MEC-7010 Post graduate course
Introduction to higher-order continuum models

2. exercise

1. Derive the dispersion relation, i.e. the phase velocity as a function of the wave number, for the dynamic Euler-Bernoulli beam model governed by the equation of motion

$$\rho A \frac{\partial^2 v}{\partial t^2} + EI \frac{\partial^4 v}{\partial x^4} = 0.$$

   Use the harmonic trial $v(x,t) = v_0 \exp[i(kx - \omega t)]$.

2. Derive the dispersion relation for the dynamic Timoshenko beam model governed by the following equations of motion

$$\rho A \frac{\partial^2 v}{\partial t^2} - GA_\mathrm{s} \left(\frac{\partial^2 v}{\partial x^2} - \frac{\partial \theta}{\partial x} \right) = 0, \tag{1}$$

$$\rho I \frac{\partial^2 \theta}{\partial t^2} - EI \frac{\partial^2 \theta}{\partial x^2} - GA_\mathrm{s} \left(\frac{\partial v}{\partial x} - \theta \right) = 0. \tag{2}$$

   Use the harmonic trial functions $v(x,t) = v_0 \exp[i(kx - \omega t)]$ and $\theta(x,t) = \theta_0 \exp[i(kx - \omega t)]$. Notice that in the case of the shear deformable Timoshenko beam there are two types of waves. Thus there are two curves to be determined.

Return at latest 9.5.2013