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A CONTINUUM DAMAGE MODEL FOR QUASI-BRITTLE MATERIALS

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Summary. A new formulation for modelling anisotropic damage of elastic-brittle materials based on Ottosen's 4-parameter failure surface and by using proper expressions for the specific Gibbs free energy and the complementary form of the dissipation potential is proposed. The formulation predicts the behaviour of the material relevantly and produces a realistic damage surface.

1 INTRODUCTION

Non-linear behaviour of quasi-brittle materials such as concrete has been the topic of various investigations in the last decades. It is well-known that the non-linear behaviour of such materials under loading is mainly due to damage and micro-cracking rather than plastic deformation.The generation of micro-cracks results in gradual loss of elasticity, volume dilatancy, strain-softening, etc. Several authors have been studied damage of concrete using a scalar, vector or higher order damage tensors.

In 1997, Murakami and Kamiya[3] modelled the anisotropic behaviour of elastic-brittle materials under multiaxial state of stress by using a second order damage tensor. They proposed a thermodynamically consistent formulation in which the Helmholtz free energy is described in terms of elastic strain and damage tensor and the damage dissipation potential is expressed in terms of damage conjugate force. A modified elastic strain tensor is introduced in the strain energy function to adjust the shape of the damage dissipation potential. However, the shape of the damage surface does not describe the well-known properties of some brittle materials such as concrete and consequently the ultimate fracture stress in uniaxial tension is approximately 25% of final uniaxial compressive stress.

In the present study, a new continuum damage formulation which has the same features as Ottosen's 4-parameter criterion[4] is proposed. The elastic behavior of the material is captured by a specific Gibbs free energy function with stress and the second rank symmetric damage tensor as the state variables and internal variable κ which controls the size of the damage surface. The complementary dissipation potential is formulated using Ottosen's 4-parameter criterion in the damage conjugate force space. The new formulation predicts a proper shape for the damage surface which is similar to the shape obtained by Ottosen's model[5].

2 FORMULATION

Assuming that deformations are fully elastic, a particular expression for the specific Gibbs free energy, describing the elastic material behaviour with the reduction effect due to damage, is given by

$$\rho_0\psi^{\mathrm{c}}(\boldsymbol{\sigma},\boldsymbol{D},\kappa) = \frac{1+\nu}{2E}\mathrm{tr}\boldsymbol{\sigma}^2 - \frac{\nu}{2E}(1+\alpha_1\mathrm{tr}\boldsymbol{D})(\mathrm{tr}\boldsymbol{\sigma})^2 + \frac{\alpha_2}{E}\mathrm{tr}(\boldsymbol{\sigma}^2\boldsymbol{D}) + H_0\left(\frac{\kappa}{\kappa_0}\right)^2\left(\frac{\kappa}{3\kappa_0}-\frac{1}{2}\right). \quad (1)$$

In equation (1), $\boldsymbol{\sigma}$ and $\boldsymbol{D}$ are the stress and damage tensors, respectively. Also, ρ_0 is the density of the material, ν denotes the Poisson's ratio, E stands for the elastic modulus and α_1, α_2, H_0 and κ_0 are newly introduced material parameters.

The damage surface is defined by reformulating the Ottosen's 4-parameter failure surface in terms of the thermodynamic force $\boldsymbol{Y}$, hardening variable K, and stress $\boldsymbol{\sigma}$ as

$$f(\boldsymbol{Y},K;\boldsymbol{\sigma}) = \frac{A\tilde{J}_2}{\sigma_{\mathrm{c0}}} + \Lambda\sqrt{\tilde{J}_2} + BI_1 - (\sigma_{\mathrm{c0}}+K) = 0, \quad (2)$$

where

$$\Lambda = \begin{cases} k_1\cos[\frac{1}{3}\arccos(k_2\cos 3\theta)] & \text{if} \quad \cos 3\theta \geq 0 \\ k_1\cos[\frac{1}{3}\pi - \frac{1}{3}\arccos(-k_2\cos 3\theta)] & \text{if} \quad \cos 3\theta \leq 0 \end{cases}. \quad (3)$$

