Kachanov-Rabotnov continuum damage model has been shown to yield mesh-independent numerical solutions only if the loading rate is below a certain threshold. The dispersion analysis shows that the wave speeds of damaging material exceeds the elastic wave velocity. Based on a preliminary numerical study, addition of second-order derivatives to the damage evolution equation seems to result in mesh-independent solutions for a larger loading rate range. Further study will be focused on the dispersion analysis as well as on developing efficient numerical methods to solve the gradient enhanced model in 3D.

References

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