MEI-55200 Numerical methods for field problems

7. Exercise: time integration methods

1. Perform the numerical solution of the Maxwell-type creep problem

$$\sigma = E(\epsilon - \epsilon_{\mathrm{c}}). \tag{1}$$

The creep strain rate $\dot{\epsilon}_{\mathrm{c}}$ is obtained from

$$\dot{\epsilon}_c = \tau_{\mathrm{pr}}^{-1}\left(\frac{\sigma}{\sigma_{\mathrm{r}}}\right),$$

where τ_{pr} is the "pseudo"relaxation time (constant) and σ_{r} is a reference stress (constant). The relaxation time is $\tau = \tau_{\mathrm{pr}}\epsilon_{\mathrm{r}}$, where $\epsilon_{\mathrm{r}} = \sigma_r/E$.

Formulate the problem as a first order ordinary differential equation for the stress and solve it numerically using the implicit Euler method. The loading is a constant strain rate: $\epsilon(t) = \tau^{-1}\epsilon_{\mathrm{r}}t$. Integrate to the final time $t = 4\tau$ by using a time step $\Delta t = 2\tau$. **Hint:** Formulate the equation (1) is a dimensionless form using a dimensionless stress $y = \sigma/\sigma_{\mathrm{r}}$. When you differentiate the equation (1) w.r.t. time, you can assume the Young's modulus E to be a constant.

2. Investigate the stability of the Crank-Nicolson scheme for the problem

$$\dot{y} + a(t)y = 0, \quad y(0) = y_0, \quad \text{where} \quad a(t) > 0.$$

Home exercise: Consider time-dependent diffusion problem

$$\begin{aligned} \rho c\dot{u} - ku'' &= 0,\ 0 < x < L, \\ q(0) = q_0 &= -\alpha_0(u - u_0), \\ q(L) = q_L &= \alpha_L(u - u_L). \end{aligned}$$

The problem describes temperature evolution e.g. in a wall.

1. Write the semi-discrete form of the diffusion problem using one linear finite element. Assume that the outdoor temperature u_0 changes in time as $u_0 = u_L\cos(\pi t/T)$ and the indoor temperature is constant u_L.
   In the boundary terms, the parameter α is the heat transfer coefficient. In general it depends on temperature of the surface, but in this case the problem will be non-linear. Let's assume it to be constant. The dimensionless ratio

   $$B_i = \frac{\alpha_i L}{k}$$

   is the Biot number.
2. Solve the problem by using (a) the explicit-, (b) the implicit Euler method and (c) the method of Crank-Nicolson in time interval $(0 < t \leq 2T)$. Perform the computations with the Biot's number 1 and 5. You can assume that the wall is made from concrete for which $\rho = 2400$ kg/m^3, $c = 880$ J/(kg K), $k = 1.7$ W/(mK), and $T = 12$ hours and the indoor temperature is $u_L = 20$°C. Use the value $L = 0.2$ m for the thickness of the wall. Use only lumped capacity matrix for the explicit method and consistent one for implicit schemes.

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