and the redefined deviatoric invariants have expressions

$$\tilde{J}_2 = \frac{1}{2}\left[\frac{E}{\alpha_2}\mathrm{tr}\boldsymbol{Y} + (\mathrm{tr}\boldsymbol{\sigma})^2\left(\frac{3\alpha_1\nu}{2\alpha_2}-\frac{1}{3}\right)\right], \quad (4)$$

$$\tilde{J}_3 = \frac{1}{3}\left[\frac{E}{\alpha_2}(\mathrm{tr}(\boldsymbol{\sigma}\boldsymbol{Y}) - \mathrm{tr}\boldsymbol{\sigma}\mathrm{tr}\boldsymbol{Y}) + \left(\frac{2}{9}-\frac{\alpha_1\nu}{\alpha_2}\right)(\mathrm{tr}\boldsymbol{\sigma})^3\right]. \quad (5)$$

Moreover, $\cos 3\theta$ is the Lode angle in terms of the redefined deviatoric invariants. In equations (2) and (3) σ_{c0} denotes the initial elastic limit in uniaxial compression and A, B, k_1 and k_2 are material constants. The constitutive equations for the model are obtained using the following equations:

$$\boldsymbol{\varepsilon} = \rho_0\frac{\partial\psi^{\mathrm{c}}}{\partial\boldsymbol{\sigma}}, \quad \boldsymbol{Y} = \rho_0\frac{\partial\psi^{\mathrm{c}}}{\partial\boldsymbol{D}}, \quad \dot{\boldsymbol{D}} = \dot{\lambda}\frac{\partial f}{\partial\boldsymbol{Y}}, \quad K = -\rho_0\frac{\partial\psi^{\mathrm{c}}}{\partial\kappa}, \quad \dot{\kappa} = -\dot{\lambda}\frac{\partial f}{\partial K}. \quad (6)$$

Above, $\dot{\lambda}$ is a multiplier that can be calculated by use of the consistency condition $\dot{f} = 0$.

3 NUMERICAL RESULTS

The constitutive equations are solved for a concrete body with the ultimate compressive strength $\sigma_{\mathrm{c}} = 32.8$ MPa under uniaxial tension, uniaxial compression and biaxial compression. The values of the material parameters are:

$$E = 32\,\mathrm{GPa}, \quad \nu = 0.2 \quad A = 2.9782, \quad B = 6.0781, \quad k_1 = 20.5560, \quad k_2 = 0.9990. \tag{7}$$

Furthermore, $\sigma_{\mathrm{c0}} = 18$ MPa and $\sigma_{\mathrm{t0}} = 1$ MPa, where σ_{t0} is the initial elastic limit in uniaxial tension. The values below for the remaining parameters are determined by using experimental results of uniaxial compression test found in literature[1]

$$\alpha_1 = 5.2, \quad \alpha_2 = 10, \quad H_0 = 1353.36\,\mathrm{Pa}, \quad \kappa_0 = 8.15 \times 10^{-6}. \tag{8}$$

In figures (1) and (2), $\varepsilon_{\mathrm{c}} = 0.212$ % denotes the value of strain when stress reaches the maximum stress σ_{c}. As can be seen in the figures, the model is able to predict the behavior of the material in uniaxial compression and tension. The ratio between the maximum tensile and compressive strengths is $\sigma_{\mathrm{t}}/\sigma_{\mathrm{c}} = 0.1$, which is characteristic of elastic-brittle materials such as concrete[2]. The results, however, are not alike in biaxial compression case.

