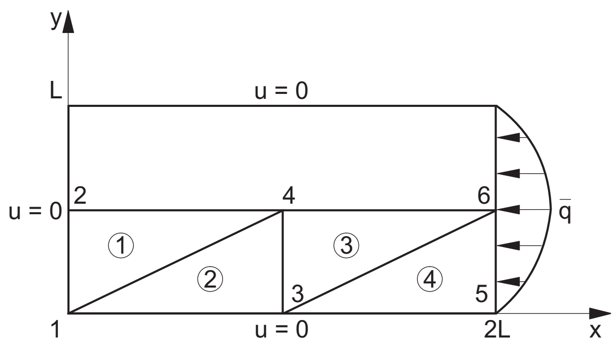
MEI-55200 Numerical methods for field problems

4. Exercise: FEM in 2-D, triangular elements

1. Solve the stationary two-dimensional heat transfer problem shown in the figure below by using linear elements. Use symmetry to reduce the problem size. The material is assumed to be homogeneous and isotropic with thermal conductivity k. The loading is given with prescribed heat flux on the boundary $x = L$ as $\vec{q}_s = -4\bar{q}_c(y/L)(1 - y/L)\vec{i}$.



2. Compute the St. Venant's torsion constant I_t for a beam having a square cross-section (side length L) and made of a homogeneous isotropic material, by using the finite element method and a triangular mesh as shown in the figure below. The problem can be formulated with St. Venant's stress function Φ as

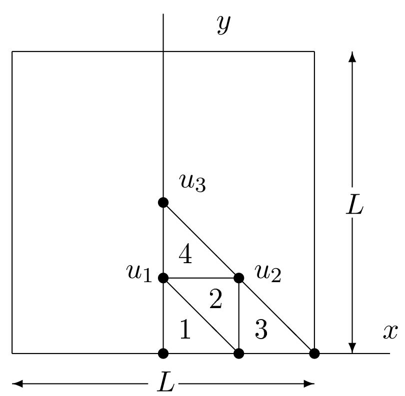
$$-\Phi_{,xx} - \Phi_{,yy} = 2G\theta,$$

with boundary conditions $\Phi = 0$. The torsional constant is obtained from equation

$$I_t = \frac{2}{G\theta} \int_\Omega \Phi(x, y) dA.$$

Determine also the shear stress distribution from a twist $\theta = 1/L$. The shear stresses can be computed from

$$\tau_{zx} = \Phi_{,y}, \qquad \tau_{zy} = -\Phi_{,x}.$$

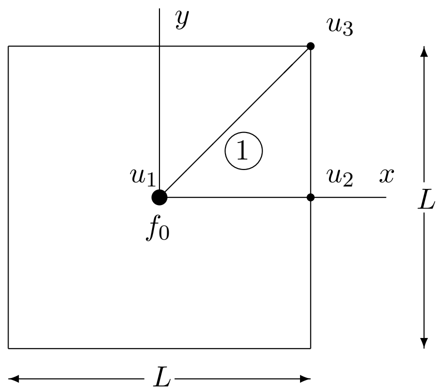


Home exercise: Let's consider a square block of concrete with a heating cable with a power of f_0 in the middle. Compute the temperature field u in the concrete using a single triangular element in one octant. Symmetry boundary conditions can be used.

- For boundary 1-2: $q_n = 0$,
- for boundary 1-3: $q_n = 0$,
- for boundary 2-3: $q_n = \alpha(u - u_{\text{air}})$.

The parameter α is the heat transfer coefficient, which in general depends on temperature, thus making the problem non-linear. Here it is assumed that α is a constant. The following values can be used in the computation.

quantity	name	value
k	thermal conductivity	0.6 W/mK
α	surface coefficient of heat transfer	8 W/mK
u_{air}	outside air temperature	5 °C
f_0	power of the cable	30 W
L	length	0.5 m



To be returned at latest in the next exercise!