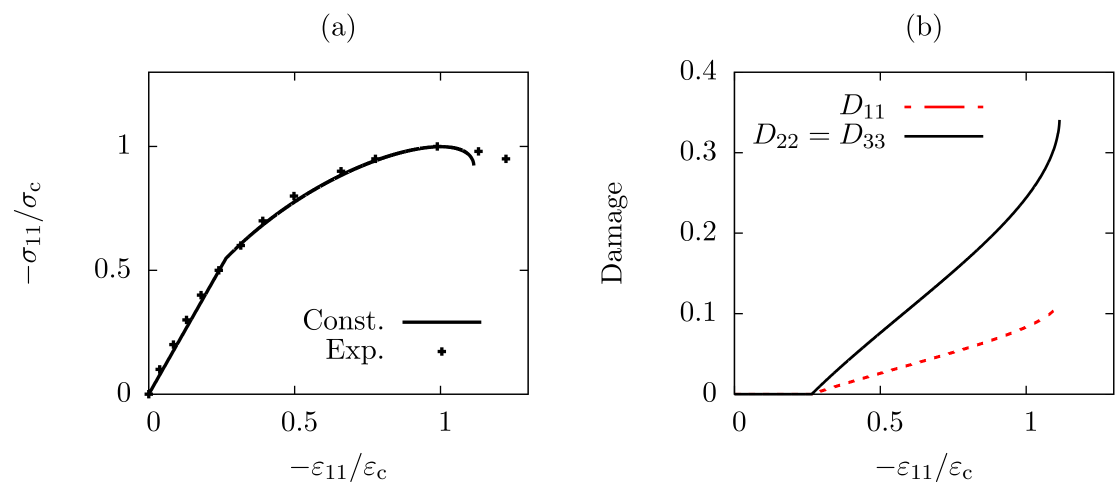

Figure 1: (a) Predicted stress-strain curve of the constitutive model for concrete specimen under uniaxial compression compared to experimental results[1], (b) Predicted damage-strain curves of the constitutive model for the concrete specimen under uniaxial compression.

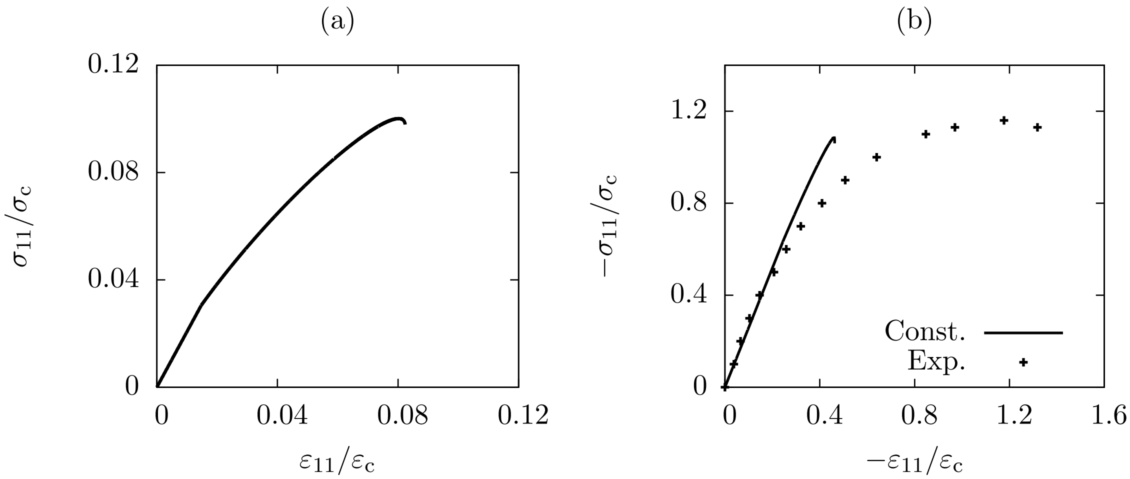

Figure 2: (a) Predicted stress-strain curve of the constitutive model for concrete specimen under uniaxial tension, (b) Predicted stress-strain curve of the constitutive model for concrete specimen under equibiaxial compression compared to experimental results [1].

4 CONCLUSIONS

In this study, the 4-parameter Ottosen's failure model is reformulated in a damage conjugate force space. Using the second order damage tensor as a state variable makes it possible to predict the anisotropic behaviour of damage for elastic-brittle materials in simple loading cases. However, the results show some shortcomings when more general loading cases are considered. For example, the model does not predict the behavior of the material under biaxial compression properly. On the other hand, the approach can be easily extended to more advanced damage models.

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